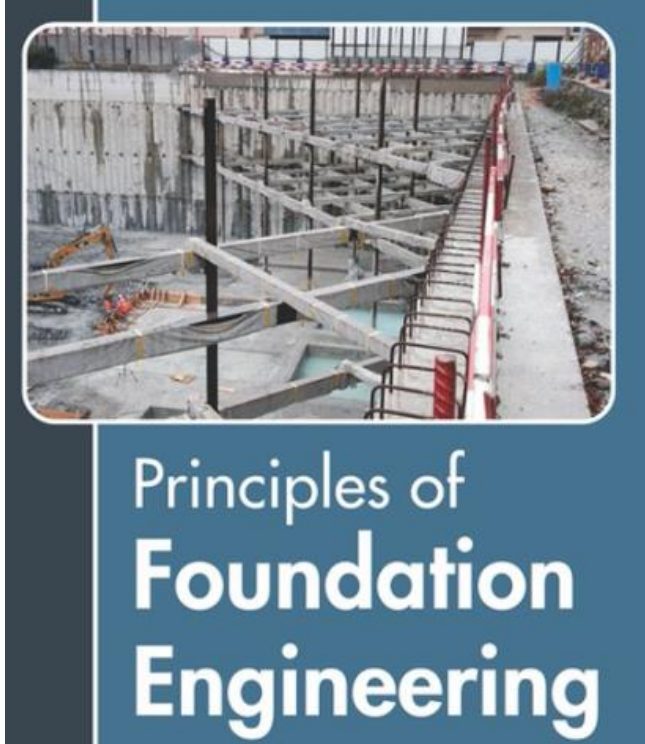

Soil Mechanics and Foundation Engineering (1)

الفرقة الثالثة - معهد المستقبل بالمنصورة



دكتور مهندس

محمد أحمد عبد اللطيف

مدرس بقسم الهندسة الإنشائية

معهد المستقبل العالي

دكتوراه الهندسة الإنشائية

جامعة المنصورة

Contents

- **Chapter 1 Introduction**

1.1 Soil Mechanics and Related Fields.....	1
1.2 Biography of Dr. Karl von Terzaghi.....	1
1.3 Uniqueness of Soils.....	4
1.4 Approaches to Soil Mechanics Problems.....	4
1.5 Examples of Soil Mechanics Problems.....	5
1.5.1 Leaning Tower of Pisa.....	5
1.5.2 Sinking of Kansai International Airport.....	5
1.5.3 Liquefaction—Sand Becomes Liquid during Earthquake.....	7
References.....	7

- **Chapter 2 Physical Properties of Soils**

2.1 Introduction.....	9
2.2 Origin of Soils.....	9
2.3 Soil Particle Shapes.....	10
2.4 Definitions of Terms with Three-Phase Diagram.....	12
2.5 Particle Size and Gradation.....	18
2.6 Summary.....	24
References.....	24
Problems.....	24

- **Chapter 3 Soil Classification**

3.1 Introduction.....	53
3.2 Unified Soil Classification System (USCS).....	53
3.2.1 For G or S.....	55
3.2.2 For C, M, O, or Pt.....	56
3.3 AASHTO Classification System.....	57
3.4 Summary.....	61
References.....	62
Problems.....	62

- **Chapter 4 Soil Compaction**

4.1 Introduction.....	65
-----------------------	----

4.2 Relative Density.....	6
4.3 Laboratory Compaction Test.....	67
4.3.1 Standard Proctor Test Procedure.....	67
4.3.2 Compaction Curve.....	68
4.3.3 Zero Air Void Curve.....	70
4.3.4 Compaction Energy.....	71
4.4 Specification of Compaction in the Field.....	72
4.5 Field Compaction Methods.....	74
4.5.1 Compaction Equipment.....	74
4.5.2 Dynamic Compaction.....	76
4.6 Field Density Determinations.....	77
4.6.1 Sand Cone Method.....	77
4.6.2 Other Field Density Methods.....	80
4.7 California Bearing Ratio Test.....	80
4.8 Summary.....	81
References.....	82
Problems.....	82

