Physics 1

(BAS021)

For

Preparatory Year Engineers

"First semester"

By

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CONTENTS

Chapter 1

Physical Measurements and Dimensions

1.1 What is Physics?

Physics is the branch of science deals with the structure of matter and the interactions between the fundamental constituents of the observable universe.

Physics can, at base, be defined as the science of matter, motion, and energy. These subjects may be divided into, *classical physics* and *modern physics*. Classical physics is concerned largely with macroscopic bodies, (can be seen with the eye). Modern physics, on the other hand, is concerned with the submicroscopic world, that is, with those phenomena in which the structure and the behavior of individual atoms and molecules are of prime importance.

1.2 Physical Measurements'

Physical quantities are often divided into:

a. **<u>Fundamental basic physical quantities</u>**,

Quantities that can not be expressed from other physical quantities such as length (l), mass (m), time (t), Temperature (T) and electric charge (q).

b. <u>Derived physical quantities</u>

Quantities that can be expressed from other physical quantities such as velocity (ν), acceleration (a), density (ρ) and volume (V).

1.3 Systems of Units

Three different systems of units are most commonly used in science and engineering. They are:

1. The meter-kilogram-second or **mks** system.
2. The Gaussian system, in which the fundamental mechanical units are the centimeter, the gram, and the second (a **cgs** system).
3. The British engineering system (a foot- pound- second or fps system).

The metric system is used universally in scientific work and provides the common units of commerce in most countries of the world.

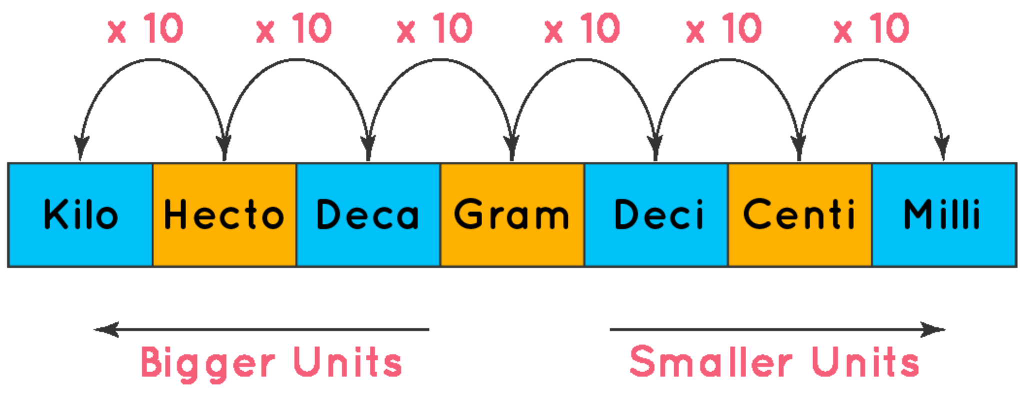
TABLE 1.4 Multiples and submultiples of metric quantities

SI Prefixes

Multiple	Prefix	Symbol
10^{15}	peta	P
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{2}	hecto	h
10	deka	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	µ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f

	mks System (International (SI))	cgs system (Gaussian system)	Hint
length	Meter	centimeter	$1m=10^2cm$
Mass	Kilogram	Gram	$1kg=10^3g$
Time	Second	second	
Force	N (Newton=Kg.m/s^2)	Dyne	$1N=10^5$dyne
Pressure	Pa (pascal=N/m^2)	Dyne/cm^2	1Pa=10dyne/cm 2
Energy	J (Joule=N.m)	Erg	$1J=10^7$erg
Power	W (Watt=J/s)	Erg/s	$1W=10^7$erg/s

Conversion Metric System for Weight



Example 1 Complete the following:

- 6.2×10^{-2} J/g=……..erg/kg
- 4.3×10^{3} dyne =………N
- 7 Å =……….mm
- 5.7 kW =……….erg/h

Solution

- 6.2×10^{-2} J/g = $6.2 \times 10^{-2} \times 10^{7}/10^{-3}$ erg/kg

  = 6.2×10^{8} erg/kg
- 4.3×10^{3} dyne =$4.3 \times 10^{3} \times 10^{-5}$ N

  = 4.3×10^{-2} N
- 7 Å = 7×10^{-10} m

  = $7 \times 10^{-10} \times 10^{3}$ mm = 7×10^{-7} mm
- 5.7 kW = 5.7×10^{3} W

  = 5.7×10^{3} J/s

  = $5.7 \times 10^{3} \times 10^{7}$ /1/(60x60) erg/h

  = $5.7 \times 10^{3} \times 10^{7} \times 60 \times 60$ erg/h

  = 20.52×10^{13} erg/h

Example 2 Determine the SI units of the kinetic energy

Solution

The kinetic energy = 1/2 mv^2

1.4 Dimensions

We need some suitable mathematical notation to calculate with dimensions like length, mass, time, and so forth. The dimension of length is written as [L], the dimension of mass as [M], the dimension of time as [T], and the dimension of temperature as [θ]. The dimension of a derived unit like velocity, which is distance (length) divided by time, then becomes $[LT^{-1}]$ in this notation. The dimension of force, another derived unit, is the same as the dimension of mass times acceleration, and hence the dimension of force is $[MLT^{-2}]$. Some common physical quantities and their dimensions are listed in the following table.

The physical quantity	SI Unit	Dimension
1- Area	m^2	$[L]^2$
2- Volume	m^3	$[L]^3$
3-The velocity	m/s	$[L][T]^{-1}$
4-The acceleration	m/s^2	$[L][T]^{-2}$
5-The force	$(N) = kg.m/s^2$	$[M][L][T]^{-2}$
6-The frequency	s^{-1}	$[T]^{-1}$
7-The density	kg/m^3	$[M][L]^{-3}$
8-The volume	m^3	$[L]^3$
9-The pressure	$N/m^2 = kg/ms^2$	$[M][L]^{-1}[T]^{-2}$
10-The work	$J = kg.m^2/s^2$	$[M][L]^2[T]^{-2}$

Example 3 Check consistency of dimensional equation of speed.

Speed = Distance/Time

Solution

$$[LT^{-1}] = L/T$$

$$[LT^{-1}] = [LT^{-1}]$$

The equation is dimensionally correct, as the dimension of speed is same on both sides.

Example 4 Check the consistency of the equation

$$x = x_0 + v_0t + (1/2)\ at^2$$

where x and x_0 are distances, t is time, v is velocity and a is an acceleration of the body.

Solution

Now to check if the above equation is dimensionally correct, we have to prove that dimensions of physical quantities are same on both sides. Also, we have to keep in mind that quantities can only be added or subtracted if their dimensions are same.

x = distance = [L]

x_0= distance = [L]

v_0t = velocity × time = $[LT^{-1}] \times [T] = [L]$

at^2 = acceleration × time^2 = $[LT^{-2}] \times [T^2] = [L]$

Since dimensions of left hand side equals to dimension on right hand side, equation is said to be consistent and dimensionally correct.

Example 5 Check whether the given equation is dimensionally correct.

$$W = 1/2\ mv^2 - mgh$$

Where W stands for work done, m means mass, g stands for gravity, v for velocity and h for height.

Solution

To check the above equation as dimensionally correct, we first write dimensions of all the physical quantities mentioned in the equation.

$$W = \text{Work done} = \text{Force} \times \text{Displacement}$$

$$= [MLT^{-2}] \times [L] = [ML^2T^{-2}]$$

$$\text{Kinetic Energy} = 1/2\ mv^2 = [M] \times [L^2T^{-2}] = [ML^2T^{-2}]$$

$$\text{Potential Energy} = mgh = [M] \times [LT^{-2}] \times [L] = [ML^2T^{-2}]$$

Since all the dimensions on left and right sides are equal it is a dimensionally correct equation.

Example 6 Suppose a bob is hanging from a ceiling and time period of oscillations depends on „length "*l*" of the thread, mass "m" of the bob and gravity "g". Find a relation between time and other physical quantities

Solution

Let's time depends on powers x, y and z of length *l*, mass m and gravity g of the bob. Then the equation becomes:

$$T = k\ l^x\ m^y\ g^z$$

where k is a proportionality constant

Writing dimensions on both sides, we get

$$[M^0L^0T] = k\,[L]^x\,[M]^y\,[LT^{-2}]^z$$

Arranging powers accordingly, we get

$$[M^0L^0T] = k\,[M^y\,L^{x+z}\,T^{-2z}]$$

Equating powers on both sides, we get three equations

$$y = 0$$

$$x + z = 0$$

$$-2z = 1$$

Solving the linear equations, we get

$$x = 1/2,\ y = 0 \text{ and } z = -1/2$$

Substituting the values of x, y and z in the equation we have derived the following relation: $T = k\,l^{1/2}m^0g^{-1/2}$

$$T = K\sqrt{\frac{l}{m}}$$

Example 7 Suppose we are told that the acceleration *a* of a particle moving with uniform speed *v* in a circle of radius *r* is proportional to some power of *r*, say r^n, and some power of *v*, say v^m. How can we determine the values of *n* and *m*?

Solution

Let : $a = k\,r^n\,v^m$

$$\text{Dim[LHS]= Dim[RHS]} \quad \rightarrow \quad \left[\frac{L}{T^2}\right] = [L^n][\tfrac{L}{T}]^m$$

$$[L][T]^{-2} = [L]^{n+m}[T]^{-m} \ \rightarrow\ \text{n+m=1, m=2} \rightarrow \text{n=-1}$$

$$a = k\,r^{-1}\,v^2 \quad \rightarrow \quad a = K\frac{v^2}{r}$$

Chapter 2

Elastic Properties of Solids

2.1 Elasticity

The subject of *elasticity* deals with the behavior of those substances, which have the property of recovering استعادة their size and shape when the forces producing deformation التشوه/تغيير الشكل are removed this can be found as in rubber rings. We find this elastic property to some extent in all solid bodies. The opposite of elasticity is plasticity.

Plasticity is the property that the substance cannot restore its original shape and size after deformation this can be found as in pinpoint دبوس.

Question Define the Elasticity and Plasticity with examples?

2.2 Stress and Strain

We shall discuss the deformation of solids in terms of the concepts of stress الاجهاد and strain الانفعال.

Stress is the external force acting on an object per unit cross-sectional area.

Strain is simply the measure of how much an object is stretched or deformed. Strain occurs when force is applied to an object. Strain deals mostly with the change in length of the object.

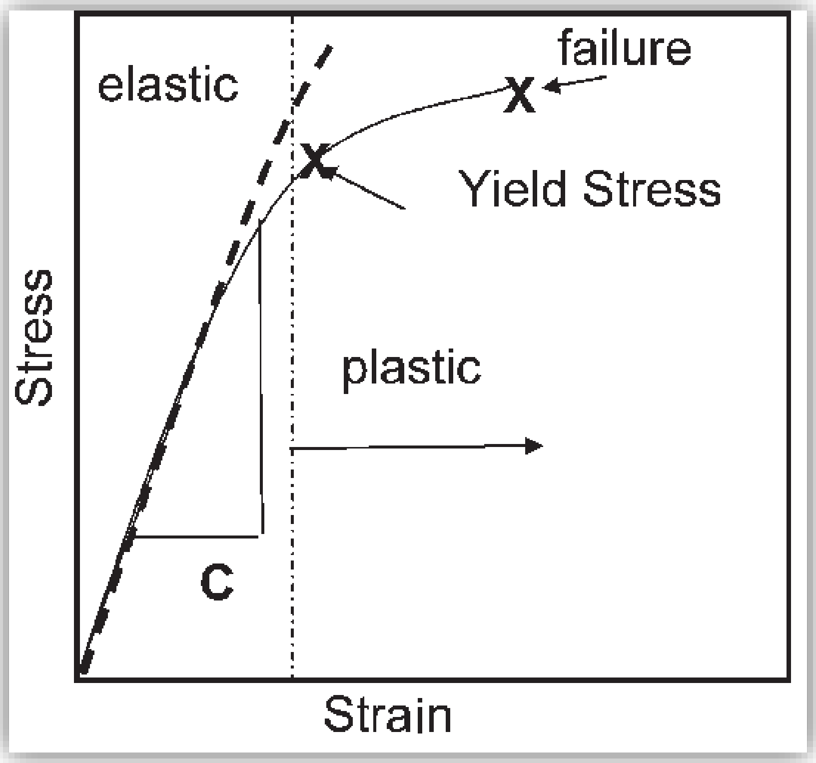
The result of a stress is strain, which is a measure of the degree of deformation.

The moduli معاملات المرونة are used to describe the elastic behavior of the objects.

The relation between stress and strain can be written as:

$$\textit{Stress} = \textit{modulus X strain} \qquad (2.1)$$

This relation can be varied according to Elastic or Plastic materials.



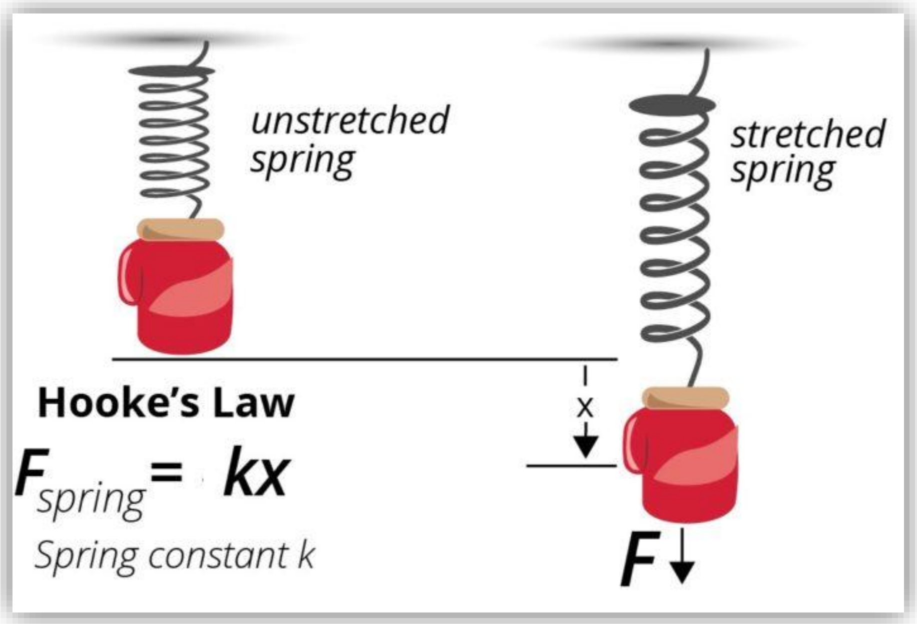
In Elastic materials, the relation between stress and strain is linear and satisfy the relation in Equation (2.1). While in Plastic materials, relation is represented as nonlinear curves.

Question: define each of Stress and Strain.

Question: Describe the Stress-Strain relation in elastic and plastic materials.

2.2.1 Hooke's law

Robert Hooke (1676) discovered a simple relation between stress and strain which is known as Hooke's law. He described the effect of tensile forces قوي الشد on the material. He observed that the increase in length of a stretched body الجسم الممتد such as spring ياي is proportional to يتناسب مع the applied force القوي المؤثرة.



Hooke's law:

"The force (F) needed to extend امتداد or compress ضغط a spring by some distance (x) scales linearly proportional to that distance."

$$F \propto x$$

$$F = kx$$

Where

F: is the applied force,

x: the elongation (displacement الازاحة)

k: The spring constant ثابت الياي

Hooke's law for a spring is sometimes, stated under the convention that F is the restoring force (قوي الاستعادة) exerted by the spring on whatever is pulling its free end which will be in the reverse direction عكس الاتجاه. In that case, the equation becomes

$$F = -kx$$

Therefore, $k = \frac{F}{x}$ (N/m) is a measure of the stiffness صلابة of the spring. It is different for different springs and materials. The larger the spring constant, the stiffer the spring and the more difficult it is to stretch.

Question: State Hook's low and investigate the Hook's equation.

Question: define the spring constant.

2.2.2 The Stress- Strain Relation Graph relation

The relation between Stress- Strain can be expressed as :

$$E = \frac{stress}{strain}$$

The constant, E, (the proportionality factor between stress and strain) is called the ***modulus of elasticity or elastic modulus***.

The E values depend on the type of the material. The SI unit of the constant E is that of stress over strain, i.e. N/m^2, as the strain is a ratio and has no units. The Stress-Strain relation graph can be shown in Fig.2.1.

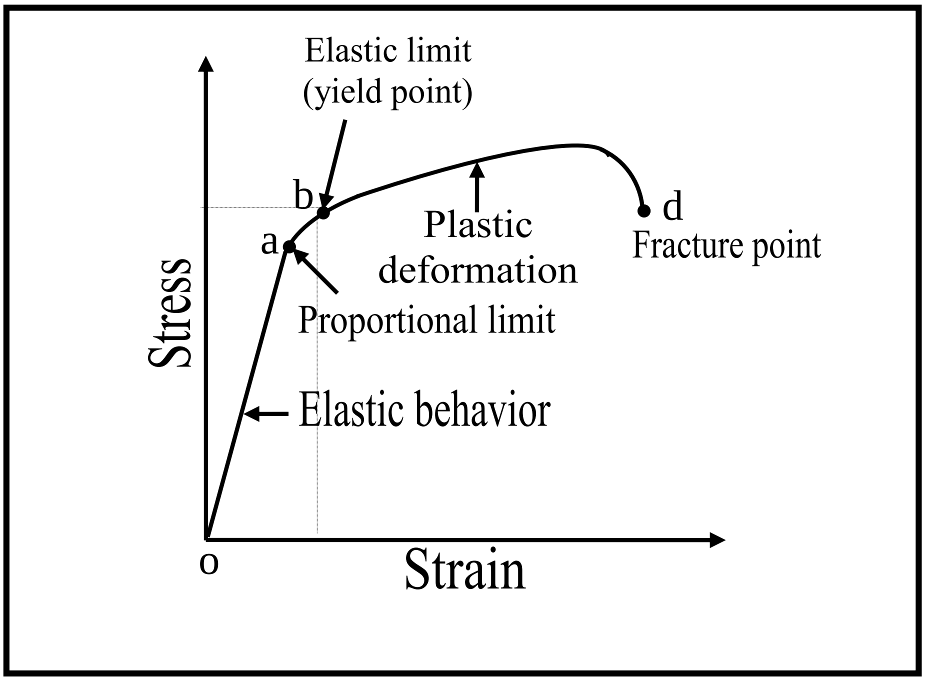


Fig.2.1 The Stress-Strain graph

- The stress and strain are linearly proportional until point **a** is reached. The point **a** is called the **<u>proportional limit</u>** حد التناسب of the material. Hook's law is obeyed until this point.
- From **a** to **b**, stress and strain are not linearly proportional.
- If the load is removed at any point between from **o** to **b**, the material will restore تستعيد its original state.
- In the region **ob**, the material is said to be **<u>Elastic</u>** or exhibit **<u>Elastic behavior</u>** and, the point **b** is called the **<u>Elastic limit,</u>** or the **yield point** نقطة الخضوع.

- Increase of the load beyond **b** produces a large increase in the strain (even if the stress decreases) until a point **d** is reached at which fracture كسر takes place.
- From **b** to **d**, the material is said **Plastic deformation**. It can't restore its original dimensions after removing the stress.

From the graph, we can define the following terms:

The Elastic limit: It is the maximum stress the material can afford تتحمل before deformation.

The Ultimate Strength: It is the greatest stress the material can afford before rupture تتمزق

Ductile Materials المواد المرنة: If large plastic deformation takes place between the elastic limit (**b**) and the fracture point (**d**), or the materials that has elastic behavior.

Brittle Material المواد الهشة: If fracture occurs soon after the elastic limit (b) is passed.

Safety Factor:

- For all engineering materials, it is not allowed to apply stress on any material beyond its elastic limit.
- The stress must be smaller, even within the elastic region to be in the linear proportional limits.

Question: Describe and explain the stress-strain graph.

Question: Define the elastic limit, the ultimate strength, ductile materials and brittle materials.

Question: What are the safety factors that taken into consideration in dealing with the engineering materials.

2.3 Elastic Modulus (E)

The ratio of stress to strain is called an elastic modulus of the material. Corresponding to the three types of strains (tensile الشد, shear القص, and volume strain), there are three elastic modulus:

- *Young's modulus of elasticity; Y:* It corresponds to tensile strain.
- *Shear modulus (or modulus of rigidity);* S: It corresponds to shearing strain.
- *Bulk modulus (or volume modulus);* B: It corresponds to volumetric strain.

2.3.1 Young's Modulus : Elasticity in Length

Consider a rod is clamped مثبت at one end and a load is applied at the other. Let L represent the wire's original length, A its cross-sectional area, and ΔL the elongation produced by the applied force F. See Fig. 2.2

By definition, stress is the force per unit area, and strain is the elongation per unit length.

let's describe the following terms:

The Tensile Stress (σ) الاجهاد الطولى

It is the normal force (A) القوي العمودية per unit cross sectional area (A). It is a scalar quantity.

$$\text{Tensile Stress } (\sigma) = \frac{F}{A} \quad (\text{Pa}=\text{N/m}^2) \tag{2.2}$$

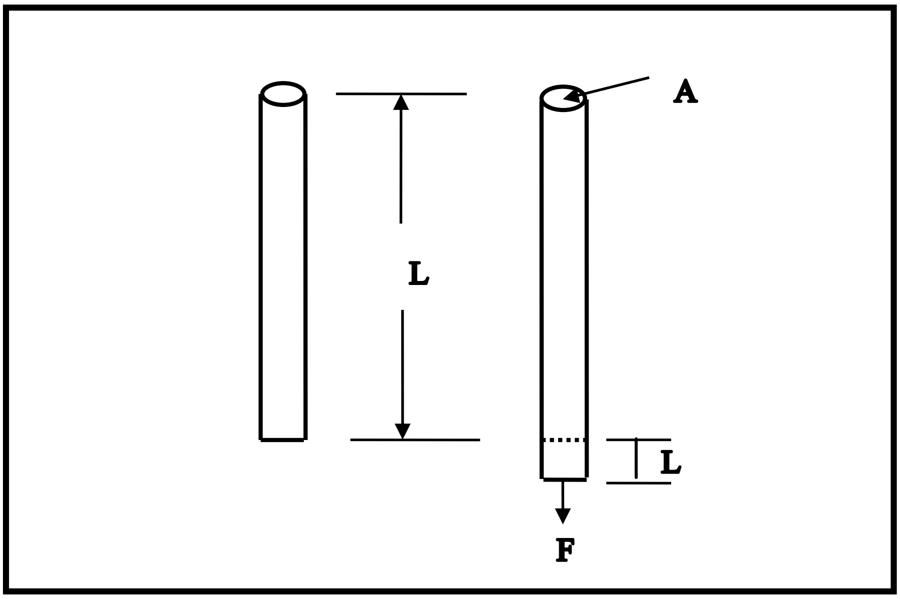


Fig.2.2

The Tensile Strain (ε) الانفعال الطولى

It is the ratio between the elongation (ΔL) and the original length الطول الاصلي of the material

$$\text{Tensile Strain } (\varepsilon) = \frac{\Delta L}{L} \tag{2.3}$$

Thus the modulus of elasticity is called Young's modulus, is the ratio between tensile stress and tensile strain. Which written as Y, and is given by:

$$Y = \frac{\sigma}{\varepsilon}$$

$$Y = \frac{F/A}{\Delta L/L}$$

Or

$$Y = \frac{FL}{A\Delta L} \quad (\text{Pa=N/m}^2) \qquad (2.4)$$

The above relation means that if the proportional limit is not exceeded, the ratio of the stress to strain is constant.

Question: Define the following terms: Tensile Stress, Tensile Strain and Young modulus.

Question: What is the physical meaning of $Y_{Steel} > Y_{Aluminium}$.

Example 1:

An 80 Kg mass is hung معلقة on a steel wire having 18 m long and 3mm diameter. What is the stress on the wire, the resulting stain and elongation of the wire, knowing Young's modulus for steel is 21x 10^{10} N/m^2.

Solution

$$L = 18m, \;\; m = 80Kg, \qquad r = 1.5 \times 10^{-3}m,$$

$$Y = 21 \times 10^{10} N/m^2$$

$$F = mg = 80 \times 9.8 = 784N$$

$$\sigma = \frac{F}{A} = \frac{784}{\pi(1.5 \times 10^{-3})^2} = 1.11 \times 10^8 \; N/m^2$$

$$\varepsilon = \frac{\sigma}{Y} = 52.8 \times 10^{-3}$$

$$\varepsilon = \frac{\Delta L}{L} = \frac{\Delta L}{18} = 52.8 \times 10^{-3}$$

$$\Delta L = 0.0095 \, m = 9.5mm$$

2.3.2 Shear modulus: Elasticity of shape

When an object is subjected to a force parallel موازي to one of its faces while the opposite face is held fixed by another force, the stress in this case is called a **shear stress**, see Fig. (2.3).

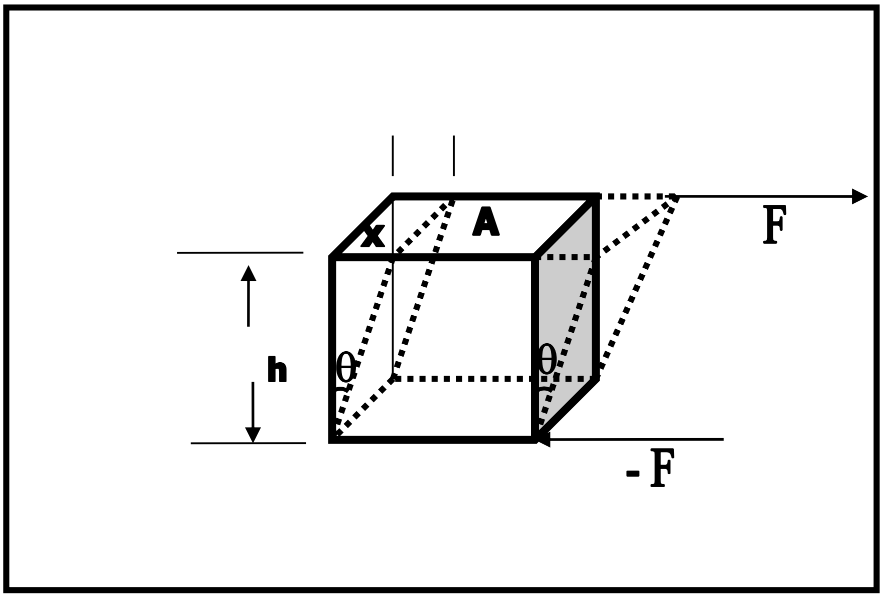


Fig.2.3

The shear stress is defined as the ratio of the tangential force **F** to the area **A** of the face being sheared.

The shear strain is defined as the ratio of the horizontal distance **x** that the sheared face moves, to the height of the object **h** (assuming that, for small distortions, no change in volume occurs with this deformation). Thus the shear modulus is:

S = Shear stress/Shear strain

$$\text{Shear stress} = \frac{F}{A}$$

$$\text{Shear strain} = \frac{x}{h}$$

$$S = \frac{F/A}{x/h} \quad , \quad \text{or}$$

$$S = \frac{Fh}{Ax} \tag{2.5}$$

The shear modulus (or modulus of rigidity صلابة), **S** has a significance for solid materials only. The SI units of shear modulus are that of stress, i.e. N/m^2.

When a material is subjected to shear stress the volume will not change.

Question: Define the following terms: Shear Stress, Shear Strain and Shear modulus.

Question: Why it is easier to punch ثقب a hole in a sheet لوح of aluminum than a sheet of steel for the same sheet thickness.

Example 2 :

A cube of gelatin is 6 cm in length when unstressed غير مضغوط. A force of 0.245 N is applied tangentially مماسه to the upper surface, causing a 0.48 cm displacement relative to the lower surface.

Find :-

a) The shear stress,

b) The shear strain, and

c) The shear modulus

Solution

$$Length = 0.06m, F = 0.245N, x = 0.48 \times 10^{-2}m$$

$$A = Length^2 = (0.06)^2 = 0.0036\text{m}^2$$

a) Shear stress $= \frac{F}{A} = \frac{0.245}{0.0036} = 68.1\ \text{N/m}^2$

b) Shear strain $= \frac{x}{h} = \frac{0.48\times10^{-2}}{0.06} = 0.8$

c) Shear modulus = $\frac{68.1}{0.8} = 85.125 N/m^2$

2.3.3 Bulk Modulus: Volume Elasticity

Bulk modulus is defined as the negative ratio of volume stress to the volume strain.

When a force is applied normally to the surface of a body and a change in volume takes place, the strain is known as volume strain.

Figure 2.5 shows that when a cube of solid is undergoes a change in volume but no change in shape, the cube is compressed ينضغط on all sides by forces normal to its six faces.

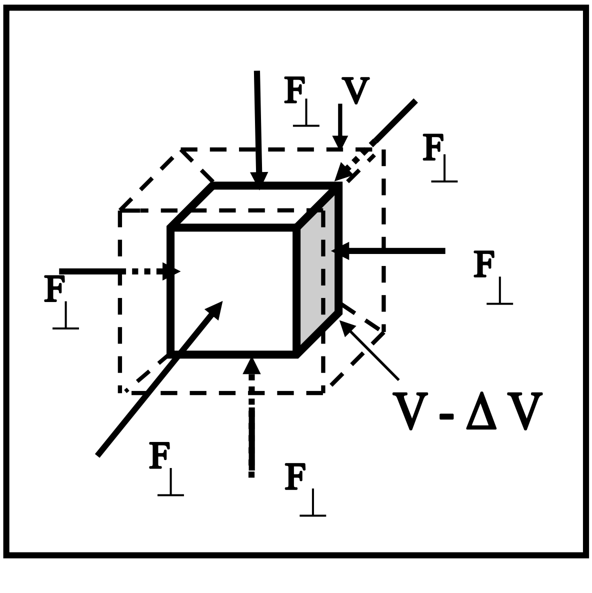


Fig.2.5

Volume Stress is measured as the ratio between the applied force F and the area A of the face. It can be called the pressure **(P)**

$$\text{Volume stress} = F/A = \Delta P$$

Volume strain is measured by the change in volume per unit volume, that is :

$$\text{Volume strain} = \Delta V/V$$

Where Δv is the change in volume produced by the force F in the original volume V.

By definition the bulk modulus of elasticity is given by :

$$B = -\text{ volume stress }/\text{volume strain}$$

$$B = -\frac{F/A}{\Delta V/V} = -\frac{\Delta P}{\Delta V/V} \qquad (2.6)$$

The minus sign is included in the definition of B because an increase in applied pressure causes a decrease in volume (negative ΔV) and vice versa. The SI units of B are the same as those of pressure, i.e. N/m^2 (or Pascal).

<u>Important</u>

- Solids and liquids have a bulk modulus.
- Liquids do not exhibit Young's modulus or shear modulus because a liquid does not sustain a shearing stress or a tensile stress. If a shearing force or a tensile force is applied to a liquid, the liquid simply flows in response.

Compressibility (K)

The reciprocal عكسي of the bulk modulus is called the *Compressibility*.

$$K = -\frac{1}{B} = -\frac{\Delta V}{PV} \qquad (2.7)$$

Which can defined as the fractional decrease in volume $(-\frac{\Delta V}{V})$ per unit increase in pressure (P).

The unit of compressibility is Pa^{-1} .

Table 2.1 shows the compressibility of liquids.

Table 2.1

Liquid	Compressibility K (Pa^{-1})
Mercury	3.7×10^{-11}
Glycerin	21×10^{-11}
Water	45.8×10^{-11}
Carbon disulfide	93×10^{-11}
Ethyl alcohol	110×10^{-11}

Question: Define the following terms: Bulk Stress, Bulk Strain and Bulk modulus.

Question: What is the physical meaning of $K_{water} < K_{Ethyl\ alcohol}$.

2.3.4 The relation between Elastic Moduli

The relation between the elastic moduli can be written as

$$Y = \frac{9BS}{3B+S} \tag{2.8}$$

Table 2.2 shows the elastic moduli for some materials

Table 2.2

Material	Young Modulus (Y) ($x10^{10}$)	Shear Modulus (S) ($x10^{10}$)	Bulk Modulus (B) ($x10^{10}$)
Aluminum	7	2.5	7.5
Brass	9	3.5	6
Copper	11	4.4	14
Crown glass	6	2.5	5
Iron	21	7.7	16
Lead	1.6	0.6	4.1
Nickel	21	7.8	17
Steel	20	7.5	16

Example 3:

A solid brass sphere is initially surrounded محاطة by air and the air pressure exerted on it is $1x10^5$ N/m^2. The sphere is submerged تم غمرها in into the ocean to a depth where the pressure is 2x 10^7 N/m^2. The volume of the sphere in air is 0.50 m^3. By how much does this volume change once the sphere is submerged?

Bulk modulus for brass = 6.1×10^{10} N/m^2

Solution

$$\Delta P = 2 \times 10^7 - 1 \times 10^5 = 1.99 \times 10^7 N/m^2$$

$$V = 0.5 m^3$$

$$B = -\frac{\Delta P}{\frac{\Delta V}{V}} = 6.1 \times 10^{10} N/m^2$$

$$\Delta V = -\frac{(0.5)(1.99 \times 10^7)}{6.1 \times 10^{10}} = -1.6 \times 10^{-4} m^3$$

The negative sign indicates that the volume of the sphere decreases

2.4 Energy Stored in a Stretched Wire (Elastic Potential Energy)

When a wire is stretched within the limits of elastic deformation, it resists the extension with internal force, which tends to bring it back to its original length.

As a result of this restoring force, the wire possesses a potential energy (known as elastic potential energy **ΔU**) whose magnitude is equal to the work (***W***) done in stretching the wire against the restoring force as in Fig. 2.6.

$$F_r = -k\, x \tag{2.9}$$

The work done in stretching the wire by an infinitismal amount dx is therefore given by :

$$W = \int F.dx$$

$$= \int -kx.dx$$

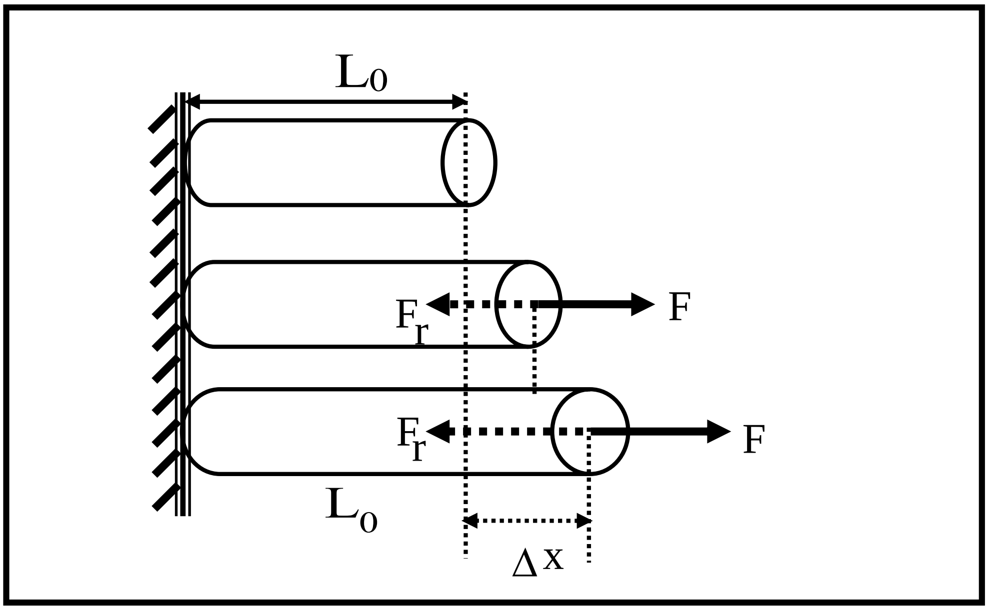


Fig. 2.6

$$= -k\int_{0}^{x} x.dx$$

$$W = -\frac{1}{2}kx^2$$

Therefore

$$\Delta U = -W$$

Question: Drive an expression for the energy stored in a stretched spring.

2.5 Poisson's ratio, μ

If a body is subjected to some force or a system of forces, then the deformation in the body is not in one direction only but throughout the body.

If a wire is stretched its length increases but its diameter decreases. This can be shown in Fig.2.7 .

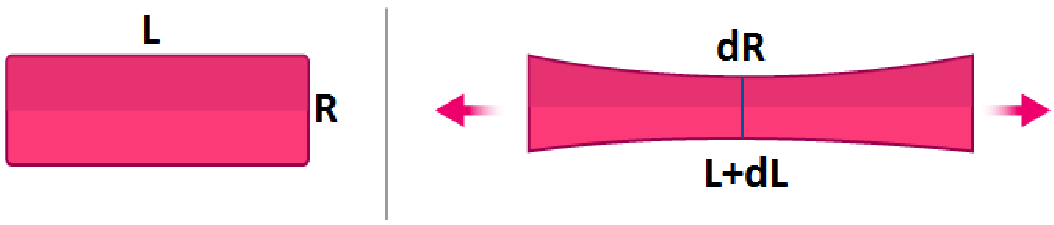


Fig.2.7 Scratched Rod

<u>For a wire</u> Poisson's **ratio** μ is defined as the negative ratio of lateral strain الانضغاط العرضي to longitudinal strain التمدد الطولي,

μ = - lateral strain / longitudinal strain

$$\mu = -\frac{\Delta R/R}{\Delta L/L}$$

Where: R is the original radius of the wire , ΔR is the decrease in Radius, L is the original length of the wire and ΔL is the increase in length .

The minus sign induced in the definition of μ because an increase in L always causes a decrease in R whereas, a decrease in L always causes an increase in R .

Poisson's ratio, μ of the cylindrical wire is equal (1/2)

$$\because V(R,L) = \pi R^2 L = Const$$

$$dV(R,L) = VdR + VdL = 0$$

$$dV(R,L) = 2\pi RLdR + \pi R^2 dL = 0$$

$$2LdR + RdL = 0$$

$$\frac{LdR}{RdL} = -\frac{1}{2} = \frac{\frac{dR}{R}}{\frac{dL}{L}} = -\mu \rightarrow \mu = \frac{1}{2}$$

Question: Define the Poisson's ratio using mathematical relations.

Question: Prove that the Poisson's ratio of the cylindrical wire is equal (1/2).

Example 4:

A copper نحاس wire has 10m long with 2 mm radius is used to carry a mass of 12 kg. How much does the wire stretch under this load? What is the minimum radius of the wire to not exceed the elasticity limit? let Y=1.2x10^7 N/cm^2, the elastic limit of copper=1.5 x10^{13}dyne/m^2.

Solution

$$r = 0.002m, L = 10m, m = 12Kg,$$

$$Y = \frac{1.2 \times 10^7 N/cm^2}{10^{-4}} = 1.2 \times 10^{11} N/m^2$$

$$Elastic\ limit = 1.5 \times 10^{13} \times 10^{-5} = 1.5 \times 10^8 N/m^2$$

$$A = \pi r^2 = 3.14 \times (0.002)^2 = 12.56 \times 10^{-6} m^2$$

$$F = mg = 12 \times 9.8 = 117.6\ N$$

$$Y = \frac{\frac{F}{A}}{\frac{\Delta L}{L}} \rightarrow \Delta L = \frac{FL}{AY} = \frac{117.6 \times 10}{12.56 \times 10^{-6} \times 1.2 \times 10^{11}}$$

$$\Delta L = 78 \times 10^{-5} m = 0.78mm$$

$$\because Elastic\ limit = \frac{F}{A_{min}}$$

$$A_{min} = \frac{F}{elastic\ limit} = \frac{117.6}{1.5 \times 10^8} = 78.4 \times 10^{-8} m^2$$

$$A_{min} = \pi r_{min}^2 = 78.4 \times 10^{-8} m^2$$

$$r_{min} = \sqrt{\frac{78.4 \times 10^{-8}}{3.14}} = 5 \times 10^{-4} = 0.5mm$$

Solved Examples

Example (1) A 80 Kg mass is hung on a steel wire having 18m long and 3mm diameter. What is the elongation of the wire, knowing Young's modulus for steel is 21 x 10^{10} N/m^2?

Solution

m=80 kg L= 18 m

D=2r=3 mm= 0.003m r=0.0015m Y= 21 x 10^{10} N/m^2

Young's modulus is given by $\Longrightarrow$ $Y = \dfrac{F/A}{\Delta L/L}$

So, the elongation is $\Longrightarrow$ $\Delta L = \dfrac{FL}{AY}$

$$\boxed{\Delta L = \frac{80x9.8}{\pi(0.0015)^2}\, x \,\frac{18}{21x10^{10}} = 0.0095m = 9.5mm}$$

Example (2) A cube of gelatin is 6 cm in length when unstressed. A force of 0.245 N is applied tangentially to the upper surface, causing 0.48 cm displacement relative to the lower surface.

Find: 1) Shear stress 2) shear strain 3) shear modulus

Solution

1) Shear stress = *F/A = 0.245/ (0.06x0.06) = 68.1 N/m²*$^{}$

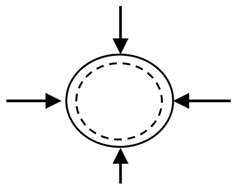
2) Shear strain = x/h = *0.0048/0.06 = 0.8*

3) Shear modulus= Shear stress / Shear strain

$= (68.1\ N/m^2)/0.8 = \mathbf{85.125\ N/m^2}$

Example (3) A solid brass sphere is initially surrounded by air and the air pressure exerted on it is 1×10^5 N/m^2. The sphere is submerged in into the ocean to a depth where the pressure is 2×10^7 N/m^2. The volume of the sphere in air is 0.50 m^3. By how much does this volume change once the sphere is submerged? Bulk modulus for brass = 6.1×10^{10} N/m^2

Solution



$$\because B = -\frac{\Delta P}{\Delta V / V_o} \Rightarrow \therefore \Delta V = -\frac{V_o \Delta P}{B}$$

$$\therefore \Delta V = -\frac{(0.5)(2X10^7 - 1X10^5)}{6.1X10^{10}} = -1.6\,X\,10^{-4}\ m^3$$

The negative sign indicates that the volume of the sphere decreases.