- **Chapter 5 Effective Stress**

5.1 Introduction.....	84
5.2 Total Stress versus Effective Stress.....	84
5.3 Effective Stress Computations in Soil Mass.....	84
5.3.1 Dry Soil Layers.....	85
5.3.2 Soil Layers with Steady Water Table.....	86
5.3.3 Totally Submerged Soil Layers.....	87
5.4 Effective Stress Change Due to Water Table Change.....	88
5.5 Capillary Rise and Effective Stress.....	89
5.6 Effective Stress with Water Flow.....	92
5.7 Quicksand (Sand Boiling).....	93
5.8 Heave of Clay Due to Excavation.....	95
5.8.1 Dry Excavation.....	97
5.8.2 Wet Excavation.....	98
5.9 Summary.....	99
References.....	99
Problems.....	99

- **Chapter 6 Stress Increments in Soil Mass**

6.1 Introduction.....	101
6.2 2:1 Approximate Slope Method.....	101
6.3 Vertical Stress Increment Due to a Point Load.....	103
6.4 Vertical Stress Increment Due to a Line Load.....	110
6.5 Vertical Stress Increment Due to a Strip Load.....	111
6.6 Vertical Stress Increment under a Circular Footing.....	112
6.7 Vertical Stress Increment under an Embankment Load.....	113
6.8 Vertical Stress Increment under Corner of Rectangular Footing.....	115
6.9 Vertical Stress Increment under Irregularly Shaped Footing.....	116
6.10 Summary.....	117
References.....	117
Problems.....	117

- **Chapter 7 Settlements**

7.1 Introduction.....	123
7.2 Elastic Settlements.....	123
7.3 Primary Consolidation Settlement.....	126
7.4 One-Dimensional Primary Consolidation Model.....	126
7.5 Terzaghi's Consolidation Theory.....	127
7.6 Laboratory Consolidation Test.....	127
7.7 Determination of C_v	128
7.7.1 Log t Method.....	128
7.7.2 t Method.....	129
7.8 e-log σ Curve.....	130
7.9 Normally Consolidated and Overconsolidated Soils.....	133
7.10 Final Consolidation Settlement for Thin Clay Layer.....	135
7.10.1 Normally Consolidated Soils.....	135
7.10.2 Overconsolidated Soils.....	137
7.11 Consolidation Settlement for Multilayers or a Thick Clay Layer.....	138
7.12 Summary of Primary Consolidation Computations.....	140
7.12.1 The "How Much" Problem.....	141
7.12.2 The "How Soon" Problem (Rate Problem).....	142
7.13 Secondary Compression.....	142
7.14 Allowable Settlement.....	144

7.15 Ground-Improving Techniques against Consolidation	
Settlement.....	145
7.15.1 Vertical Drain (Paper Drain, Wick Drain, and Sand Drain) Techniques.....	146
7.15.2 Preloading Technique.....	147
7.15.3 Vacuum Consolidation Technique.....	148

• **Chapter 8 Shear Strength of Soils**

8.1 Introduction.....	151
8.2 Failure Criteria.....	151
8.3 Direct Shear Test.....	156
8.4 Unconfined Compression Test.....	157
8.5 Triaxial Compression Test.....	159
8.5.1 General Concept and Test Setup.....	159
8.5.2 Initial Consolidation Process and Drainage Condition during Shear.....	161
8.5.3 Consolidated Drained (CD) Triaxial Test.....	161
8.5.4 Consolidated Undrained (CU) Triaxial Test with Pore Water Pressure Measurement.....	163
8.5.5 Effective Stress Parameters from CU and CD Tests.....	167
8.5.6 Unconsolidated Undrained (UU) Test.....	169
8.6 Other Shear Test Devices.....	171
8.6.1 Vane Shear Device.....	172
8.6.2 Tor-Vane Shear Test.....	173
8.6.3 Pocket Penetrometer.....	174
8.7 Summary of Strength Parameters for Saturated Clays.....	176
8.7.1 UU Test.....	177
8.7.2 CD Test and CU Test (Effective Stress).....	179
8.7.3 CU Test (Total Stress).....	180