Example (4): A steel sphere was carried to a planet on which atmospheric pressure is much higher than on the earth, the higher pressure causes the radius of sphere to decrease. To describe the relationship between stress and strain for the sphere, you would use:

a) Young's modulus.

b) ***Shear modulus.***

c) ***Bulk modulus.***

d) ***None of these.***

Solution

(c)Bulk modulus

Example (5) A block of iron is sliding across a horizontal floor, the friction force between the block and the floor causes the block to deform. To describe the relationship between stress and strain for the block, you would use:

a) Young's modulus.

b) Shear modulus.

c) Bulk modulus.

d) None of these

Solution

(b) Shear modulus

Chapter 3

Fluid Mechanics

3.1 Fluid Static

3.1.1 Fluids

A fluid is a substance that can flow. Hence the term fluid includes liquids and gases. Generally, a fluid is a collection of molecules that are randomly arranged and held together by weak cohesive forces and by forces exerted by walls of a container. Fluid static is the study of fluids at rest.

3.1.2 Pressure

When fluids are at rest, there are no shear forces. The only stress that can be exerted on an object submerged مغمور in a static fluid is one that tends to compress يضغط the object from all sides, this what we called pressure ***(P)*.** and it is always perpendicular عمودي to the surfaces of the object. See Fig. 3.1

.

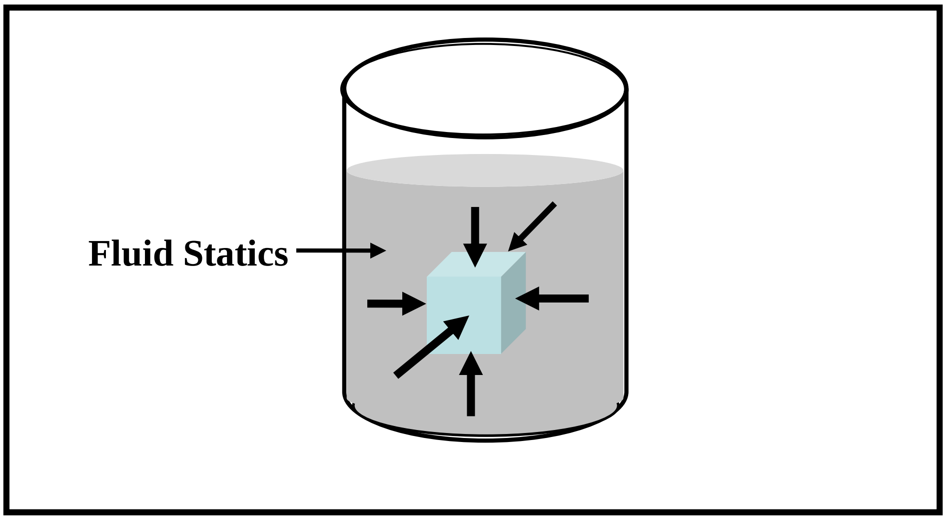


Fig. 3.1

Pressure is defined as the magnitude of the normal force القوي العمودية on a surface area A. It can be calculated as:

$$P = F/A \qquad \text{Pascal} \qquad (3.1)$$

The Pressure P is a scalar quantity**.**

3.1.3 Variation of Pressure with depth

Water pressure increases with depth. Atmospheric pressure decreases with increasing altitudes الارتفاعات.

Now we show how the pressure in a liquid increases with depth. Consider a cylinder of a liquid of density **ρ** at rest as shown in Fig.3.2 with cross-sectional area **A** extending from depth **d** to depth **d + h**.

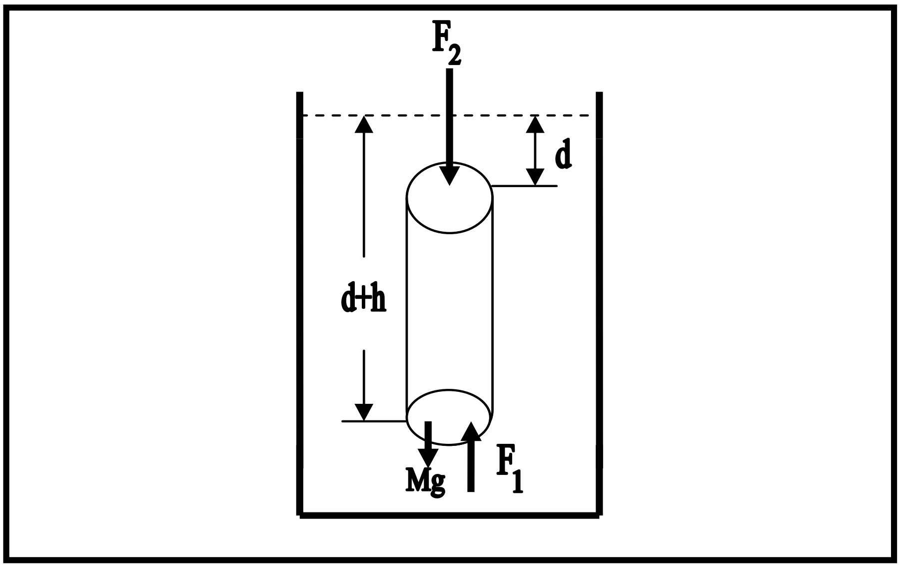


Fig. 3.2

Several forces that affect the cylindrical liquid, they can be summarized as:

1. The downward force exerted by the outside fluid on the top of the cylinder has a magnitude:

$$F_1 = P_1 A \quad \downarrow$$

2. The upward force exerted on the bottom of the cylinder has a magnitude:

$$F_2 = P_2 A \quad \uparrow$$

3. The weight of the liquid in the cylinder and it acts downward:

$$F_g = m\,g = \rho V g = \rho A h g \;\downarrow$$

Where

m: the mass of the cylinder, g: the gravity acceleration, V: the cylinder volume.

Because the cylinder is in equilibrium, the net force acting on it must be zero. see that:

$$F_1 + F_2 + F_g = 0$$

$$F_2 = F_1 + F_g$$

$$P_2 A = P_1 A + \rho A h g$$

$$P_2 = P_1 + \rho h g \tag{3.2}$$

If the liquid is open to the atmosphere and P_0 is the pressure at the surface of the liquid , then P_0 is atmospheric pressure, we usually take atmospheric pressure to be P_0 = 1.00 atm = 1.013 x 10^5 Pa. then $\boldsymbol{P_2}$ called the **absolute pressure**, it depends on the fluid density **ρ** and the depth in the fluid ***h***.

$$P = P_0 + \rho h g$$

Question Determine the dependence of the pressure on the depth in a static fluid?

Example 1 What is the pressure due to water at a depth of 7.5 Km below sea level ? the water density $\rho_w = 1.025 \times 10^3$ Kg / m^3

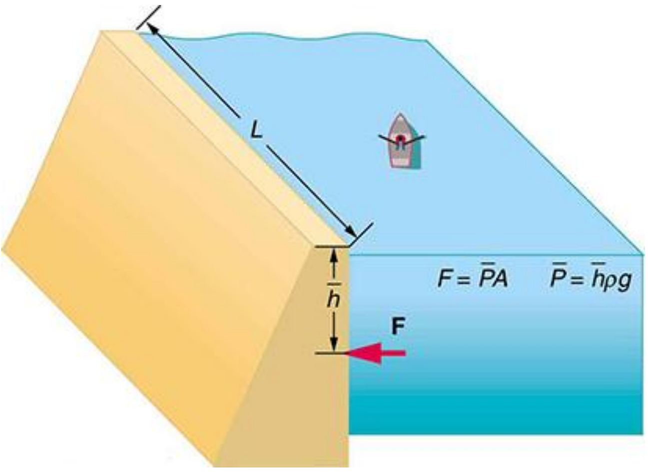
Solution

$$P = P_0 + \rho_w gh$$

$$P = 1.025 \times 10^3 \times 9.8 \times 7.5 \times 10^3$$

$$P = 7.53 \times 10^7 \text{ N/m}^2 = 75.3 \text{ MPa}$$

Example 2 The dam is 500 m wide, and the water is 80.0 m deep at the dam. (a) What is the average pressure on the dam due to the water? (b) Calculate the force exerted against the dam



Solution

The average pressure P due to the weight of the water is the pressure at the average depth h of 40.0 m, since pressure increases linearly with depth.

$$P = \rho g h$$

$$P = 10^3 \times 9.8 \times 40 = 3.92 \times 10^5\ N/m^2$$

$$\boldsymbol{F = PA = 3.92 \times 10^5 \times (80 \times 500) = 1.57 \times 10^{10} N}$$

Example 3 What is the pressure at a point 2000m high above sea level assuming that the density of air is approximately constant and $\rho_{air} = 1.22\ Kg / m^3$?

Solution

$$P = P - \rho_{air} g h$$

$$= 1.013 \times 10^5 - 1.22 \times 9.8 \times 2000$$

$$= 7.74 \times 10^4\ N/ m^2$$

3.1.4 Pascal's Principle مبدأ باسكال

Pascal's principle is given as follows: "Pressure applied to confined incompressible fluid مائع محبوس غير قابل للانضغاط is transmitted undiminished ينتقل غير منقوص to every portion of the fluid and the walls of the containing vessel الاناء" .

An important application of Pascal's law is the ***hydraulic press*** illustrated in Fig.3.3. A force of magnitude F_1 is applied to a small piston of surface area A_1. The pressure is transmitted through an incompressible liquid to a larger piston of surface area A_2.

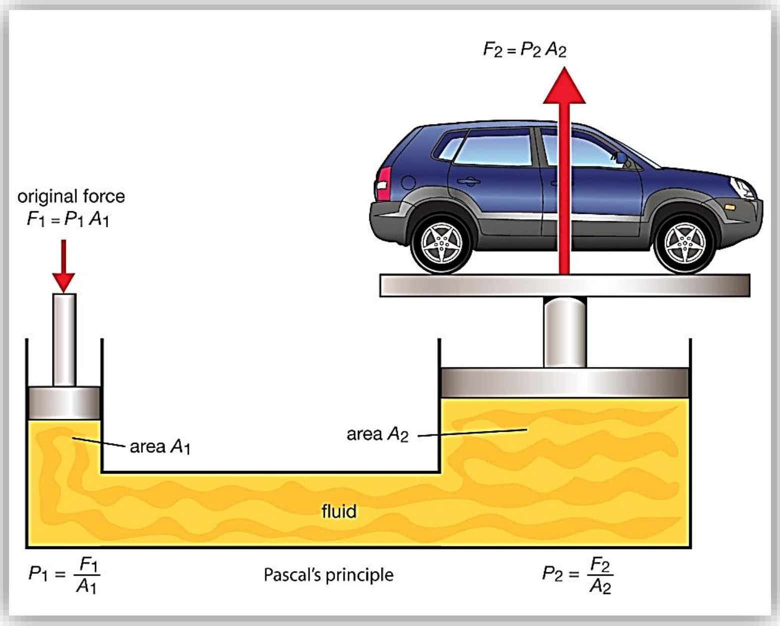


Fig.3.3

$$P_1 = P_2$$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

Therefore, the force **F_2** is greater than the force **F_1** by the factor **A_2 /A_1**. By designing a hydraulic, a large output force can be applied by means of a small input force.

- The efficiency الكفاءة of the press

$$\eta = \frac{F_2 x_2}{F_1 x_1}$$

Where x_1 and x_2 are the input and output pistons displacements respectively.

- The mechanical advantages الفائدة الآلية of the press

$$\varepsilon = \frac{F_2}{F_1}$$

Question State Pascal's principle, and what is it use?

Question Describe with the aid of mathematical expressions the hydraulic press.

Example 4 Small piston of a hydraulic press has a diameter of 8cm, and the larger piston has a diameter of 160cm. determine the minimum force to lift 1500kg load. Let the efficiency 80%. By how much is the load lifted, assume the small piston moves 1m.

Solution

$$r_1 = 4cm,\ \ r_2 = 80cm,\ \ m_1 = 1500kg$$

$$A_1 = \pi r_1^2 = 3.14 \times 16 = 50.24\ cm^2$$

$$A_2 = \pi r_2^2 = 3.14 \times 6400 = 20.096 \times 10^3\ cm^2$$

$$F_2 = m_2 g = 14700\ N$$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$F_1 = \frac{A_1}{A_2} F_2 = \frac{50.24}{20.096 \times 10^3} \times 14700 = 36.75N$$

With efficiency η=80%

$$\eta = \frac{F_2 x_2}{F_1 x_1}$$

$$x_2 = \eta \frac{F_1 x_1}{F_2} = 0.8 \times \frac{36.75 \times 1}{14700} = 2mm$$

3.1.5 Archimedes' Principle مبدأ ارشميدس

Archimedes' principle states "the upward buoyant force قوة الطفو that is exerted on a body immersed مغمور in a fluid is equal to the weight of the fluid that the body displaces وزن السائل المزاح.

Verification of Archimedes principle

Consider a cylinder body of height **($h=h_2-h_1$)**, area **A** and density ρ_s immersed in a liquid as shown in Fig.3.4.

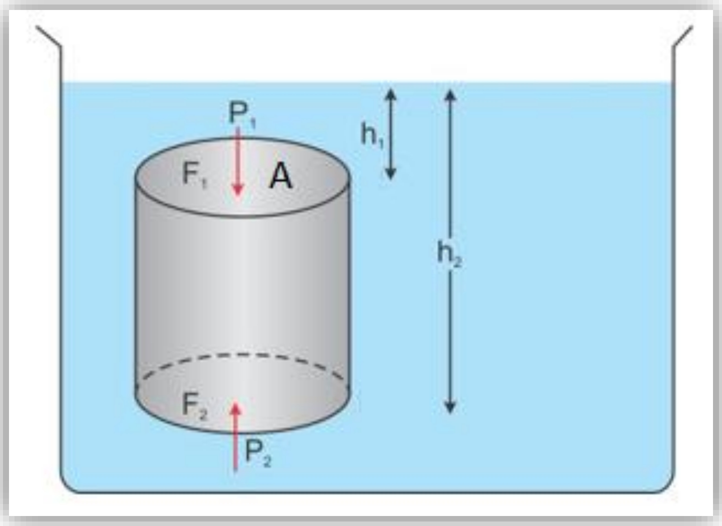


Fig.3.4

If the pressure at the top of the cylinder is P_1 with force F_1, then the pressure at the bottom of the cylinder is P_2 with force F_2. Three forces affect the cylinder as F_1, F_2 and the cylinder weight F_g. As Archimedes principle, the buoyant force F_B (upward) is the resultant force of the forces F_1 and F_2, this can be written as:

$$F_B = F_2 - F_1$$

$$F_B = P_2 A - P_1 A$$

$$F_B = A(P_2 - P_1)$$

$$P_1 = \rho_l g h_1, \quad P_2 = \rho_l g h_2$$

$$F_B = \rho_l g A(h_2 - h_1) = \rho_l g A(h)$$

$$F_B = \rho_l g V_{im} = W_l$$

ρ_l: the liquid density, V_{im}: the volume of the immersed cylinder, W_l: the weight of the displaced liquid by the cylinder.

Question State and prove Archimedes' principle?

The net forces that acts on the immersed body can be calculating from the body weight (F_g) and the buoyant force F_B.

$$F_T = F_g - F_B$$

$$F_g = \rho_s g V, \qquad F_B = \rho_l g V$$

$$F_T = F_g(1 - \frac{F_B}{F_g})$$

$$F_T = F_g(1 - \frac{\rho_l}{\rho_s})$$

Question Investigate the net force that acts on the immersed body in a static liquid.

Example 5 A rectangle tub حوض made of a thin shell غشاء رقيق of poured cement الاسمنت المصبوب has length l=120cm, width w=110cm, depth y=90cm, and mass M=188 kg. The tub floats in a lake. How many people of mass m=100kg can stand in the tub before it sinks تغرق. ($\rho_w = 1000 kg/m^3$)

Solution

The tub volume $V_{tub} = 1.2 \times 1.1 \times 0.9 = 1.188 m^3$

The total mass $M_{total} = M + Nm = 188 + 100N$

Where N: number of persons.

وزن الجسم الطافي = وزن السائل المزاح في الحوض

$$M_{total} = \rho_w g V_{tub}$$

$$188 + 100N = 1188$$

$$N = 10$$

3.1.6 Surface Tension التوتر السطحي

Surface Tension: "It is the phenomena ظاهرة of liquid surfaces اسطح السوائل to act as stretched membrane غشاء مشدود under tension".

To understand that phenomena assume a rectangular frame ABCD as in Fig.3.5. The frame side CD is movable while other sides are fixed. The frame is immersed in a soap solution محلول صابون. The free side CD will slide into the rectangle under the effect of the surface tension force.

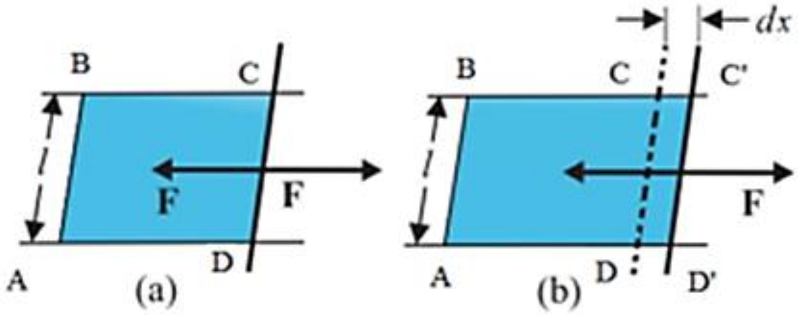


Fig.3.5

It can be calculated experimentally as:

$$F \propto 2l = 2\gamma l$$

$$\gamma = \frac{F}{2l}$$

where

2l: is the effective length of the membrane.

γ: is the surface tension coefficient, it defined as the force per unit length (N/m).

So, the work done (*W*) is defined as the work done per unit increase in the surface area (J/m^2), it can be written as:

$$W = F\Delta x$$

$$W = 2\gamma l\Delta x = \gamma\Delta A$$

$$\gamma = \frac{W}{\Delta A}$$

Question Describe and explain the surface tension phenomena and give an expression for the surface tension coefficient.

3.1.7 Capillarity الخاصية الشعرية

It is the phenomena that liquids can ascend ترتفع or descend تهبط in the capillary tubes الانابيب الشعرية. This depends on the cohesive forces قوي التماسك and adhesive forces قوي التلاصق. Water ascends in the glass tubes to a height h, as shown in Fig.3.6

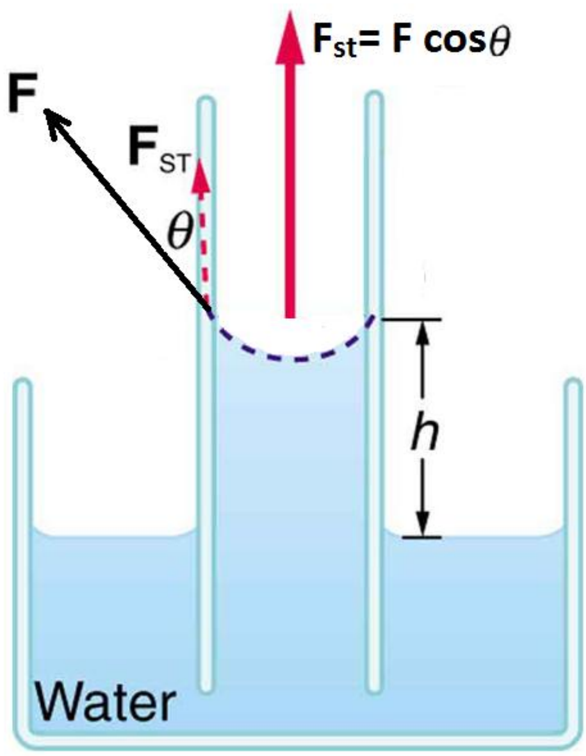


Fig.3.6

Let $\boldsymbol{F_g}$ is the weight of the water column, $\boldsymbol{F_{st}}$ is the vertical surface tension force, ***r*** is the inner radius of the tube and $\boldsymbol{\theta}$ is the angle of contact.

$$F_g = mg = \rho V g$$

$$F_g = \rho(\pi r^2 h)g$$

$$\because F_{st} = F cos\theta = \gamma(2\pi r)cos\theta$$

For balancing system,

$$F_{st} = F_g$$

$$\gamma(2\pi r)cos\theta = \rho(\pi r^2 h)g$$

$$\therefore h = \frac{2\gamma cos\theta}{\rho g r}$$

Question Describe and explain the capillarity and give an expression for the height of the water in capillarity tube in terms of the tube radius.

Example 6 The capillaries are typically 0.2mm radius. It immersed in water, if the contact angle is 45°. Find the maximum height to which water can rise in the tube according to the surface tension alone. ($\gamma = \frac{73dyne}{cm}, \rho_w = 1000kg/m^3$)

Solution

$$\therefore h = \frac{2\gamma cos\theta}{\rho gr}$$

$$\therefore h = \frac{2 \times 73 \times 10^{-3} \times 0.707}{1000 \times 9.8 \times 2 \times 10^{-5}} = 0.527m$$

3.2 Fluid Dynamics

3.2.1 General concepts of fluids flow

Fluid dynamics is the study of fluids in motion .Fluid flow can be steady ثابت (laminar سطحي) or non-steady. When the fluid velocity ***v*** at any given point is constant in time, the fluid motion is said to be steady. In non-steady غير ثابت flow, the velocity *v* is function of the time.

- Fluid flow can be rotational دوامي or irrotational غير دوامي. If the element of fluid at each point has no net angular velocity ليس له سرعة دورانية about that point, the fluid flow is irrotational.
- Fluid flow can be compressible قابل للانضغاط or incompressible غير قابل للانضغاط. Liquids can usually be considered as flowing incompressible. But even a highly compressible gas may sometimes undergo unimportant changes in density. Its flow is then practically incompressible.
- Finally fluid flow can be viscous لزج or nonviscous غير لزج . Velocity in fluid motion السرعة في الموائع is the analog of friction in the motion of solids تقابل الاحتكاك في حركة المواد الصلبة. Viscosity introduces tangential forces قوى مماسية

between layers of fluid طبقات المائع in relative motion and results in dissipation of mechanical energy.

We shall confine our discussion of fluid dynamics for the **Ideal fluid flow** **التدفق المثالي للسوائل**, therefore the following four concepts are considered :

1. **The fluid is steady**. In steady (laminar) flow, the velocity of the fluid at each point remains constant.
2. **The fluid is non viscous**. In this case, internal friction is neglected. An object moving through the fluid experiences no viscous force.
3. **The fluid is incompressible**. The density of an incompressible fluid is constant.
4. **The flow is irrotational**. In irrotational flow, the fluid has no angular momentum about any point.

Question What are the features of the ideal fluid flow.

Streamlines انسيابي

The path taken by a fluid particle under a steady flow is called a *streamline*. The velocity of the particle is always tangent مماس to the streamline, as in Fig.3.7. Consider the point **P** within the fluid, since ***v*** at **P** does not change in time, every particle arriving at **P** will pass on with the same speed in the same direction.

The same is true about the point **Q** and **R**. The curve in Fig.3.7 is called <u>streamline</u>.

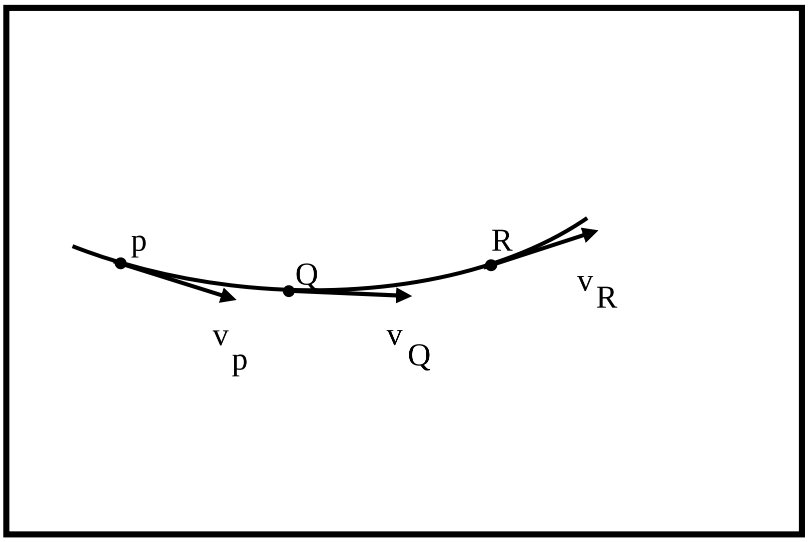


Fig 3.7

In steady flow التدفق المستقر, streamlines cannot intersect لا تتقاطع (otherwise a particle reaching the intersection could follow either of two paths and the flow would not be steady). Therefore, in steady flow, streamlines illustrate a fixed pattern of the flow نظام ثابت من السريان. In principle we can draw a streamline through every point in the fluid.

Let us assume steady flow and select a finite number of streamlines to form a bundle, this tubular انبوبي region is called a <u>stream tube or a tube of flow.</u> Thus, a stream tube is a region in a fluid bounded by streamlines, as seen in Fig.3.8. In steady flow, a particle within stream tube cannot pass outside the tube

3.2.2 The Continuity Equation

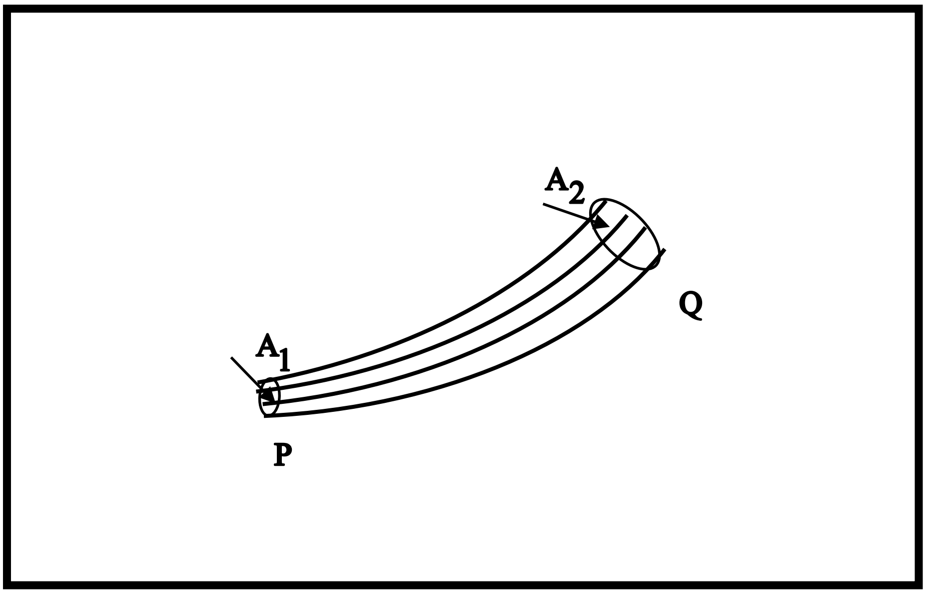


Fig.3.8

In Fig.3.8, the velocity of the fluid inside the tube of flow may have different magnitudes at different points (although it parallel to the tube at any point التدفق للمائع موازي للانبوبة) . Let the speed be $\boldsymbol{v_1}$ for fluid particles at P and $\boldsymbol{v_2}$ for fluid particles at Q. Let $\mathbf{A_1}$ and A_2 be the cross-sectional areas of the tube perpendicular to the streamlines at the points P and Q respectively. In the time interval $\Delta \boldsymbol{t}$ a fluid element travels approximately the distance $\boldsymbol{v}\Delta \boldsymbol{t}$.

Then the mass of fluid $\Delta \boldsymbol{m_1}$ crossing $\mathbf{A_1}$ in the time interval $\Delta \mathbf{t}$ is كتلة المائع التي تمر خلال فترة زمنية:

$$\Delta m_1 = \rho_1 A_1 v_1 \Delta t$$

The mass of fluid Δm_2 crossing A_2 in the same time interval Δt is:

$$\Delta m_2 = \rho_2 A_2 v_2 \Delta t$$

Where ρ_1 and ρ_2 are the fluid densities at P and Q , respectively . Because the fluid is incompressible , $\rho_1 = \rho_2 = \rho$ and because the flow is steady ,then, $\Delta m_1 = \Delta m_2$

Then

$$A_1 v_1 = A_2 v_2 = \text{constant} \qquad (3.3)$$

Or

$$A v = \text{constant}$$

This expression is called the equation of continuity for fluids. It states that:

"The product of the area and the speed at all points along a pipe is constant for an incompressible fluid"

The equation (3.3) shows that the speed is high where the tube is constricted متقلص (small A) and low where the tube is wide (large A) . The product ***Av*** gives the volume flux or flow rate معدل السريان and it has the dimensions of volume per unit time

The condition Av = const. is equivalent to the statement that the volume of fluid that enters one end of a tube in a given time interval equals the volume leaving the other end of the tube in the same time interval if no leaks are present.

Question State the continuity fluid equation, showing that volume of fluid that enters one end of a tube in a given time equals to the volume of fluid that exit.

3.2.3 Bernoulli's Equation

When a fluid moves through a region where its speed and /or elevation ارتفاعه above the Earth's surface changes, the pressure in the fluid varies with these changes.

عندما يتحرك سائل عبر منطقة تتغير فيها سرعته و / أو ارتفاعه فوق سطح الأرض

Bernoulli's equation is a general expression that relates the <u>pressure difference</u> between two points in a flow tube to both <u>velocity changes</u> تغيرات السرعة and <u>elevation changes</u>. تغيرات الارتفاع

Consider the flow of a segment of a nonviscous, steady, incompressible flow of a fluid through a nonuniform pipeline أنبوب غير منتظم or tube of flow shown in Fig.3.9.

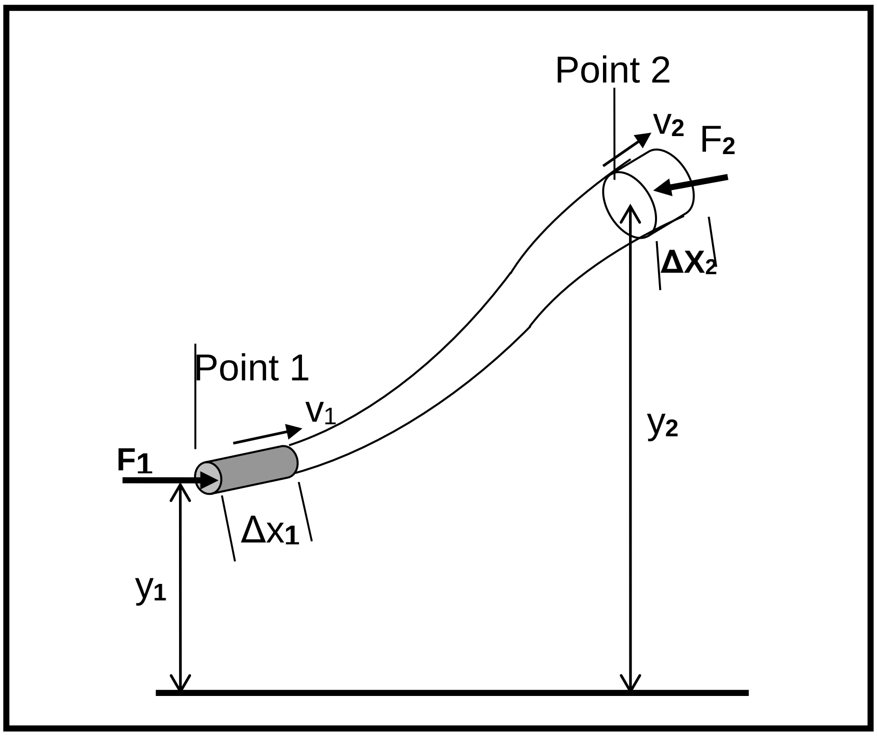


Fig.3.9

At the beginning of the time interval **Δt**, the segment of fluid قطاع من المائع consists of the gray portion (portion 1) at the left and the uncolored portion at the upper right. During the time interval Δt, the left end of the segment moves to the right by a distance Δx_1.

At the same time, the right end of the segment moves to the right through a distance Δx_2, so that the volume element,

$$A_1 \, \Delta x_1 = A_2 \, \Delta x_2$$

- At the end of the time interval, the segment of fluid consists of the uncolored portion and the gray colored portion at the upper right as shown in Fig.3.10

- At the end of the time interval, the segment of fluid consists of the uncolored portion and the gray colored portion at the upper right as shown in Fig.3.10

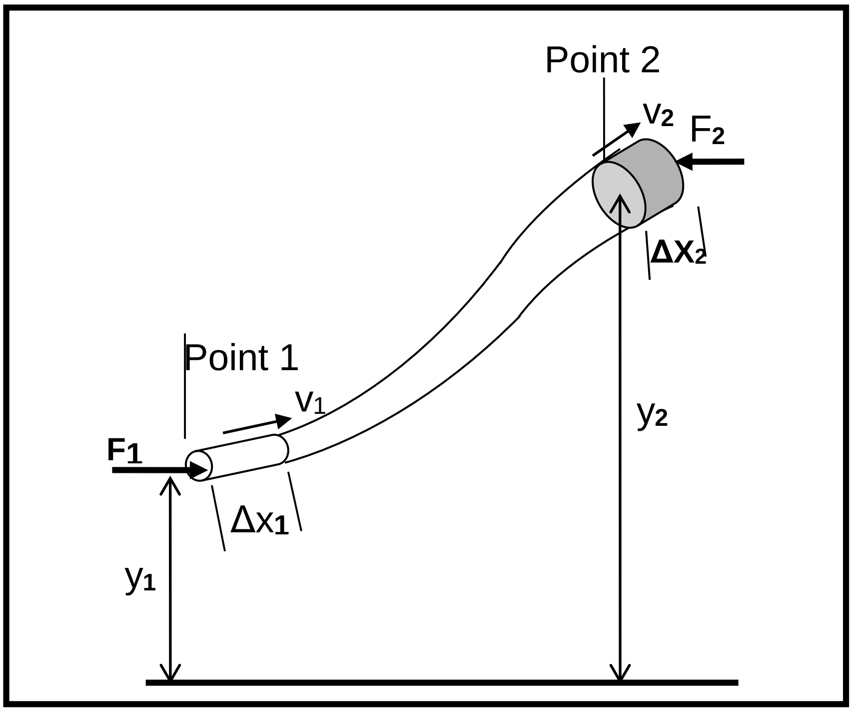


Fig.3.10

The work الشغل المبزول done on the system by the resultant force is determined as follows:

- 1- The work done at point 1 to push the entering fluid (input) into the tube is the work done on the system by the pressure force $\mathbf{F_1}$ is given by

$$W_1 = P_1 A_1 \Delta x_1 \tag{3.4}$$

- 2- The work done at point 2 to push forward the fluid out the tube (output) is the work done by the system by the pressure force $\mathbf{F_2}$ is given by

$$W_2 = - P_2 A_2 \Delta x_2 \tag{3.5}$$

- 3- The work done on the system by gravity is associated with lifting the gray portion of fluid from height y_1 to height y_2 is given by

$$W_3 = - mg (y_2 - y_1) \tag{3.6}$$

The work done on the system by the resultant force is found by adding these three terms, thus

$$W = P_1 A_1 \Delta x_1 - P_2 A_2 \Delta x_2 - mg (y_2 - y_1)$$

$$= P_1 V_1 - P_2 V_2 - mg (y_2 - y_1)$$

where $V_1 = V_2 = m/ \rho$, then

$$W = (P_1 - P_2) m/\rho - mg (y_2 - y_1) \tag{3.7}$$

The work energy theorem for a particle states that:

"**The work done on a particle by the resultant force is always equal to the change in the kinetic energy of the particle**".

That is to say $W = \Delta K$

$$(P_1 - P_2)\, m/\rho \; - mg\,(y_2 - y_1) = \tfrac{1}{2}\, m v_2^2 - \tfrac{1}{2}\, m\, v_1^2$$

$$(P_1 - P_2) - \rho\, g\,(y_2 - y_1) = \tfrac{1}{2}\,\rho\,(v_2^2 - v_1^2)$$

$$P_1 + \tfrac{1}{2}\,\rho\, v_1^2 + \rho\, g\, y_1 = P_2 + \tfrac{1}{2}\,\rho\, v_2^2 + \rho\, g\, y_2 \qquad (3.8)$$

$$P + \tfrac{1}{2}\,\rho\, v^2 + \rho\, g\, y = \text{constant}. \qquad (3.9)$$

Equation (3.9) is known as Bernoulli's equation for steady, nonviscous , incompressible fluid . This expression shows that:

- The pressure of a fluid decreases as the speed of the fluid increases.يقل الضغط كلما زادت السرعة
- Pressure of a fluid decreases as the elevation increases. يقل الضغط كلما زاد الارتفاع.

This explains why water pressure ضغط المياه from faucets صنابير on the upper floor طوابق العليا of a tall building is weak ضعيف unless measures are taken to provide higher pressure for these upper floors .

Question Investigate استنتج the Bernoulli's equation.

Special cases:

1- The fluid is at rest, $v_1 = v_2 = 0$, Eq. 3.8 becomes

$$P_1 - P_2 = \rho\, g\, (y_1 - y_2) = \rho\, g\, h$$

2- Horizontal tube , $y_1 = y_2$,

$$P_1 + ½\, \rho\, v_1^2 = P_2 + ½\, \rho\, v_2^2$$

In Eq (3.9) ,The pressure $p + \rho\, g\, h$, which would be present even if there were no flow ($v = 0$) is called the static pressure ; the term $1/2\, \rho\, v^2$ is called the dynamic pressure .

Example1

A pipe has a diameter of 16 cm at point 1 (P_1 = 200 KPa) and 10 cm at point 2 that is 6 m higher than portion 1. When oil of density 800 kg/ m3 flows in this pipe at a rate of 0.03 m3 /s . Find the pressure at point 2 ?

Solution

$$A_1\, v_1 = A_2\, v_2 = 0.03 \quad \text{, then}$$

$$v_1 = 0.03 / \pi\, (0.08)^2 = 1.49 \text{ m/ s}$$

$$v_2 = 0.03 / \pi\, (0.05)^2 = 3.82 \text{ m/s}$$

From Bernoulli's Eq.

$$P_1 + ½\, \rho\, v_1^2 + \rho\, g\, y_1 = P_1 + ½\, \rho\, v_2^2 + \rho\, g\, y_2$$

$$P_2 = P_1 + ½\, \rho\, (v_1^2 - v_2^2) + \rho\, g\, (y_1 - y_2)$$

$$= 2 \text{ x } 105 + ½\, 800 \{ (1.49)2 - (3.82)2 \} + 800 \text{ x}9.8 \text{ x}6$$

$$= 1.48 \text{ x}105 \text{ pa .}$$

3.2.4 Application of Bernoulli's Equation and the Equation of Continuity.

Bernoulli's equation can be used to determine fluid speeds by means of pressure measurements يمكن من خلال قياس ضغط المائع تحديد سرعة السريان. The principle generally used in such measuring devices is the following: The Eq. of continuity requires that the speed of the fluid at a constriction increases; Bernoulli's equation then shows that the pressure must fall there. That is for a horizontal tube (or pipe) :

$$1/2\rho\, v^2 + P = \text{constant} ;$$

If v increases and the fluid is incompressible, P must decreases.

The Venturi meter

This is a gauge مقياس in a flow tube to measure the flow speed سرعة التدفق of a liquid.

Figure 3.11 is a horizontal pipe known as Venturi tube , it can be used to determine the flow speed (v_2) at point 2 if the difference $P_1 - P_2$ is known as follows :

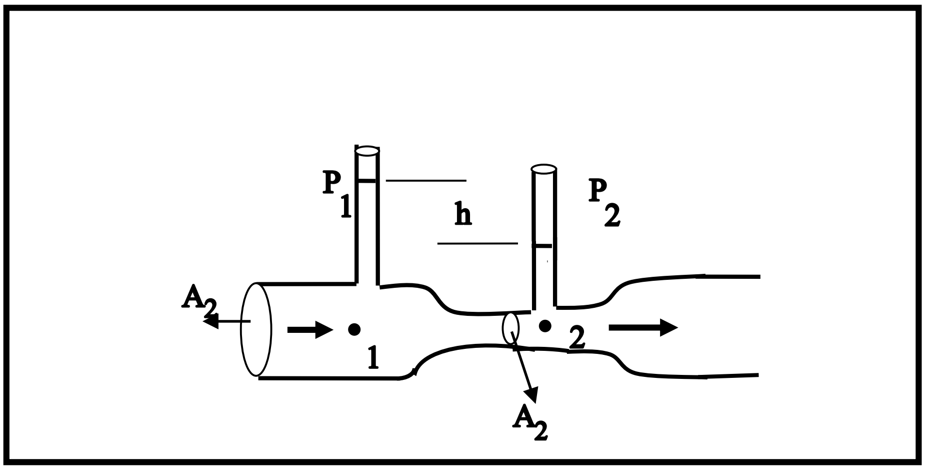


Fig.3.11

Applying equation (3.8) to points 1 and 2

$$P_1 + 1/2\,\rho\, v_1^2 + \rho\ g\, y_1 = P_2 + 1/2\,\rho\, v_2^2 + \rho\ g\, y_2$$

Putting $y_1 = y_2$ because the pipe is horizontal ,gives

$$P_1 + 1/2\,\rho\, v_1^2 = P_2 + 1/2\,\rho\, v_2^2 \qquad (3.10)$$

From the equation of continuity ,

$$A_1 v_1 = A_2\, v_2$$

Then $\qquad v_1 = A_2\, v_2 / A_1$

Substituting this expression into Eq. (3.10) gives

$$P_1 + 1/2\,\rho\,(A_2/A_1)^2\, v_2^2 = P_2 + 1/2\,\rho\, v_2^2$$

$$(P_1 - P_2) = 1/2\ \rho\, v_2^2\,(A_1^2 - A_2^2)/A_1^2$$

Then

$$v_2 = A_1[\,2(P_1 - P_2)/\rho\,(A_1^2 - A_2^2)\,]^{1/2}$$

or $\qquad v_2 = A_1[\,2gh/\ (A_1^2 - A_2^2)\,]^{1/2}$

Similarly, we can obtain

$$v_1 = A_2[\,2(P_1 - P_2)/\rho\,(A_1^2 - A_2^2)\,]^{1/2}$$

or $\qquad v_1 = A_2[\,2gh/(A_1^2 - A_2^2)\,]^{1/2}$

Example 2 A Venturi meter reads height $h_1 = 30$ cm, and $h_2 = 10$ cm. Find the velocity of flow in the pipe. $A_1 = 7.85 \text{ X } 10^{-3}\ m^2$ and $A_2 = 1.26 \text{ x } 10^{-3}\ m^2$.