• **Chapter 9 Bearing Capacity and Shallow Foundations**

9.1 Introduction.....	182
9.2 Terzaghi's Bearing Capacity Theory.....	182
9.3 Generalized Bearing Capacity Equation.....	183
9.3.1 Shape Factors f_{cs} , f_{qs} , $f_{\gamma s}$	184
9.3.2 Depth Factors f_{cd} , f_{qd} , $f_{\gamma d}$	185

9.3.3 Inclination Factors f_{ci} , f_{qi} , $f_{\gamma i}$	186
9.4 Correction Due to Water Table Elevation.....	187
9.5 Gross versus Net Bearing Capacity.....	189
9.6 Factor of Safety on Bearing Capacity.....	189
9.6.1 F.S. for Gross Bearing Capacity.....	189
9.6.2 F.S. for Strength Parameters.....	190
9.7 Shallow Foundation Design.....	191
9.7.1 Footing Depth.....	192
9.7.2 Design Method.....	193
9.8 Summary.....	194
References.....	195
Problems.....	197

- **Chapter 10 Deep Foundations**

10.1 Introduction.....	199
10.2 Types of Piles.....	198
10.3 Load Carrying Capacity by Static Analytical Methods.....	199
10.3.1 Tip Area A_p and Perimeter of Pile “p”	200
10.4 Static Pile Capacity on Sandy Soils.....	203
10.4.1 Tip Resistance.....	203
10.4.2 Skin Friction Resistance.....	203
10.5 Static Pile Capacity in Cohesive Soils.....	204
10.5.1 Tip Resistance.....	204
10.5.2 Skin Frictional Resistance.....	205

CIS 351 Foundations Engineering 1

Foundation Types; Selection of Foundations; Allowable Movements; Soil Mechanics Review; Soil Description, Classification, Effective Stress, In Situ Tests, Shear Strength, Shallow Foundations: Bearing Capacity, Evaluation of Settlements, Isolated Foundations, Combined Foundations, Mat Foundations; Deep Foundations: Pile foundations, Drilled shaft foundation, Load transfer mechanisms, Axial Load Capacity of Deep Foundations, Static Capacity Analysis for Piles, Static Capacity Analysis for Drilled Shafts, Field Load Tests, Group Effects, Settlement, Construction of Deep Foundations, Pile Driving, Pile Driving Formula, Wave Equation Analysis, Drilled shaft construction, Foundations on Expansive Soils.

Chapter 1 Introduction

1.1 SOIL MECHANICS AND RELATED FIELDS

Soil mechanics is one of the engineering disciplines that deal with soils as an engineering material. Since ancient ages, engineers have been handling soils as an engineering material for various construction projects. Construction of the Egyptian pyramids, Mesopotamian ziggurats, Roman aqueducts, and China's Great Wall are a few of such magnificent historical achievements. However, those ancient projects were mostly accomplished by accumulated experiences of ancient engineers. During the eighteenth and nineteenth centuries, some modern engineering theories were employed in this field, following the development of Newtonian mechanics. Coulomb's and Rankine's lateral earth pressure theories (Chapter 12) are some examples of such theories.

The modern era of soil mechanics had to wait until 1925, when **Dr. Karl von Terzaghi** published a book called *Erdbaumechanik* (1925). Especially, his then-new concept of "effective stress," which deals with interaction with pore water, has revolutionized the mechanics of soils. The development of modern soil mechanics is due to his great contribution. He is now regarded as the **father of modern soil mechanics**.