Solution

$$v_1 = A_2[\,2(P_1 - P_2)/\rho\,(A_1^2 - A_2^2)\,]^{1/2}$$

$$v_1 = A_2\,[\,2gh / A_1^2 - A_2^2\,]^{1/2}$$

$$= 1.26 \times 10^{-3} \times 10^{3} \left[2 \times 20 \times 10^{-2} \times 9.8 / (7.85)^{2} - (1.26)^{2} \right]^{1/2}$$

$$= 0.32 \text{ m/s}$$

3.2.5 Viscosity اللزوجة

In real fluids there exists internal friction احتكاك داخلي between adjacent moving layers of the fluid الطبقات المتحركة المتجاورة للسائل. Viscosity may be thought of as the internal friction of a fluid. Because of viscosity, a force must be exerted to cause one layer of a fluid to slide past another, or to cause one surface to slide past another if there is a layer of fluid between the surfaces. Both liquids and gases exhibit viscosity, although liquids are much more viscous than gases.

Viscosity can be defined as the resistance to flow a liquid معاوقة تدفق المائع. The flow process is one which involves molecules عملية السريان تعتمد علي انزلاق جزيئات المائع sliding past each other under the influence of some applied stress. The rate of flow will depend upon: معدل السريان يعتمد علي

- the magnitude of the stress, قيمة الاجهاد
- the shape of the molecules, and شكل الجزيئات
- the magnitude of the forces of intermolecular attraction قوي الجذب بين جزيئات المائع

Question What are the factors that the flow rate depend on for non-ideal fluids.

3.2.6 Coefficient of Viscosity

Consider a layer AB of a liquid moving with a velocity v with respect to a parallel layer CD that is at a distance r from it.

Consider that the force required to produce the motion **F** acting on an area A and this force is acting along the direction AB, i.e. along the direction of motion. An equal force will, therefore, act on it in the backward direction due to viscosity .

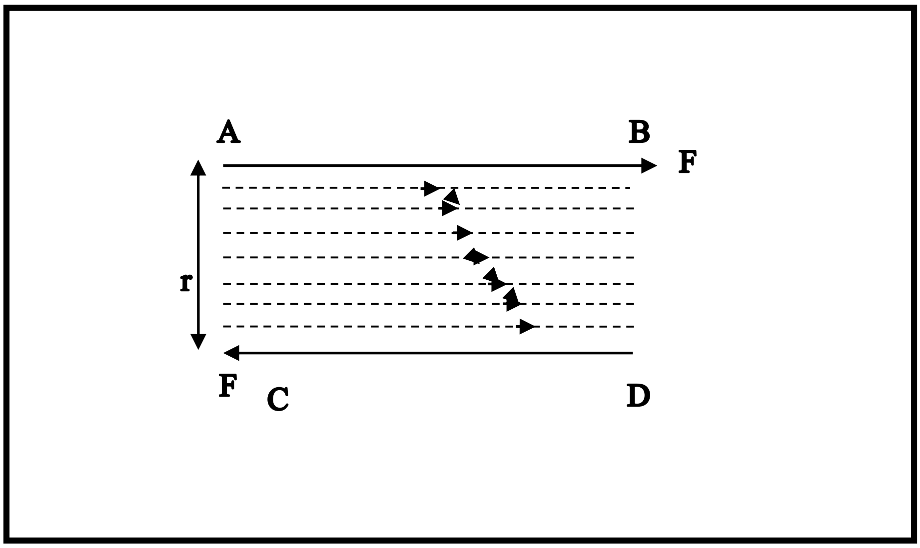


Fig.3.12

The backward force القوي العكسية **F** will depend on the following factors:

1. The relative velocity v, it is found that the magnitude of the force **F** is directly proportional to v and acts in the direction opposite to the direction of motion, i.e.

$$\mathbf{F} \propto -v$$

2. The area on which **F** acts. It is found that the magnitude of F is directly proportional to A, i.e.

$$\mathbf{F} \propto A$$

3. The distance *r*. It is found that the magnitude of **F** is inversely proportional to r, i.e.

$$\mathbf{F} \propto 1/r$$

Then , it follows that

$$\mathbf{F} \propto vA/r$$

Or

$$\mathbf{F} = -\eta A v/r \qquad (3.11)$$

Where the constant of proportionality η is called the coefficient of viscosity and it depends on the nature of the fluid.

The negative sign must be introduced because *v* decreases as r increases. If the two layers AB and CD are very close to each Other, the relation 3.11can be written as:

$$\mathbf{F} = -\eta A\, dv/dr \qquad (3.12)$$

Where dv/dr is called the rate of change of velocity with distance .

From equation (3.11) the coefficient of viscosity η can be written as

$$\eta = -Fv/Ar$$

$$= -(F/A)/(v/r)$$

The quantity F/A is the shear stress exerted on the fluid and the quantity v/r is the rate of change of shear strain , therefore,

$$\eta = -\ \text{shear stress} / \text{rate of change of shear strain}$$

The SI unit of the of the coefficient of viscosity η is Nsm^{-2} , this unit is called Pascal second (Pa .sec) or Poiseuilles (PI) . In cgs system the unit of η is dyne second cm^{-2} and it is the commonly used unit , and is called poise , where :

$$1 \text{ poise} = 1 \text{ dyne s cm}^{-2}$$
$$= 10^{-1} \text{ Nsm}^{-2}$$

Question Prove that the viscosity coefficient can be found as a relation with the shear stress and shear strain.

3.2.7 Poiseuille's Formula

Consider a viscous liquid that flow in a cylindrical tube of length *l* and radius R such that :

- The flow of liquid is parallel to the axis of the tube.
- The flow is steady, i.e. no acceleration of the flow exist.
- The velocity of the liquid layer in contact with the walls is zero and increases regularly and continuously towards the inner side, it being maximum along the axis of the tube.

This flow of a viscous liquid is called the laminar, in which the velocity is greatest at the center of the tube and decreases to zero at the walls تزداد سرعة المائع في منتصف الانبوب و تقل كلما اتجهنا ناحية جدار الانبوب لتصل الي صفر عندها .

Stoke's Formula for the Velocity of a small sphere falling through a viscous liquid:

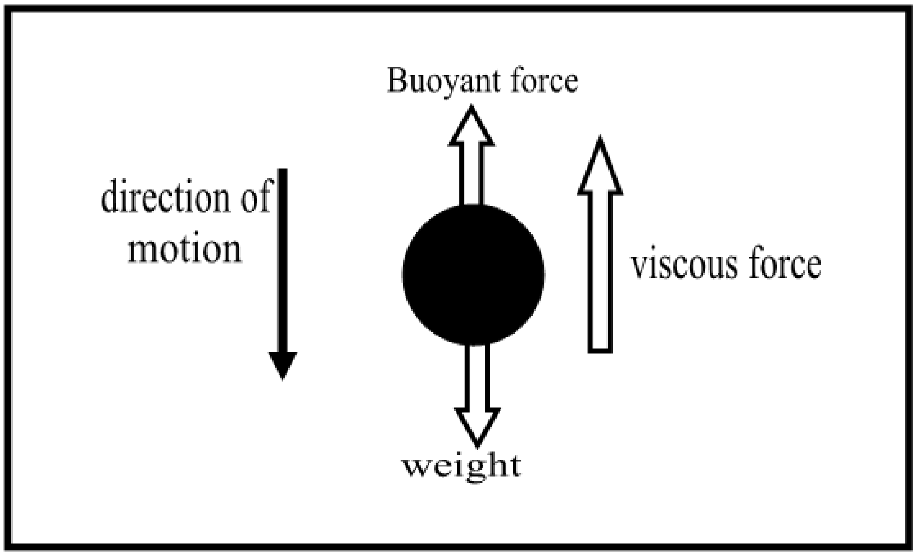


Fig. 3.13

When a steel ball is dropped into a viscous liquid in a tall jar برطمان, it begins to move down with acceleration under gravitational pull. The motion of the ball in the liquid is opposed by viscous forces. These viscous forces increase as the velocity of the ball increases تزداد قوي اللزوجة مع زيادة سرعة الكرة في المائع. Finally, a velocity will be attained when the apparent weight of the ball becomes equal to the retarding viscous forces acting on it. ستتحقق السرعة عندما يصبح الوزن الظاهري الكرة مساويًا للقوى اللزجة المثبطة المؤثرة عليها

For a small sphere falling through a viscous fluid, the opposing force is depends on :

a. The terminal velocity v of the ball

b. The coefficient of viscosity η

c. The radius r of the sphere

Combining all these factors, we have

$$F \propto v \eta r$$

Or

$$F = K v \eta r$$

Where K is dimensionless constant. Stoke found experimentally that:

$$F = 6 \pi v r \eta \qquad (3.13)$$

Chapter 4

Gravitation

4.1 Introduction

Gravitation is the force of attraction التجاذب between any two bodies. All the objects in the universe الكون like plants كواكب, moons اقمار and all celestial bodies الاجرام السماوية attract تنجذب each other with a certain amount of force.

In most of the cases, the force is too weak ضعيفة to be observed due to the very large distance of separation.

4.2 Newton's Low of Gravitation

According to Newton's law of gravitation, "Every particle in the universe attracts every other particle with a force whose magnitude is:

- Directly proportional to the product of their masses i.e.

$$F \propto (m_1 m_2)$$

- Inversely proportional to the square of the distance between their centre i.e.

$$F \propto 1/r^2$$

where M_1 and M_2 are the masses of the two objects, r is the distance between them. This can be shown in Fig.4.1

$$F = G\frac{m_1 m_2}{r^2}$$

hence, G is the gravitational constant ($G=6.67 \times 10^{-11}\ Nm^2/Kg^2$)

The gravitational force is only an attractive force.

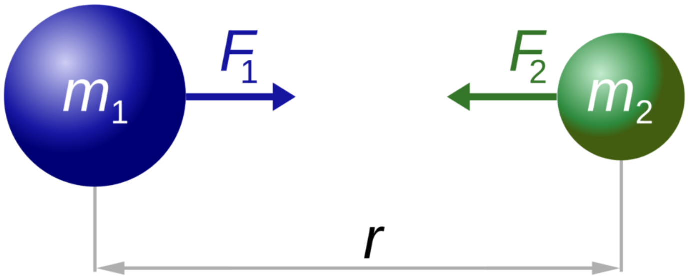


Fig.4.1

4.2 Kepler's Laws

4.2.1 The 1st law of kepler (The orbit Law)

The orbit مدار of a planet is an ellipse بيضاوي with the Sun at one of the two foci. احدي البؤرتين

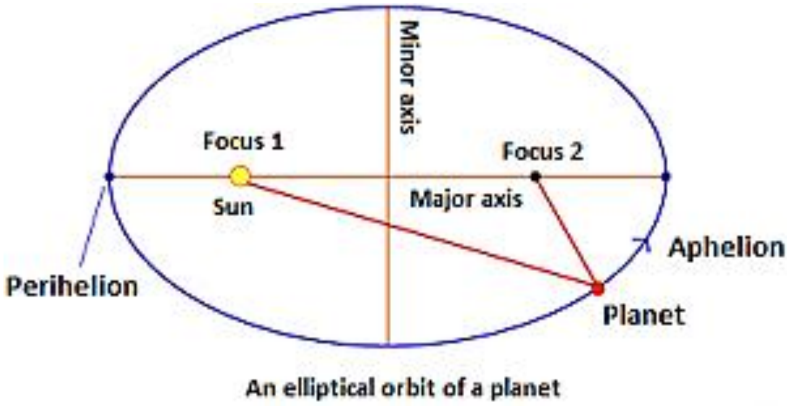


Fig.4.2

So, the planet will sometimes be far from the sun and sometimes near from it.

4.2.1 The 2nd law of kepler (The Areas Law)

A radius vector from any planet to the Sun sweeps out equal areas مساحات متساوية in equal lengths of time خلال فترات زمنية متساوية

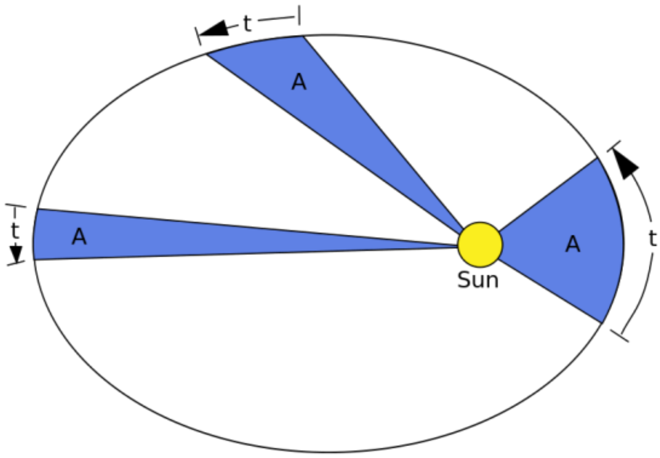


Fig.4.3

Therefore, the planet which is near from the sun must move faster to pass the track in the same time as it passes far from the sun.

4.2.1 The 3rd law of kepler (The Periods Law)

The square of a planet's orbital period مربع زمن الدوران is proportional to the cube مكعب of the length of the semi-major axis محور الدوران of its orbit.

<u>*In circular orbits:*</u>

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$$

Where ***T*** is the periodic time, ***M*** is the mass of sun, ***r*** is the radius of the circular orbit.

<u>*In elliptical orbits:*</u>

$$T^2 = \left(\frac{4\pi^2}{GM}\right) a^3$$

Where ***a*** is the semi major axis of the elliptical orbit.

Find the relation of the period time of the orbits.

From newton's law, hence m is the mass of the planet

$$F = G\frac{Mm}{r^2}$$

The magnitude of the centripetal force

$$F = ma = m\frac{V^2}{r} \rightarrow V = rw \rightarrow F = mrw^2$$

where *w* is the angular speed

$$mrw^2 = G\frac{Mm}{r^2}$$

$$w^2 = G\frac{M}{r^3}$$

Expressed using the orbital period T for one revolution of the circle

$$w = \frac{2\pi}{T}$$

$$\frac{4\pi^2}{T^2} = G\frac{M}{r^3}$$

$$\therefore T^2 = \left(\frac{4\pi^2}{GM}\right)r^3$$

Example1 Calculate the mass of the sun, assuming the earth's orbit around the sun is circular with radius 2X10^{12} m.

Solution

$$M = \left(\frac{4\pi^2}{GT^2}\right)r^3$$

$$T = 365.25 \times 24 \times 60 \times 60 = 31.5576 \times 10^6 sec$$

$$M = 4.755 \times 10^{13} kg$$

Question Proof the third law of kepler, showing that the periodic time depends on the orbit radius.

4.3 Properties of the elliptical orbits

The elliptical orbit is shown in Fig 4.4

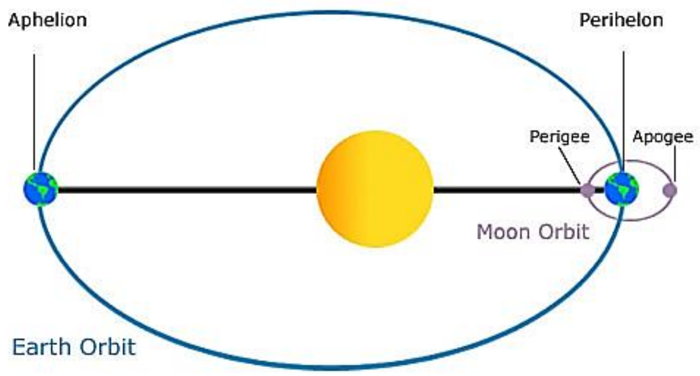


Fig.4.4

Aphelion:

It is the point where the planet makes its farthest distance ابعد مسافة from the sun.

Perihelion:

It is the point where the planet makes its closest approach اقرب ما يكون to the sun.

The eccentricity (*e*): الاختلاف المركزي

the ratio of the distance between its to foci to the length of the major axis. هي النسبة بين البعد بين البؤرتين الي طول القطر الرئيسي

As show in Fig.4.5.

$$e = \frac{r_{max}-r_{min}}{r_{max}+r_{min}} = \frac{r_{max}-r_{min}}{2a}$$

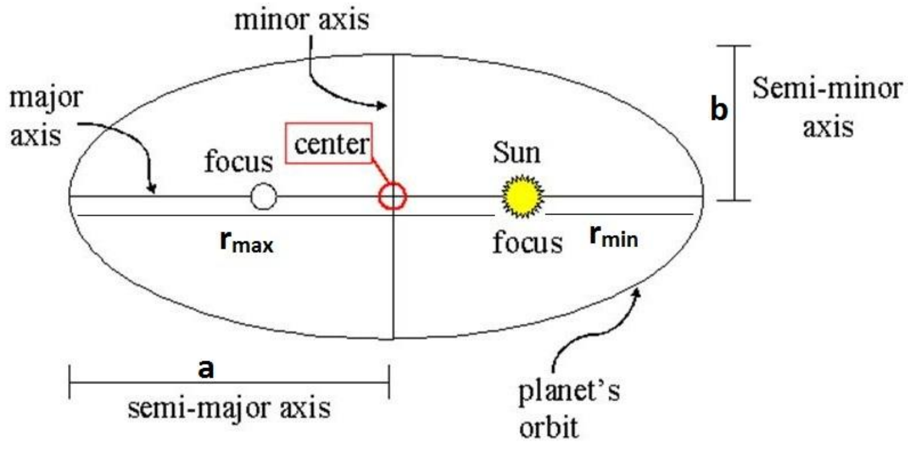


Fig.4.5

The eccentricity is dimension less with range $0 \le e \le 1$.

To satisfy the newton's law

$$V_{Aph} r_{max} = V_{per} r_{min}$$

V_{Aph}: The speed of the planet at aphelion.

V_{Per}: The speed of the planet at perihelion.

Example2 Consider the motion of a comet مذنب in an elliptical orbit around star. The eccentricity of the orbit is given by e=0.2. The distance between the aphelion and perihelion is 1.2×10^{14} mm.

i) Find the distance of the nearest and farthest approaches of the comet.

ii) If the speed of the comet is 80 km/s at perihelion, what is its speed at aphelion.

Solution

$$r_{min} = a - ae = 4.8 \times 10^{10} m$$

$$r_{min} = a + ae = 7.2 \times 10^{10} m$$

$$V_{Aph} r_{max} = V_{per} r_{min}$$

$$V_{Aph} = 53.33 \times 10^{3} m/s$$

4.4 The Acceleration of the gravity

i. At the surface of planet

For an object with mass m as shown in Fig.4.6.

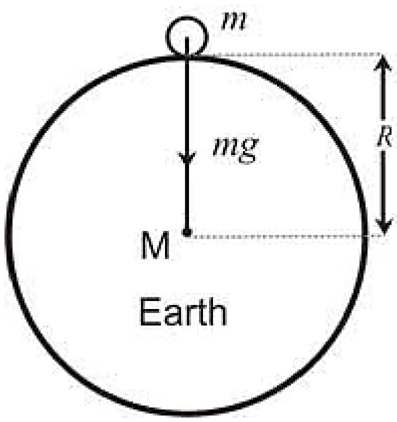


Fig.4.6

$$F = G\frac{Mm}{R^2} = mg_0$$

$$g_0 = G\frac{M}{R^2}$$

ii. At distance *h*

As shown in Fig.4.7

$$r = R + h$$

$$g = G\frac{M}{(R+h)^2}$$

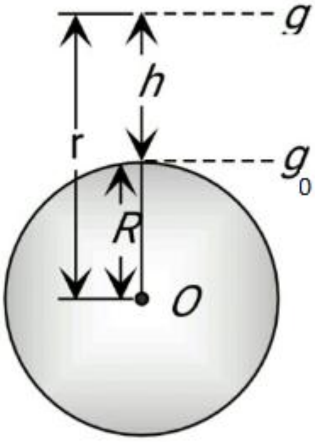


Fig.4.7

$$g = G\frac{M}{R^2(1+\frac{h}{R})^2}$$

$$g = G\frac{M}{R^2}.\frac{1}{(1+\frac{h}{R})^2}$$

$$g = g_0.(1+\frac{h}{R})^{-2}$$

h: the altitude of the particle.