Related terminologies of soil mechanics are foundation engineering, geotechnical engineering, and geoenvironmental engineering. **Foundation engineering** is the field of designing safe foundations, including building footings and retaining structures, and the construction of earth structures such as embankments, earth and rockfill dams, safe earth slopes, etc., based on the knowledge of soil mechanics. Thus, the discipline has been called **soil mechanics and foundation engineering** for many years. The new term, **geotechnical engineering**, was coined around 1970 to merge rock mechanics into soil mechanics and foundation engineering, and it is the most popularly used terminology in this field at present. In the 1980s, environmentally related geotechnical engineering became a great engineering concern, and the term **geoenvironmental engineering** was created. This includes the design and construction of solid- and liquid-waste containment facilities and any other environmentally related geotechnical engineering problems.

1.2 BIOGRAPHY OF DR. KARL VON TERZAGHI

Dr. Karl von Terzaghi (Figure 1.1), the father of modern soil mechanics, was born in Prague, Austria, in 1883. At the age of 10, Terzaghi was sent to a military boarding school. He developed an interest in astronomy and geography. He entered the Technical University in Graz to study mechanical engineering in 1900. He graduated with honors in 1904. Terzaghi then fulfilled a compulsory year-long military service.



FIGURE 1.1 Karl von Terzaghi at age 43.

He returned to the university for 1 year after this and combined the study of geology with courses on subjects such as highway and railway engineering.

His first job was as a junior design engineer for a firm in Vienna. The firm was becoming more involved in the relatively new field of hydroelectric power generation, and Karl became involved in the geological problems the firm faced. He embarked on an ambitious and challenging project to construct a hydroelectric dam in Croatia and an even more chaotic project in St. Petersburg, Russia. During 6 months in Russia, he developed some novel graphical methods for the design of industrial tanks, which he submitted as a thesis for his PhD at the university. His growing list of achievements began to open more opportunities to him. He then resolved to go to the United States in 1912.

There, he undertook an engineering tour of major dam construction sites on the West Coast. This was no ordinary tour, but rather was his opportunity to gather reports and firsthand knowledge of the problems of many different projects, and he used it to the fullest before returning to Austria in December 1913. When World War I broke out, he found himself drafted into the army. He faced combat in Serbia and witnessed the fall of Belgrade. After a short stint managing an airfield, he became a professor in the Royal Ottoman College of Engineering in Istanbul (now Istanbul Technical University). He began a very productive period, in which he began his lifelong work of bringing true engineering understanding to the subject of soil as an engineering material. He set up a laboratory for measurements of the force on retaining walls. The results were first published in English in 1919 and were quickly recognized as an important new contribution to the scientific understanding of the fundamental behavior of soils.

At the end of the war, he was forced to resign his post at the university, but managed to find a new post at Robert College in Istanbul. This time he studied various experimental and quantitative aspects of the permeability of soils and was able to work out some theories to explain the observations. In 1925, he published much of this in *Erdbaumechanik*, which revolutionized the field to great acclaim and resulted in the offer of a position from the Massachusetts Institute of Technology (MIT), which he immediately accepted.

One of his first tasks in the United States was to bring his work to the attention of engineers. He entered a new phase of prolific publication, and a rapidly growing and lucrative involvement as an engineering consultant on many large-scale projects.

In 1928, Terzaghi was determined to return to Europe. He accepted a chair at the Vienna Technische Hochschule in the winter of 1929. Using Austria as his base, he traveled ceaselessly throughout Europe, consulting and lecturing, and making new professional contacts and collaborations. Terzaghi then returned to America, where he gave a plenary lecture at the First International Conference on Soil Mechanics and Foundation Engineering at Harvard University in 1936. He served as the first president of the International Society of Soil Mechanics and Foundation Engineering from 1936 to 1957.

He made a lecture tour of many other universities but discovered that prospects for employment were dim. He returned to Vienna in November 1936. There, he was caught up in a nasty professional and political controversy. He escaped from Vienna frequently by extended consulting trips to major construction projects in England, Italy, France, Algeria, and Latvia, adding greatly to his store of practical engineering experience.

In 1938, Terzaghi immigrated to the United States and took up a post at Harvard University. Before the end of the war, he consulted on the Chicago subway system and the Newport News shipways construction, among others. He became an American citizen in March 1943. He remained as a part-timer at Harvard University until his mandatory retirement in 1953 at the age of 70. In July of the next year, he became the chairman of the consulting board for the construction of the Aswan High Dam. He resigned this post in 1959 after coming into conflict with the Russian engineers in charge of the project, but continued to consult on various hydroelectric projects, especially in British Columbia. He died in 1963.