R: the radius of the planet.

Example3 Determine the acceleration gravity on moon surface. If the moon mass is 7.3554X10^{22} kg, and its radius 1.739X10^6 m. (G=6.67X10^{-11} Nm^2/Kg^2)

Solution

$$g_{Moon} = G\frac{M_{Moon}}{R_{Moon}^2} = 6.67\times10^{-11}\times\frac{7.3554\times10^{22}}{1.739\times10^6}$$

$$= 1.622m/s^2$$

Chapter 5

Oscillatory Motion

5.1 Oscillations الذبذبات

Any motion that repeats تتكرر itself in equal intervals of time علي فترات زمنية متساوية is called **periodic motion الحركة الدورية or harmonic motion الحركة المتناسقة.**

- If a particle in periodic motion moves back and forth over the same path اذا كان الجسم يتحرك حركة دورية ليعود الي نفس مكانة في كل حركة, the motion is called **oscillatory or vibratory تسمي الحركة التذبذبية.**
- The **period T الزمن الدوري** of a harmonic motion is the time required to complete one oscillation or cycle لعمل ذبذبة او دورة كاملة. The **frequency** التردد of the motion ***f*** is the number of oscillations عدد الذبذبات (or cycles) per unit time في الثانية. The frequency is therefore the reciprocal مقلوب of the period, or

$$f = 1/T \qquad (5.1)$$

- The position at which no net force acts لاتوجد قوي مؤثرة on the oscillating particle is called its equilibrium position موقع الاتزان.
- The displacement الازاحة is the distance of the oscillating particle from its equilibrium position at any instant.

Let us focus our attention on a particle oscillating back and forth along a straight line between fixed limits. Its displacement x changes periodically دوريا in both magnitude and direction. Its velocity ***v*** and acceleration ***a*** also vary periodically in magnitude and direction, and in view of the relation F = ma , so does the force acting on the particle .

Figure 5.1a shows a particle oscillating between the limits x_1 and x_2, **O** being the equilibrium position. Fig.5.1b shows the corresponding potential energy curve طاقة الوضع/الكامنة , which has a minimum value at that position.

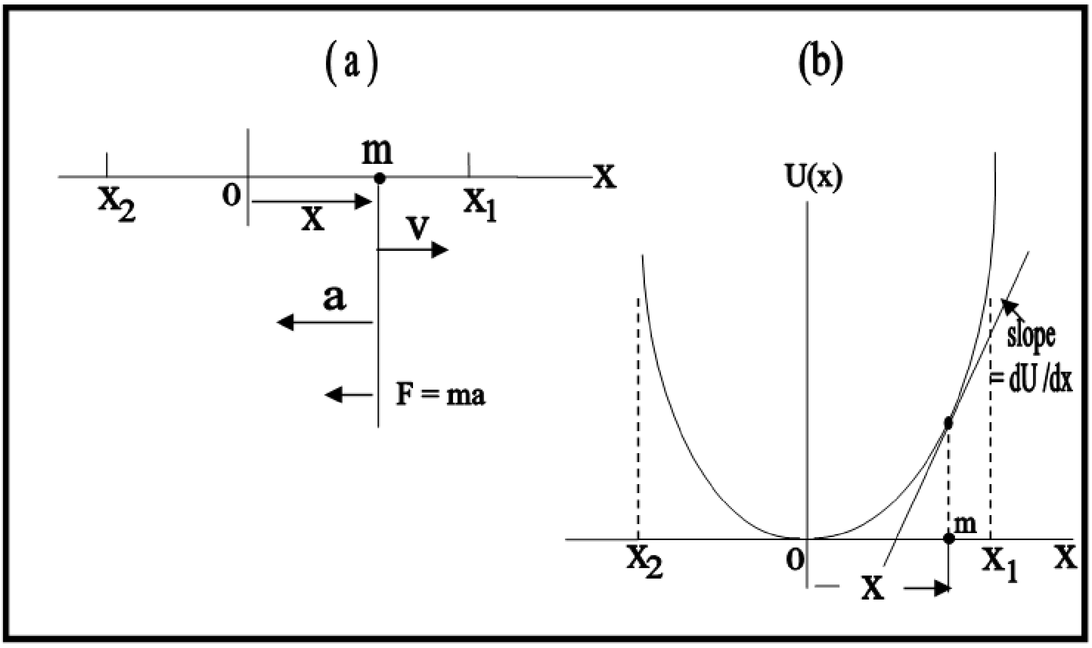


Fig .5.1

The force acting on the particle at any position is derivable تشتق من from the potential energy function; it is given by :

$$F = - dU /dx \qquad (5.2)$$

The total mechanical energy E الطاقة الميكانيكية for an oscillating particle is the sum of its kinetic energy طاقة الحركة and potential energy, or

$$E = K + U \qquad (5.3)$$

In which E remains constant if nonconservative forces قوي مقاوم للتغيير, such as the force of friction , are acting .

5.2 Simple Harmonic Motion

Let us consider an oscillating particle moving back and forth about an equilibrium position through a potential that varies as

$$U(x) = 1/2\ kx^2 \qquad (5.3)$$

In which k is a constant. The force acting on the particle is given by:

$$F(x) = -\ dU/dx = -\ d(1/2kx^2)/\ dx$$

$$F(x) = -\ kx \qquad (5.4)$$

Such an oscillating particle is called a **simple harmonic oscillator (S.H.O)** التذبذب التوافقي بسيط and the motion is called **simple harmonic motion (S.H.M)** الحركة التناسقية البسيطة**.** In such a motion, as Eq. (5.4) shows, the force acting on the particle is proportional to the displacement but is opposite to it in direction. In S.H.M. the limits of oscillation are equally spaced about the equilibrium position.

Applying Newton's law, F = ma to the S.H.O , gives:

$$-kx = m\frac{d^2x}{dt^2}$$

or

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \qquad (5.5)$$

Let us now solve the Eq. of motion of the S.H.O (Eq.5.5). We can rewrite Eq. (5.5) as

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x \tag{5.6}$$

The solution of Eq. (5.6) is

$$x = A\cos(\omega t + \delta) \tag{5.7}$$

and

$$dx/dt = -\omega A \sin(\omega t + \delta)$$

$$d^2x/dt^2 = -\omega^2 A \cos(\omega t + \delta)$$

Putting this into Eq. (5.6) one gets

$$-\omega^2 A \cos(\omega t + \delta) = -k/m\ A \cos(t + \delta)$$

Therefore, If we choose the constant ω such that

$$\omega^2 = k/m \tag{5.8}$$

Then $x = A\cos(\omega t + \delta)$ is in fact a solution of the equation of a simple harmonic oscillator.

The physical significance of the constant ω:

If the time *t* in Eq. (5.7) is increased by $2\pi/\omega$, the function becomes:

$$x = A\cos[(\omega(t + 2\pi/\omega) + \delta)$$

$$= A\cos[(\omega t + 2\pi + \delta)$$

That is the function merely repeats itself after a time $2\pi/\omega$. Therefore $2\pi/\omega$ is the period of the motion T . Since $\omega^2 = k/m$, We have:

$$T = 2\pi/\omega = 2\pi\sqrt{\frac{m}{k}} \qquad (5.9)$$

The frequency *f* of the oscillator is the number of complete vibrations per unit time and given by

$$f = 1/T = \omega/2\pi = 1/2\pi\sqrt{\frac{k}{m}} \qquad (5.10)$$

and the angular frequency ω is given by

$$\omega = 2\pi f = 2\pi/T = \sqrt{\frac{k}{m}} \qquad (5.11)$$

it has the dimension of reciprocal of time and its unit is the radian/sec.

The quantity (ωt + δ) is called the **phase طور** of the motion. The constant δ is called the **phase constant.**

Thus, in a simple harmonic motion the relation between the displacement , the velocity and the acceleration of the oscillating particle is given by :

$$x = A\cos(\omega t + \delta)$$

$$v = dx/dt = -\omega A\sin(\omega t + \delta)$$

$$a = d^2x/dt^2 = -\omega^2 A\cos(\omega t + \delta)$$

Note that the maximum displacement is $x_{max} = A$, the maximum velocity is $v_{max} = \omega A$ and the maximum acceleration is $a_{max} = A\omega^2$.

5.3 Energy Considerations in Simple Harmonic Motion

For the simple harmonic motion the displacement is given by

$$x = A\cos(\omega t + \delta) \tag{5.12}$$

The total energy is given by

$$E = K + U \tag{5.13}$$

The potential energy U at any instant is given by

$$U = 1/2\, k\, x^2$$

$$= 1/2\, k\, A^2\cos^2(\omega t + \delta) \tag{5.14}$$

The kinetic energy K at any instant is 1/2 mv^2, where

$$V = dx/dt = -\,\omega A \sin(\omega t + \delta)$$

Then $$K = 1/2\, m\omega^2 A^2 \sin^2(\omega t + \delta)$$

Where $$\omega^2 = k/m$$

$$K = 1/2\, kA^2 \sin^2(\omega t + \delta)$$

Therefore

$$E = 1/2\, kA^2 \sin^2(\omega t + \delta) + 1/2\, kA^2 \sin^2(\omega t + \delta)$$

or

$$E = 1/2\, kA^2 \tag{5.16}$$

Example1 A horizontal spring is found to be stretched 3 in . from its equilibrium position when a force of 0.75 lb acts on it . Then a 1.5 lb body is attached to the end of the spring and is pulled 4 in. along a horizontal frictionless simple harmonic motion .

a) what is the force constant of the spring ?
b) what is the force exerted by the spring on the 1.5 lb body just before it is released ?
c) what is the period of oscillation after release ?
d) what is the amplitude of the motion ?
e) what is the maximum speed of the vibrating body ?

Solution

a) The displacement x = 3/12 = 0.25 ft

$$K = F/x = 0.75 / 0.25 = 3 \text{ lb/ft}$$

b) The spring is stretched 4.0 in or 1/3 ft . Hence

$$F = - kx = - 3.0 \times 1/3 = - 1.0 \text{ lb}$$

c)
$$T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{w/g}{k}} = 2\pi \sqrt{\frac{1.5/32}{3}}$$

$$= \pi / 4 = 0.79 \text{ sec} .$$

d) The amplitude is the maximum displacement which corresponds to zero kinetic energy and a maximum potential energy . This is the initial condition before release , so that the amplitude is the initial displacement of 4.0 in , hence

$$A = 4.0 \text{ in.} = 4/12 = 1/3 \text{ ft.}$$

e)
$$v_{max}=\omega A=(2\pi/T)A=(2\pi/\pi/4)=2.7 \text{ ft/sec.}$$

5.4 Application of Simple Harmonic Motion

1- Simple Pendulum

A simple pendulum consists of a point mass, suspended by a light inextensible cord. When pulled to one side of its equilibrium position and released, the pendulum swings in a vertical plane under the influence of gravity. The motion is periodic and oscillatory.

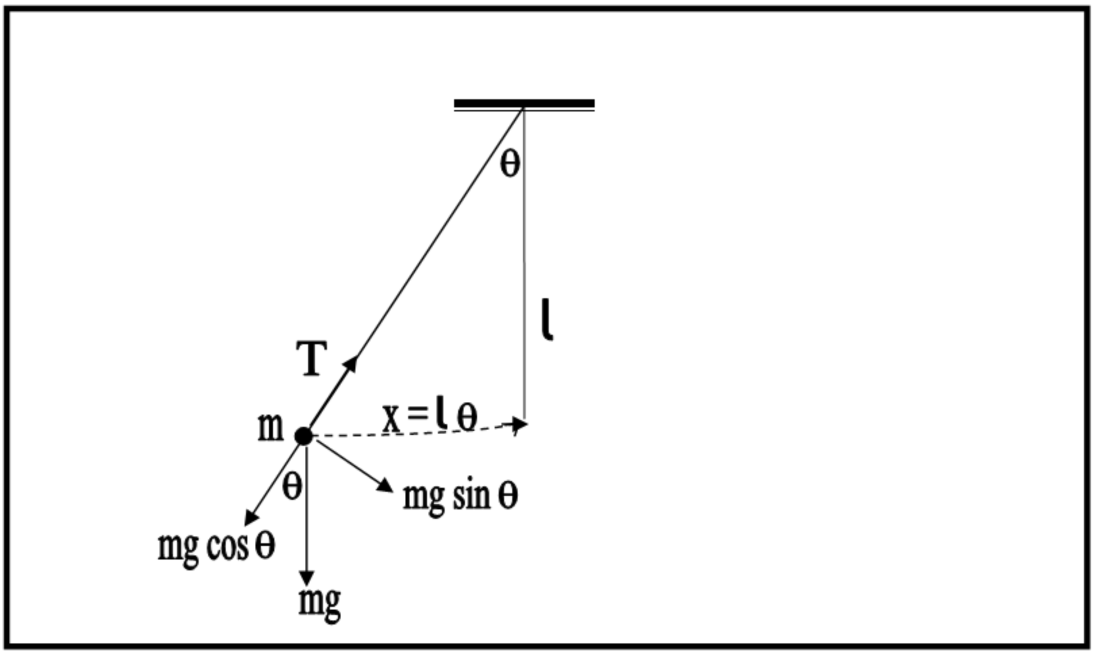


Fig.5.2

Figure 5.2, shows a pendulum of length ℓ , particle mass m , making an angle θ with the vertical . The forces acting on m are mg, the gravitational force, and T, the tension in the cord. Resolve mg into a radial component of magnitude mg cosθ , and a tangential component of magnitude mg sinθ. The radial component of the forces supply the necessary centripetal acceleration to keep the particle moving on a circular arc. The tangential component is the restoring force acting on m tending to return it to the equilibrium position. Hence, the restoring force is

$$F = - mg \sin \theta$$

If the angle θ is very small , then

$$\sin\theta \sim \theta$$

and we obtain

$$F = - mg\,\theta = - mg\, x/\ell = -(mg/\ell)\, x$$

For small displacements, the restoring force is proportional to the displacement and is oppositely directed. The constant (mg/ℓ) represents the constant k in $F = -kx$, that is, $k = mg/\ell$

The period of a simple pendulum when its amplitude is small is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{\ell}{g}} \qquad (5.17)$$

The simple pendulum provides a convenient method for measuring the value of g, the acceleration due to gravity.

Chapter 6

Coulomb's law

6.1 Properties of Electric Charges:

(1) There are two kinds of electric charges, positive and negative charges

- Unlike charges attract each other
- Like charges repel تتنافر each other

(2) Charges are conserved ((i.e.) it does not create but it transfers from body to other).

(3) Charges are quantized.

$q=Ne$, where N is some integer

لاحظ أن وحدة قياس الشحنة الكهربية هي الكولوم ويرمز له بالرمز C

$nC=10^{-9}C$ $\mu C=10^{-6}C$

Materials are divided into three types

1. ***Conductors*** : الموصلات

   In these materials, electric charges move freely.

2. ***Insulators*** : العوازل

   In this material, there are no free charges.

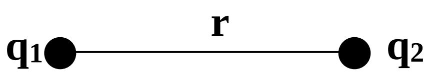
3. ***Semiconductors*** : اشباه الموصلات

   Their electrical properties are intermediate between conductors and insulators.

6.2 Coulomb's Law:

1. The force is directly proportional to the product of charges (q_1q_2)

$$F_e \propto q_1\, q_2$$



2. The force (repulsion or attraction التنافر او التجاذب) is inversely proportional to the square of the separation (r) between charges.

$$F_e \propto \frac{1}{r^2}$$

3. The force is attractive if the charges of opposite sign and repulsion if the charges have the same sign.

Then, $$\boldsymbol{F_e \propto \frac{q_1\, q_2}{r^2} \Longrightarrow F_e = K\,\frac{|q_1||q_2|}{r^2}}$$

Where K is Coulomb constant and can be given by

$$K = \frac{1}{4\pi\varepsilon_0} = \ 9\,x10^9\ Nm^2/C^2$$

F_e has SI unit as Newton

Note **<u>Newton's *universal law of gravity:*</u>**

Gravitational force F_G between two masses m_1 and m_2 separated by a distance r is given by:

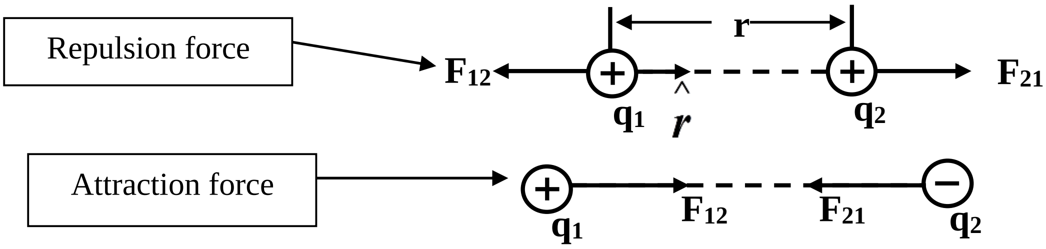
$$\boldsymbol{F_G = G\,\frac{m_1\, m_2}{r^2}}$$ Its unit also is Newton.

Where, G (is constant) = $6x10^{-11}$ N.m^2/kg^2.

The force is a vector quantity, so we can write the force on q_2 due to q_1 as:

$$\overrightarrow{F_{21}} = K\frac{q_1 q_2}{r^2}\hat{r}_{21}$$

where $\hat{\boldsymbol{r}}_{21}$: is the unit vector directed from q_2 to q_1



<u>Notes</u>

- The electric force exerted by q_2 on q_1 is equal in magnitude to the force exerted by q_1 on q_2 and in the opposite direction

$$\vec{F}_{21} = -\vec{F}_{12}$$

1. If q_1 and q_2 have the same sign, the product q_1q_2 is positive and the force is repulsive.
2. If q_1 and q_2 are of the opposite sign, the product q_1q_2 is negative and the force is attractive.

Example 1 Object A has a charge of +2µC, and object B has a charge of +6 µC. Which statement is true.

(a) $F_{AB}=-3F_{BA}$ (b) $F_{AB}= -F_{BA}$ (c) $3F_{AB}= -F_{BA}$

Solution

$$\overrightarrow{F_{AB}} = -\overrightarrow{F_{BA}}$$

- If there are four charges q_1, q_2, q_3, and q_4, the resultant force on q_1 is:

$$F_1 = F_{21} + F_{31} + F_{41}$$

Example 2 The electron and proton of a hydrogen atom are separated by a distance 5.3 x 10^{-11}m. Find the magnitude of the electrical force and the gravitational force between the two particles.

Solution

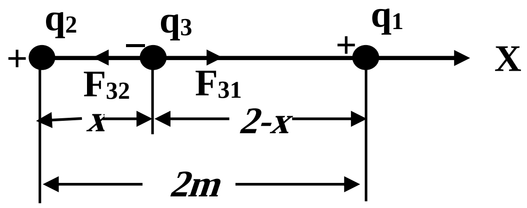
$$F_e = K\frac{e^2}{r^2} = 9\,x10^9\,\frac{(1.6\,x10^{-19})^2}{(5.3\,x10^{-11})^2} = \boxed{8.2\,x10^{-8}\ N}$$

$$F_G = G\frac{m_e m_p}{r^2} = 6\,x10^{-11}\,\frac{(9.11\,x10^{-31})(1.67\,x10^{-27})}{(5.3\,x10^{-11})^2} = \boxed{3.6\,x10^{-47}\ N}$$

$$\therefore \quad \frac{F_e}{F_G} = 2\,x10^{39}$$

Therefore, the gravitational force is negligible compared with electric force.

Example 3 Three charges lie along the X axis as in the figure. The positive charge $q_1 = 15\mu C$ is at x=2m and the positive charge $q_2 = 6\mu C$ is at the origin. Where a negative charge q_3 must be placed on the X-axis such that the resultant force on it is zero?



Solution

- If the net force acting on q_3 is zero, then the force F_{31} must be equal in magnitude and opposite in direction to the force F_{32}.
- The two forces acts on q_3 are F_{31} and F_{32} (Attractive forces)
- Let x be the coordinate of q_3 between q_1 and q_2.

$$\therefore \quad F_{31} = K\frac{q_3 q_1}{(2-x)^2} \qquad and \qquad F_{32} = K\frac{q_3 q_2}{x^2}$$

$$\therefore \quad \sum F_x = 0 \quad \Rightarrow \therefore F_{31} - F_{32} = 0 \quad \Rightarrow \quad F_{31} = F_{32}$$

$$\therefore \quad K\frac{q_3 q_1}{(2-x)^2} = K\frac{q_3 q_2}{x^2} \quad \Rightarrow \quad \frac{q_1}{(2-x)^2} = \frac{q_2}{x^2}$$

$$\therefore \quad (4-4x+x^2)q_2 = q_1 x^2$$

$$(4-4x+x^2)6\text{x}10^{-6} = 15x10^{-6}x^2 \Rightarrow (4-4x+x^2)2 = 5x^2$$

$$3x^2+8x-8=0 \qquad \Rightarrow x = \frac{-8+\sqrt{64-4*3*(-8)}}{2*3}$$

$$x = \frac{-8+\sqrt{64+96}}{6} = \frac{-8+\sqrt{64+96}}{6}$$

We find that x = 0.775 m. Then, q_3 must be placed at **0.775**m from q_2.

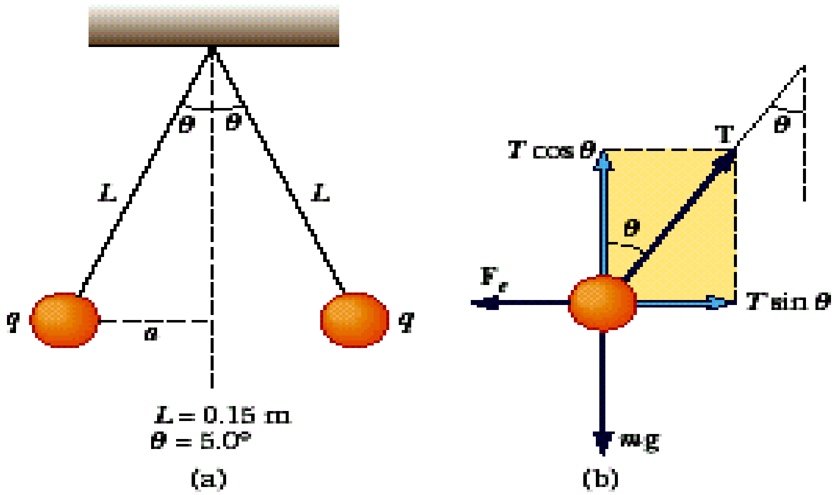
Note That:

The general form of quadratic equation is

$$ax^2+bx+c=0$$

Its solution is $$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Example 4 Two identical small charged spheres, each having a mass of 3x 10^{-2} kg, hang in equilibrium as shown in Figure. If the length of each string is 0.15 m, and the angle is 5°. Find the magnitude of the charge on each sphere.



Solution

From figure (a) $$\sin\theta = \frac{a}{L}$$

$$\therefore \quad a = L\sin\theta = 0.15 \ x \ \sin(5) = 0.013m$$

Then the separation of the two charged spheres is r = 2a = 0.026m.

From fig (b) because the two spheres are in equilibrium then the resultant force in the X and Y directions must be zero.

$$\sum F_x = \sum F_y = 0$$

$$(1) \quad \sum F_x = T\sin\theta - F_e = 0 \quad \Rightarrow F_e = T\sin\theta$$

$$(2) \quad \sum F_y = T\cos\theta - m\,g = 0 \quad \Rightarrow T = \frac{m\,g}{\cos\theta}$$

$$F_e = m\,g\frac{\sin\theta}{\cos\theta} = m\,g\tan\theta$$

$$\therefore \quad F_e = (3\,x10^{-2}\,Kg)(9.8\,m/s^2)\tan(5) = 2.57\,x10^{-2}\,N$$

From coulomb's law:

$$F_e = K\frac{q^2}{r^2} \quad \Rightarrow q^2 = \frac{F_e r^2}{K} = \frac{(2.57\,x10^{-2})(0.026)}{9\,x10^9} = 1.93\,x10^{-15}\,C^2$$

So, the magnitude of the charge on each sphere is q=4.4 x^8C.

Chapter 7

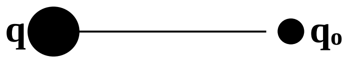
Electric Field

7.1 The Electric Field المجال الكهربي

The electric field **E** is defined as: "the electric force F acting on a positive test charge divided by the magnitude of the test charge q_o"

$$\boldsymbol{E = \frac{F}{q_o} \quad but \quad F = K\frac{q\,q_o}{r^2}}$$

Then,



$$E = K\frac{q}{r^2}\hat{r}$$

وحدة قياس المجال الكهربي هي (V/m) او (N/C)

The direction of the electric field:

- If q is positive (+ve), the direction of E is outward from q
- If q is (-ve), the direction of E is toward q as shown in the Fig.7.1.

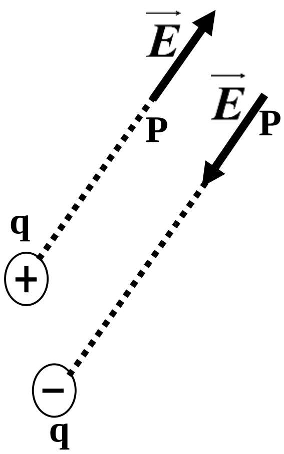


Fig.7.1

Note

At any point P, the total electric field due to a group of charges equals the vector sum of the electric fields of the all charges.

$$\vec{E} = \vec{E_1} + \vec{E_2} + \vec{E_3} + ... = K\sum_i \frac{q_i}{r_i^2}\hat{r_i}$$

Where $\hat{r_i}$ is a unit vector directed from q_i towards point P.

Example 1 Find the electric force on a proton placed in an electric field of $2x10^4$ N/C directed along the positive X-axis.

Solution

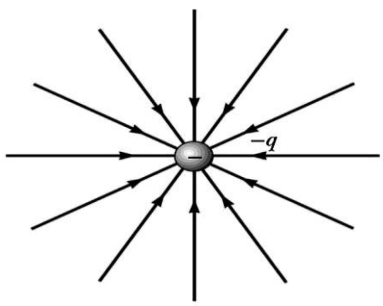
E=F/q , q=e

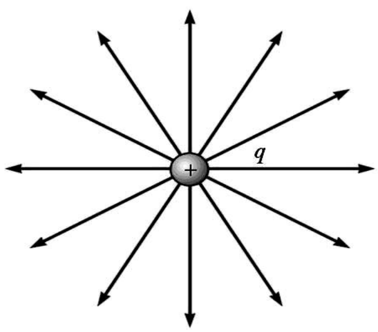
The force on a proton is

$$F = q\,E = e\,E = (1.6x10^{-19})(\,2 \,x\, 10^4\,) = 3.2 \,x\, 10^{-5}\ N$$

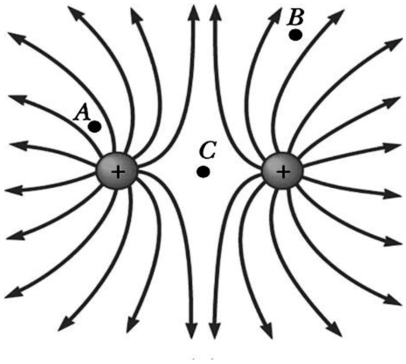
7.2 Electric field lines

- The electric field lines are drawing to point to the direction of electric field vector.
- The rules of drawing the electric field lines:
    1. The lines must begin on positive charges and enter to the negative charges.



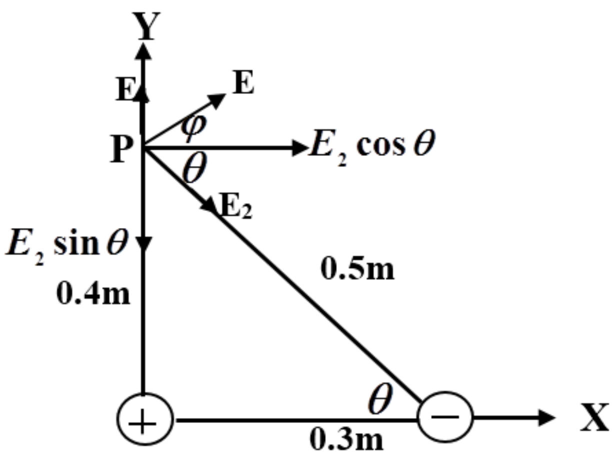


2. These lines represent the direction of electric field.
3. No two lines can cross.



4. The direction of electric field vector is tangent to the electric field line at each point.

Example 2 A charge $q_1 = 7\mu c$ is located at the origin. In addition, a second charge $q_2 = -5\mu c$ is located on the X-axis at 0.3m from the origin. Find the electric field at the point P with the coordinates (0,0.4) m.



Solution

- The electric field for q_1 is:

$$E_1 = K\frac{q_1}{r_1^{\,2}} = (9x10^9)\frac{7x10^{-6}}{(0.4)^2} = 3.94x10^5\ N/c$$

- The electric field for q_2 is:

$$E_2 = K\frac{q_2}{r_2^{\,2}} = (9x10^9)\frac{5x10^{-6}}{(0.5)^2} = 1.8x10^5\ N/c$$

<u>The y-component of E is</u> :

$$E_y = E_1 - E_2\sin\theta = 3.94x10^5 - 1.8x10^5\,x\,\frac{0.4}{0.5} = 2.5x10^5\ N/C$$

<u>The x-component of E is</u> :

$$E_x = E_2\cos\theta = 1.8x10^5\,x\,\frac{0.3}{0.5} = 1.1x10^5\ N/C$$

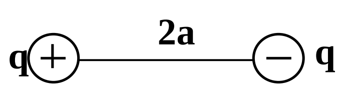
Then, the total field E has the magnitude:

$$E = \sqrt{E_x^{\,2} + E_y^{\,2}} = \sqrt{(1.1x10^5)^2 + (2.5x10^5)^2} = 2.7x10^5\ N/C$$

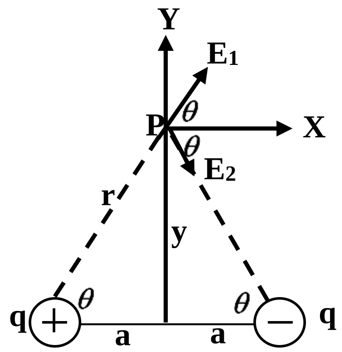
The Field direction: $\varphi = \tan^{-1}\dfrac{E_y}{E_x} = \dfrac{2.5x10^5}{1.1x10^5} = 2.27 \quad \Rightarrow \quad \varphi = 66^o$

7.3 Electric field of a Dipole

An electric dipole "Consists of a positive charges (q) and a negative charge (– q) separated by a small distance 2a".



Example 3 Find the electric field E due to a dipole along the Y-axis at a point P, which is a distance y from the origin. Assume that y>>a.



Solution

At point P, the fields E_1 and E_2 due to the charges of dipole, which are equal in magnitude:

$$E_1 = E_2 = K\frac{q}{r^2} = K\frac{q}{y^2 + a^2}$$

$$(1) \quad E_y = E_1 \sin\theta - E_2 \sin\theta = 0$$

Therefore, the y-component cancel each other.

$$(2) \quad E_x = E_1 \cos\theta + E_2 \cos\theta = 2E_1 \cos\theta$$

$$\because E = E_x = 2E_1 \cos\theta \qquad but \quad \cos = \frac{a}{r} = \frac{a}{(y^2 + a^2)^{1/2}}$$

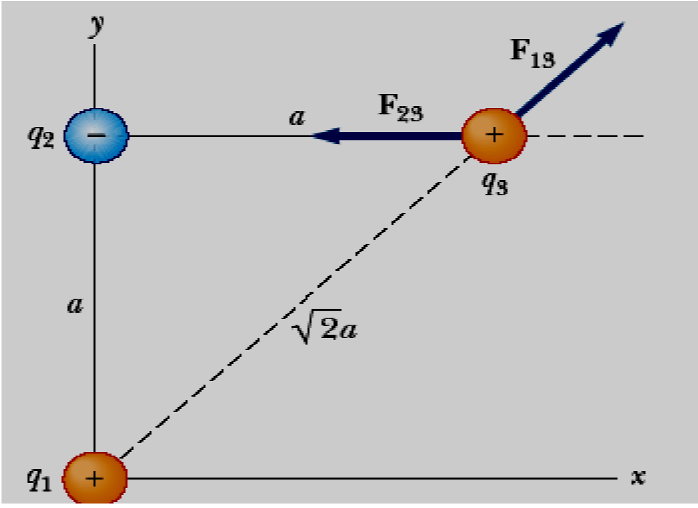
$$\therefore \; E = 2K\frac{q}{(y^2 + a^2)} \cdot \frac{a}{(y^2 + a^2)^{1/2}} = K\frac{2qa}{(y^2 + a^2)^{3/2}}$$

$$for \; y >> a \quad \Rightarrow \quad y^2 + a^2 \approx y^2$$

$$\therefore \quad E = K\frac{2qa}{y^3} \quad i.e. \quad for\ dipole \quad E \propto \frac{1}{y^3}$$

Problems

Problem 1 Consider three point charges located at the corners of a right triangle as shown in Figure below, where $q_1=q_3=5$ µc, $q_2=-2$ µc, and a= 0.1 m. Find the resultant force exerted on q_3.



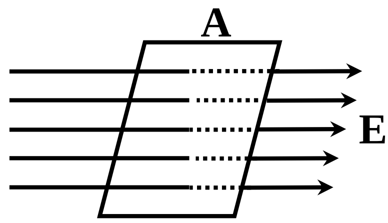
Problem 2 Three charges q_1 = 12 µc, q_2 = 5 µc and q_3 = -2 µc are placed at the corners of a triangle of equal sides L=0.2 m. Find the force acting on charge q_3 due to q_1 and q_2.

Chapter 8

The Electric Flux

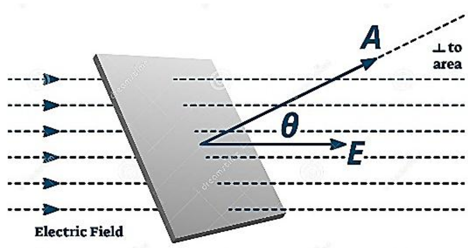
8.1 The Electric Flux "ϕ" الفيض الكهربي

The Electric Flux is the number of field lines crossing normally an area A.



- If $E \perp A$: ϕ = EA

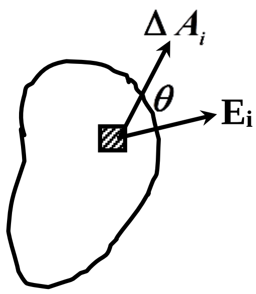
- If the electric field lines makes an angle θ with the normal to the area A then, $\phi = E\,A\cos\theta$



- ***If*** $E \,//\, A \Rightarrow \theta = 0 \Rightarrow \cos\theta = 1$
- ***If*** $E \perp A \Rightarrow \theta = 90 \Rightarrow \cos\theta = 0$

Due to, $0 \le \cos\theta \le 1$, so the maximum value of ϕ is EA.

For general surface



The surface will be divided into small elements of area ΔA_i, so the flux due to this element is $\Delta\phi_i = E_i \,.\Delta A_i = E_i \,\Delta A_i \cos\theta$

Then the total flux is

$$\phi = \int E \,.\, d\,A = \int E\; d\,A\cos\theta$$

8.2 Gauss's law

Gauss' law relates the net flux of an electric field through a closed surface.

It can given by

$$\emptyset = \frac{q_{in}}{\varepsilon_0}$$

Hence, q_{in} is the net charge inside a surface

$$q_{in} = q_1 + q_2 + q_3 + \cdots$$

ε_0 is the permittivity of free space

$$\varepsilon_0 = 8.85 \times 10^{-12}\ C^2/N.m^2$$

The Gauss's Law depend only on the enclosed charges inside the Gaussian surface, with don't care to the surface shape.

8.3 Applications of Gauss's Law

a. The Electric Field of Infinite charged rod length

Figure 8.1 shows a section of an infinitely long cylindrical plastic rod with a uniform charge density λ. It is desired to find an expression for the electric field magnitude (E) at radius *r* from the central axis of the rod, outside the rod.

The net flux through the cylinder is then:

$$\emptyset = EAcos\theta = E(2\pi rh)cos0 = E(2\pi rh)$$

Gauss's law we have the charge q_{in} enclosed by the cylinder.

$$\emptyset = \frac{q_{in}}{\varepsilon_0}$$

For linear charge density (charge per unit length, remember) is uniform, the enclosed charge is q=λh

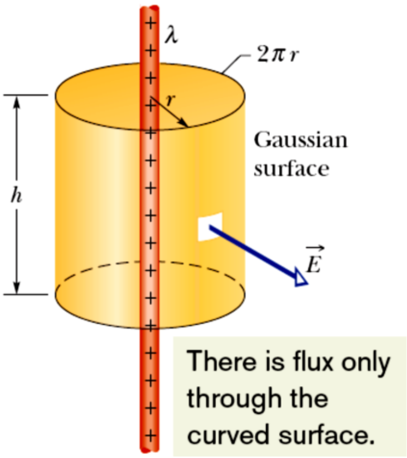


Fig.8.1

$$E(2\pi rh) = \frac{\lambda h}{\varepsilon_0} \rightarrow E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Example 1 Assume a narrow vertical cylinder of height h=1.8 m and radius r=0.10 m. The charge Q was uniformly distributed along the cylinder and that electrical breakdown would have occurred if the electric field magnitude along the cylinder is E=2.4 x10^6 N/C. What is value of Q.

Solution

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{Q/h}{2\pi\varepsilon_0 r} \rightarrow Q = E2h\pi\varepsilon_0 r$$

$$Q = 2.4 \times 10^6 \times 1.82 \times 3.14 \times 0.1 \times 8.85 \times 10^{-12} \cong 24\mu c$$

b. The Electric Field of Infinite charged Plane Area

Figure 8.2, shows a portion of a thin, infinite, non-conducting sheet with a uniform (positive) surface charge density **σ**. A sheet of thin plastic wrap, uniformly charged on one side, can serve as a simple model. Let us find the electric field $\vec{E}$ a distance **r** in front of the sheet.

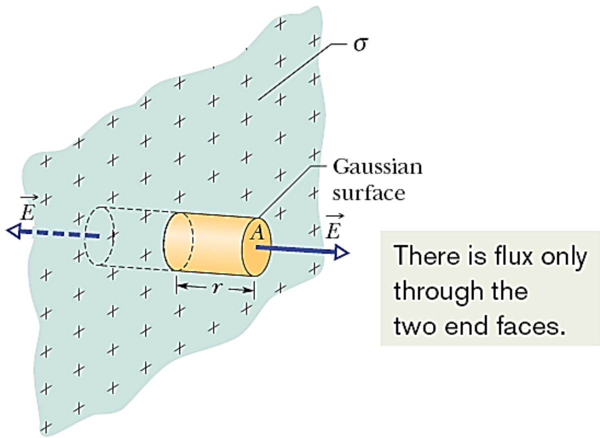


Fig.8.2

From symmetry, $\vec{E}$ must be perpendicular عمودي to the sheet and hence to the end caps. Furthermore, since the charge is positive, is directed away from the sheet, and thus the electric field lines pierce تخترق the two Gaussian end caps in an outward direction.

$$q_{in} = \varepsilon_0 \oint \vec{E}.d\vec{A} = \varepsilon_0 \oint EAcos\theta dA$$

$$q_{in} = \varepsilon_0 \oint_{First\ end} EdA + \varepsilon_0 \oint_{Second\ End} EdA$$

$$= \varepsilon_0(EA + EA) = 2\varepsilon_0 EA$$

Where σA is the charge enclosed by the Gaussian surface. This gives:

$$E = \frac{\sigma}{2\varepsilon_0}$$

c. The Electric Field of uniformly charged Sphere

Figure 8.3, shows a charged spherical shell of total charge q and radius R and two concentric spherical Gaussian surfaces, S_1 and S_2. Apply Gauss' law to surface S_2 for which $r \geq R$,we would find that

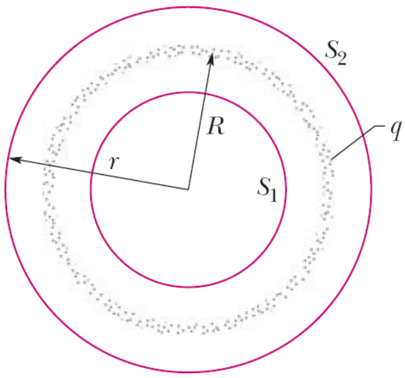


Fig.8.3

$$\emptyset = EAcos\theta = \frac{q_{in}}{\varepsilon_0}$$

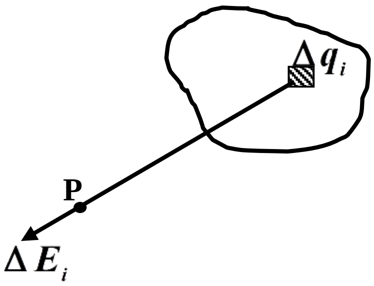
$$E = \frac{q}{A\varepsilon_0} = \frac{q}{4\pi r^2 \varepsilon_0}$$

For inside the sphere q=0, so E=0.