In honor of his great contribution in the field, the American Society of Civil Engineers (ASCE) established the Karl Terzaghi Award in 1960 to be awarded to an “author of outstanding contributions to knowledge in the fields of soil mechanics, subsurface and earthwork engineering, and subsurface and earthwork construction,” and the Terzaghi lectures are delivered and published annually as a highest honor in the field (abbreviated and modified from Wikipedia).

Goodman (1999) provides a detailed biography of Dr. Karl von Terzaghi that is strongly recommended for all geotechnical engineers and geologists to learn more about his great contributions and many lessons on professional practice.

His contribution is throughout this book, including effective stress, consolidation, shear strength, and bearing capacity theory.

1.3 UNIQUENESS OF SOILS

As this book shows, soil is a very unique material and complex in nature. The unique characteristics of soils are as follows:

1. It is not a solid, continuous material, but rather is composed of three different constituents: solid (grain), water, and air, and is thus an aggregated material.
2. Particle sizes have significant influence on soil behavior from granular soil to clay.
3. The amount of water also plays a very important role in soil behavior.
4. Its stress–strain relation is not linear from the small strain levels.
5. Its pore spaces possess the capability of water flow.
6. It has time-dependent characteristics; that is, it is susceptible to creep.
7. It swells when wetted or shrinks when dried.
8. It is an anisotropic material due to the particle shapes and the depositional direction under gravity.
9. It is also spatially nonhomogeneous.

To handle this unique nature, the discipline utilizes many different areas of mechanics. For the various phases, it uses solid mechanics as well as discrete mechanics. The water flow characteristics are explained by knowledge of fluid mechanics such as Darcy's law and Bernoulli's law. Physicochemical knowledge is required to understand swell and shrinkage characteristics. Understanding its anisotropic characteristics requires a high level of knowledge in mechanics and material science. Some statistical approaches are also needed to treat the nonhomogeneity of soils.

As briefly seen earlier, soil is a very unique material, and its engineering properties vary a lot depending on the particle sizes, origins, and many other factors. Their constitutive models are not as simple as Hooke's law, which is used in some other materials.

1.4 APPROACHES TO SOIL MECHANICS PROBLEMS

Complexity and spatial variation of soil make the field observation and laboratory testing very significant. **Field observation** ranges from geological study of the site to soil sampling and sometimes in-situ testing of properties, such as well tests for permeability, vane shear tests for strength determination, etc. Sampled specimens are brought back to **laboratories** for various physical and mechanical tests. The former includes the grain size test, Atterberg limits tests, specific gravity test, etc., and the latter includes a compaction test, permeability test, consolidation test, and various shear strength tests.

Based on field observations and laboratory test data, geotechnical engineers classify soils, determine design properties, and **design** safe foundations and earth structures, by fully utilizing modern soil mechanics knowledge and foundation engineering concepts. Construction companies carry out **construction** of the project according

to specifications made by design engineers. Usually, design engineers monitor construction practices carefully for proper execution.

The last stage is **field monitoring** of the performance of earth structures. At present, large construction projects always come with instrumentation and performance monitoring. Simple or complex theories are available in most cases. However, those are not always perfect due to complexity of soils and variations in material properties. Thus, the monitoring and reevaluation of design based on the feedback of the data are very crucial for the success of projects.

1.5 EXAMPLES OF SOIL MECHANICS PROBLEMS

Engineers have to deal with many challenging soil mechanics problems even at present, as well as in the past. A few historical and interesting cases are presented in the following subsections.

1.5.1 LEANING TOWER OF PISA

This famous building illustrates historical soil mechanics problems. The 56 m high bell tower at Pisa, Italy, leans about 3.97° or 3.9 m at top toward the south. The construction of the tower started in 1173 and was completed in 1372. It was reported that the tower started to sink unevenly after the construction progressed to the third floor in 1178 and more floors were built up to accommodate for the tilt.