8.4 Electric Field of a Continuous Charge Distribution

1. If the total charge Q is uniformly distributed on a volume V, The Volume charge density:

$$\rho = \frac{Q}{V} \quad (C/m^3)$$



2. If the total charge Q is uniformly distributed on a surface area A, The surface charge density:

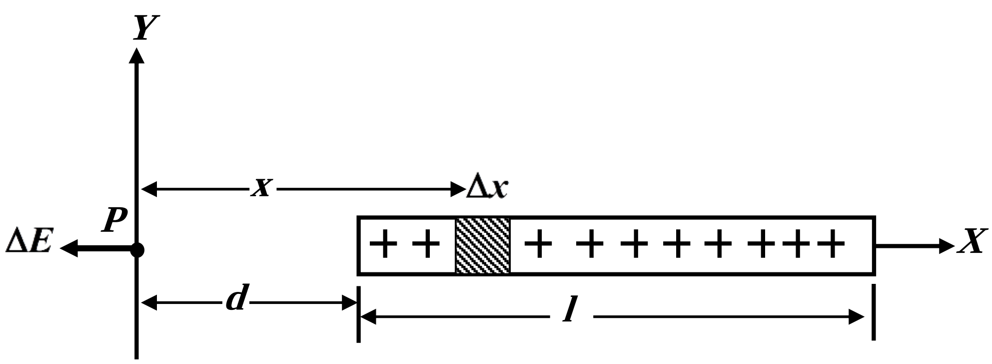
$$\sigma = \frac{Q}{A} \quad (C/m^2)$$

3. If the total charge Q is uniformly distributed along a line of length *l*, The linear charge density:

$$\lambda = \frac{Q}{l} \quad (C/m)$$

The Electric Field of a Finite Line Charge:

A rod of length *l* has a uniform positive charge per unit length λ and total charge Q. Calculate the electric field at a point P along the axis of the rod.



$$\because \quad \lambda = \frac{Q}{l} = \frac{\Delta q}{\Delta x} \quad \Rightarrow \quad \Delta q = \lambda \Delta x$$

Δq is the charge of the element of length Δx

The electric field due to the element (ΔE) at P:

$$\Delta E = K\frac{\Delta q}{x^2} = K\frac{\lambda \Delta x}{x^2}$$

Thus, the total electric field E:

$$E = \int_{d}^{l+d} K\lambda \frac{dx}{x^2} = K\lambda \int_{d}^{l+d} \frac{dx}{x^2} = K\lambda \left[-\frac{1}{x}\right]_{d}^{l+d} = K\lambda\left[-\frac{1}{l+d} + \frac{1}{d}\right] = K\lambda \frac{l}{d(d+l)}$$

$$\because \quad \lambda = \frac{Q}{l} \quad \Rightarrow \quad Q = \lambda l$$

$$\therefore \quad E = K\frac{Q}{d(d+l)}$$

If d>>l $\quad \Rightarrow \quad E = K\frac{Q}{d^2}$ (The rod appear as a point charge)

Chapter 9

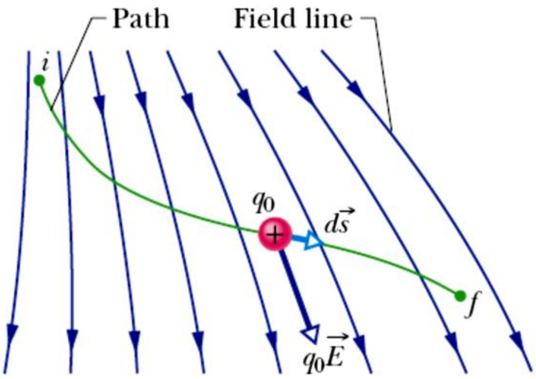
The Electric Potential

9.1 The Electric Potential Difference

The electric potential difference ΔV at a point P is the difference in the electric potential energy per unit charge. Its unit is 1 volt (J/C)

$$\Delta V = \frac{\Delta U}{q} = -\frac{W}{q}$$

A test charge q_0 moves from point *i* to point *f* along the path shown in a non-uniform electric field. During a displacement $\vec{s}$, an electric force $\vec{F}$ acts on the test charge. This force points in the direction of the field line at the location of the test charge.



The work W done on a particle can be given as:

$$W = \int_i^f \vec{F}.ds = \int_i^f F cos\theta.ds$$

where θ is the angle between the force and the displacement.

$$\because F = q_0 E$$

$$\therefore W = q_0 \int_i^f E cos\theta.ds$$

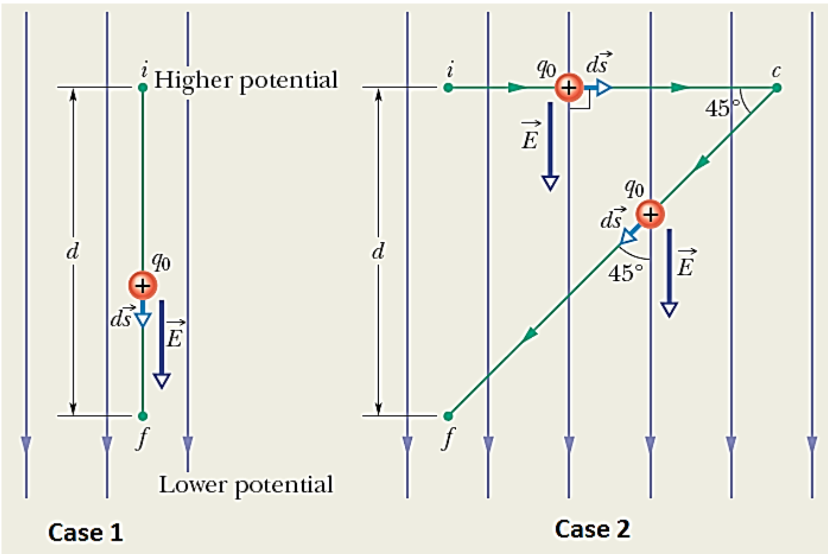
$$\therefore \Delta V = -\int_i^f E cos\theta.ds$$

So in a uniform electric field when the force in the same direction of the electric field, this relation can be written as:

$$\Delta V = V_f - V_i = -Ed$$

Where d is the total displacement for the charge.

Example 1 Find the electric potential of the following charge path that shown in Fig.9.1



Solution

Case 1

The motion is in the same direction of the electric field lines, θ=0 then:

$$\Delta V = V_f - V_i = -Ed$$

Case 2

The first path from *i* to *c*, the motion is normal to the electric field line so θ=90°, and there is no potential difference $V_c = V_i$

The second path from *c* to *f*, θ=45°,

$$\therefore \Delta V = -\int_c^f E\cos\theta . ds = -E\cos\theta \int_c^f ds$$

The distance from *c* to *f* can be calculated as $\frac{d}{\cos\theta}$

$$\therefore \Delta V = -\int_c^f E\cos\theta . ds = -E\cos\theta \frac{d}{\cos\theta} = -Ed$$

9.2 Potential Due to a Charged Particle

Consider a test charge q_0 in a non-uniform electric field as shown in Fig.9.2. It moves in the same direction of the electric field line to infinity through point P at a distance R.

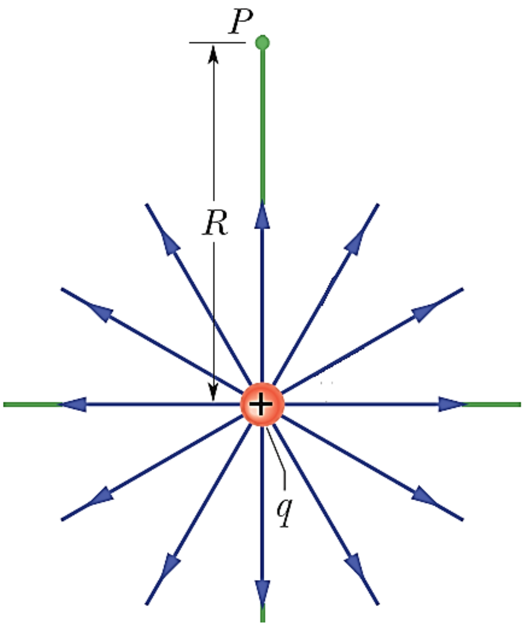


Fig.9.2

$$\Delta V = -\int_R^\infty E\cos\theta . dr$$

for θ=0 and cosθ=1

$$\because E = k\frac{q}{r^2}$$

$$\therefore \Delta V = -kq \int_R^{\infty} \frac{1}{r^2} dr$$

$$\Delta V = kq[\frac{1}{r}]_R^{\infty} = 0 - V$$

Hence at infinity V=0

$$V = k\frac{q}{r}$$

Example 2 What is the electric potential at point P, located at the center of the square of charged particles shown in Fig. 9.3. The distance d is 1.3 m, and the charges are: ($k=9X10^9$)

q_1=+12nC , q_2=-24nC, q_3=+31nC, q_4=+17nC

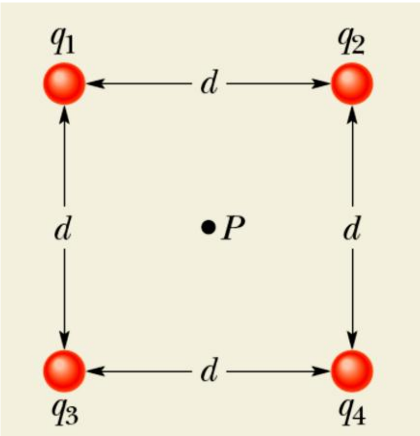


Fig.9.3

Solution

$V=V_1+V_2+V_3+V_4$

$$V = k[\frac{q_1}{r_1} + \frac{q_1}{r_2} + \frac{q_1}{r_3} + \frac{q_1}{r_4}]$$

For the same distance $\rightarrow$ $r_1=r_2=r_3=r_4=\sqrt{2\times\frac{d^2}{4}}=\frac{d}{\sqrt{2}}$

$$V=\frac{9\times10^9}{1.3/\sqrt{2}}[12-24+31+17]\times10^{-9}=350V$$

Example 3 Two parallel charged plates have potential differenve 12V (Battery). If the distance between the plates 0.5cm. Find the magnitude of the electric field.

Solution

For a uniform electric field

$$\Delta V=-Ed$$

ΔV=12V, d=0.005m

$$E=-2.4\times10^3\ V/m$$

Chapter 10

Capacitance

10.1 The Capacitor

Capacitor is two isolated conductors of any shape separated by dielectric material.

Figure 10.1 shows a capacitor with two plates separated by distance **d** with sectional area **A**. The two plates are charged with two opposite charges **q** which induced by a potential difference V.

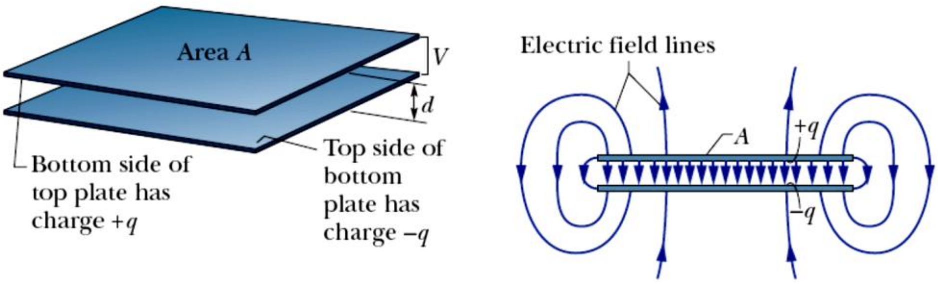


Fig.10.1

$$Q \propto V \rightarrow Q = CV$$

$$C = \frac{Q}{V}$$

where Q the magnitude of the charge on the plate, C the capacitance of the capacitor its unit called **Farad** (**F**).

10.2 The Capacitance Calculating

To relate the electric field E between the plates of a capacitor to the charge q on either plate, use Gauss' law:

$$q = \varepsilon_0 \oint E.dA$$

For a uniform electric field

$$q = \varepsilon_0 EA$$

$$\Delta V = -\int_{-}^{+} E.ds$$

let the potential difference ΔV labeled as V

$$V = Ed$$

$$C = \frac{\varepsilon_0 EA}{Ed} = \varepsilon_0 \frac{A}{d}$$

Hence ε_0 is the permittivity of the dielectric material (almost in the course 8.85X10^{-12} F/m).

Example 1 Two parallel plates capacitor have a separated distance 5mm with area 2m^2. A potential difference of 10kV is applied to the capacitor. Find:

a) The capacitance,

b) The Plate charge,

c) The electric field.

Solution

d=0.005m, A=2m^2, V=10000 volt

a)

$$C = \varepsilon_0 \frac{A}{d} = 8.85 \times 10^{-12} \frac{2}{0.005} = 3.54 \times 10^{-9} F = 3.54 nF$$

b)

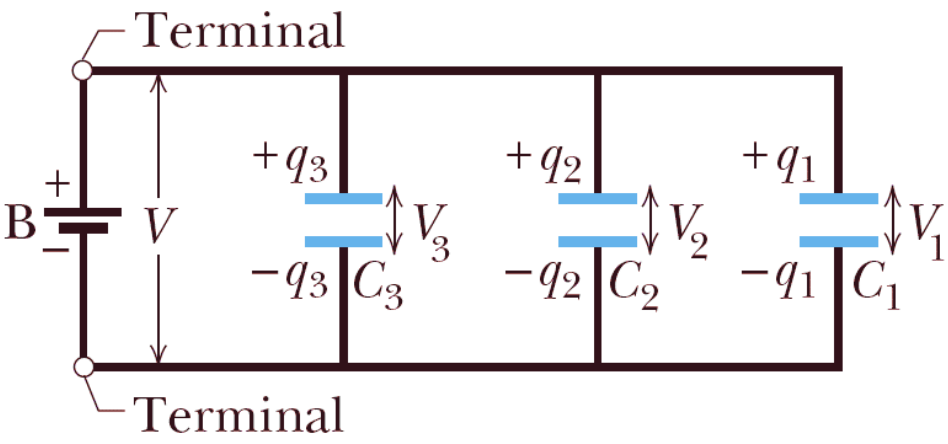
$$Q = CV = 3.54 \times 10^{-9} \times 10^4 = 35.4 \mu C$$

c)

$$E = \frac{V}{d} = 2 \times 10^6 V/m$$

10.3 The Capacitors Connections

a. Capacitors in Parallel



$$q_1 = C_1V, \quad q_2 = C_2V, \quad \text{and} \quad q_3 = C_3V.$$

$$q = q_1 + q_2 + q_3 = (C_1 + C_2 + C_3)V.$$

$$C_{eq} = \frac{q}{V} = C_1 + C_2 + C_3,$$

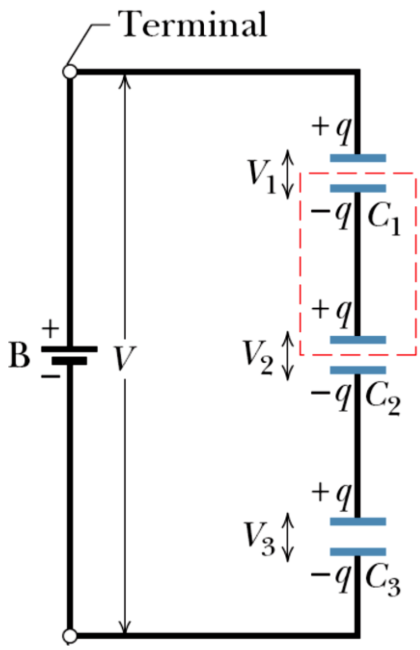
a. Capacitors in Series

$$V_1 = \frac{q}{C_1}, \quad V_2 = \frac{q}{C_2}, \quad \text{and} \quad V_3 = \frac{q}{C_3}.$$

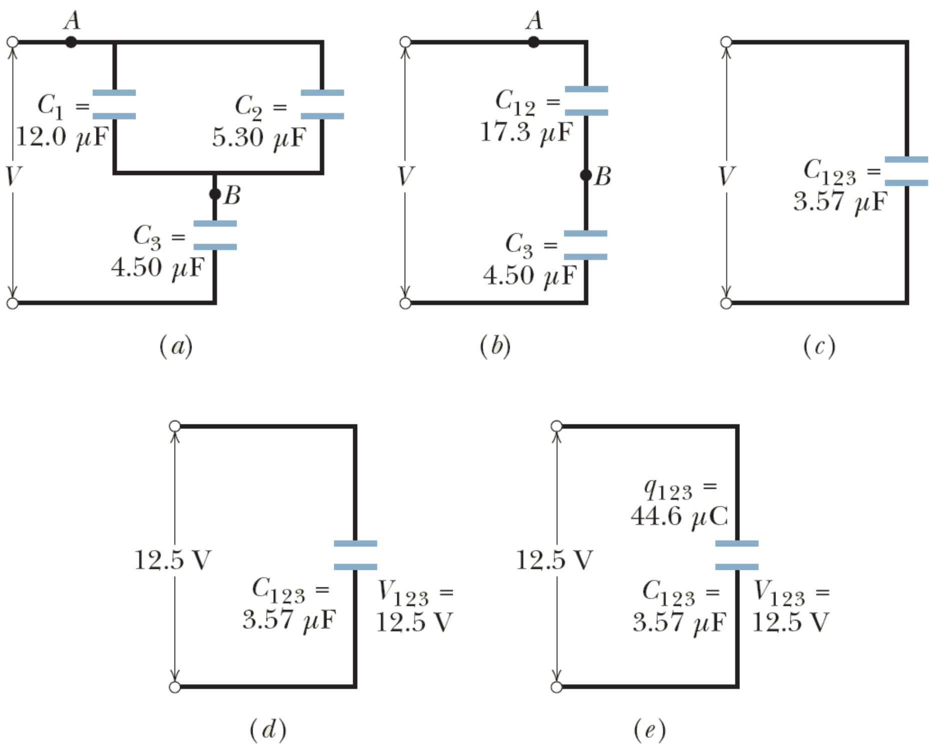
$$V = V_1 + V_2 + V_3 = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right).$$

$$C_{eq} = \frac{q}{V} = \frac{1}{1/C_1 + 1/C_2 + 1/C_3},$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$



Example 2 Find the equivalent capacitor of the following, then if the supply voltage 12.0v, find the total charge.:



$$Q = CV = 12.5 \times 3.57 \times 10^{-6} = 44.6\mu C$$

10.4 The Energy Stored in Capacitors

The energy stored in capacitor equals the work done to charge the capacitor.

Suppose that, at a given instant, a charge q has been transferred from one plate of a capacitor to the other. The potential difference V- between the plates at that instant will be q/C. If an extra increment of charge dq is then transferred، the increment of work required will be,

$$dW = Vdq = \frac{1}{c} qdq$$

The work required to bring the total capacitor charge up to a final value Q is

$$W = \frac{1}{C}\int_0^Q qdq$$

$$W = \frac{Q^2}{2C}$$

This work is stored as potential energy U in the capacitor, so that

$$U = W = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

The Energy Density

The energy density ***u*** is the potential energy per unit volume between the plates.

$$u = \frac{U}{Ad}$$

$$u = \frac{\frac{1}{2}CV^2}{Ad} = \frac{\frac{1}{2}(\varepsilon_0 \frac{A}{d})E^2 d^2}{Ad}$$

$$u = \frac{1}{2}\varepsilon_0 E^2$$

Example 3 An isolated conducting sphere whose radius R is 6.85 cm has a charge Q =1.25 nC. Find:

a) How much potential energy is stored in the electric field of this charged conductor?

b) What is the energy density at the surface of the sphere?

Solution

a)

$$U = \frac{Q^2}{2C} = \frac{Q^2}{2\varepsilon_0 \frac{A}{d}}$$

For sphere $A = 4\pi R^2, d = R$

$$U = \frac{Q^2}{8\varepsilon_0 \pi R} = \frac{1.25 \times 10^{-9}}{8\pi(8.85 \times 10^{-12})(0.0685)} = 1.03 \times 10^{-7} J$$

b)

$$E = k\frac{Q}{R^2}$$

$$u = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0 k^2 \frac{Q^2}{R^2} = 2.54 \times 10^{-5}\, J/m^3 = 25.4 \mu J/m^3$$

10.5 Dielectrics

- It is a non-conducting material such as glass, plastic, paper,… etc.

If the space between the plates of a capacitor is completely filled with a dielectric material, the capacitance C in (vacuum or effectively, in air) is multiplied by the material's dielectric constant k, which is a number greater than 1.

The dielectric constant of the insulating material is shown in the following table.

Material	Dielectric Constant κ	Dielectric Strength (kV/mm)
Air (1 atm)	1.00054	3
Polystyrene	2.6	24
Paper	3.5	16
Transformer oil	4.5	
Pyrex	4.7	14
Ruby mica	5.4	
Porcelain	6.5	
Silicon	12	
Germanium	16	
Ethanol	25	
Water (20°C)	80.4	
Water (25°C)	78.5	
Titania ceramic	130	
Strontium titanate	310	8

For a vacuum, κ = unity.

$$\varepsilon_{dielectric} > \varepsilon_{air}$$

$$\varepsilon_{dielectric} = k\varepsilon_{air} \rightarrow C_{dielectric} = kC_{air}$$

where k is the dielectric constant

Example 4 A parallel-plate capacitor whose capacitance C is 13.5 pF is charged by a battery to a potential difference V=12.5 V between its plates. The charging battery is now disconnected and a porcelain slab (k=6.50) is slipped between the plates.

a) What is the potential energy of the capacitor before the slab is inserted?

b) What is the potential energy of the capacitor–slab device after the slab is inserted?

Solution

a)

$$U = \frac{1}{2}CV^2 = \frac{1}{2}(13.5 \times 10^{-12})(12.5)^2 = 1055pJ$$

b) Because the battery has been disconnected, the charge on the capacitor cannot change when the dielectric is inserted.

$$U = \frac{Q^2}{2C} = \frac{Q^2}{2kC} = \frac{U_{air}}{k} = \frac{1055}{6.5} = 162pJ$$

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