The lean is obviously due to uneven settlement of the foundation soil. This time-dependent settlement phenomenon is called **consolidation** settlement of clay and is discussed later in this book. In March 1990, the tower was closed to the public due to the possibility of collapse in the near future. Engineering remediation procedures were discussed to stop further leaning. An early attempt was made to put heavy load (800 metric tons of lead counterweight) on the north side of the tower foundation to compensate for the larger settlement on the south. A more drastic measure was taken later to extract soil mass (38 m^3) under the north side of the foundation soil by angled auger holes so that the north side experienced extra settlement. In December 2001, the tower was reopened to the public and has been declared stable for at least another 300 years (Figures 1.2 and 1.3).

1.5.2 SINKING OF KANSAI INTERNATIONAL AIRPORT

The first-phase construction of **Kansai International Airport**, a man-made island (4.5 km long and 1.1 km wide) near Osaka, Japan, began in 1987, and the airport became operational in 1994. It was an amazingly fast-paced construction for a project of this magnitude. The massive earth filling at an average water depth of 12 m on Osaka Bay required $208,000,000 \text{ m}^3$ of reclaimed soil and rock (82 times the volume of the Great Pyramid of Giza). The filling materials were brought from excavations of three mountains nearby. Geotechnical engineers anticipated quite a large consolidation settlement due to this massive fill over a large area on soft bay foundation soil. Settlement, which immediately started, was carefully monitored, and results were compared with computed values. It sank 50 cm in 1994; settlement



FIGURE 1.2 Leaning tower of Pisa.



FIGURE 1.3 Lead counterweight.

was reduced to 20 cm in 1999, and was 9 cm in 2006. Originally, engineers estimated 12 m total settlement in 50 years, but, in fact, it had already settled 11.5 m by 2001. Because of anticipated uneven ground settlement, the terminal building was equipped with jacks in each column so that uneven settlement could be adjusted not to have extra stress on individual columns. It is still sinking.



FIGURE 1.4 Kansai International Airport during phase II construction in 2002. (Photo courtesy of Kansai International Airport Land Development Co.)

Amazingly, the phase II runway, the second island on deeper water, has been constructed (Figure 1.4 in 2002 and the cover page picture in 2003) and opened to operation in 2007. This is a magnificent mega-construction project in recent years with very challenging geotechnical engineering problems.

1.5.3 LIQUEFACTION—SAND BECOMES LIQUID DURING EARTHQUAKE

Can you believe that soil transforms into liquid? Yes, it does. During Japan's 1964 Niigata earthquake with a Richter magnitude of 7.5, apartment buildings lost their foundation support and sank and tilted (Figure 1.5). Foundation soil was transformed into viscous liquid due to earthquake vibration. A similar phenomenon was also observed in the Alaska earthquake that occurred in March 1964. Liquefied soil triggered massive landslides in Anchorage. This phenomenon is called **soil liquefaction**. Soil liquefaction describes the behavior of soils that,



FIGURE 1.5 Building tilt and settlement due to liquefaction during the 1964 Niigata earthquake.

Chapter 2 Physical Properties

of Soils

2.1 INTRODUCTION

In this chapter, the answers to “What is soil?” and “How is it formed?” are presented first. Key terms used in soil mechanics are then defined by using the three-phase diagram (i.e., solid, water, and air phases). Lastly, soil grain shapes, sizes, and grain size distribution are discussed. Laboratory methods to determine the grain size distribution are presented.

2.2 ORIGIN OF SOILS

Soil can be defined as *an assemblage of nonmetallic solid particles (mineral grains)*, and it consists of three phases: **solid**, **liquid (water)**, and **gas (air)**. Commonly used terms such as *gravel*, *sand*, *silt*, and *clay* are the names of soils based on their particle grain sizes. The names quartz, mica, feldspar, etc. are based on their crystal names.

The **rock cycle** in Figure 2.1 illustrates the origins of a variety of soils on the earth. Most original rock starts forming from molten magma (liquid) in the deep earth (to the depth of 2885 km from the base of crust). Magma cools down and solidifies when it approaches the earth’s crust (about 4–6 km thick under the deep oceans and 25–60 km thick on the continent) due to tectonic and volcanic activities. Thus, **igneous rocks**, such as basalt, granite, pumice, and olivine, are formed. The next process is **weathering**. Solid igneous rocks on the earth’s surface are subjected to all kinds of attacks in the environment, such as erosion by water and air, thermal expansion and contraction, intrusion of plant roots in the cracks, icing on the cracks, and chemical attack on the surface. The surface of igneous rocks is weathered and broken down to smaller sizes. The next process in the cycle is **transportation**. Broken fragments of rock are displaced by means of water run, glacier, and sometimes by wind, and they eventually settle down at a certain distance from the original location (**deposition**). During the transportation process, particles further suffer physical and chemical attacks, and they become smaller and rounder. Deposited materials (**sediments**) are one type of soil. Thus, this type of soil originates from igneous rocks, and particles have crystals similar to those in igneous rocks.

Deposited soils on many occasions are subjected to many geological years of cementation and compression that transform them to **sedimentary rocks** such as sandstone, shale, limestone, and dolomite, as well as many others. Rocks might go through further transformation due to high heat and pressure in deeper earth but

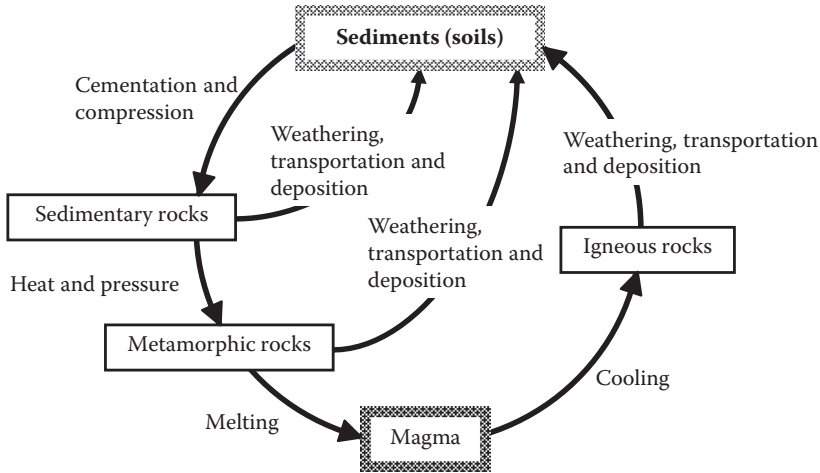


FIGURE 2.1 Rock cycle.

without melting. This process is called **metamorphism**, and **metamorphic rocks** are formed. Transformation of marble from limestone by metamorphism is a good example of this. Metamorphic rocks could be melted into magma to complete a rock cycle when they go more deeply into the earth.

Sedimentary rocks and metamorphic rocks are also subjected to weathering, transportation, and deposition processes to form sediments (soils) as shown in Figure 2.1. Thus, these rocks can become the origins of soil particles in addition to the ones from igneous rocks. All the preceding processes, including origin of soils, weathering, transportation, and deposition, make different types of soil in terms of particle sizes, shapes, mineral compositions, etc. Note that the particle size becomes smaller and smaller, and thus it yields a larger surface area per the same weight of soils. Smaller particles with larger surface area will be more vulnerable to chemical attack, and their original crystal structures may be changed to form different clay minerals as will be discussed in Chapter 3.

2.3 SOIL PARTICLE SHAPES

Weathering and transportation processes produce a variety of particle shapes, from angular to rounded, as shown in Figure 2.2 (*Müller 1967*). Since soil is an assemblage of particles, interlocking of those particles and their contact mechanism—in particular, for larger particles—determines many important mechanical properties of soils such as strength, rigidity, permeability, and compaction. For example, angular particle assembly will give more resistance to sliding deformation (higher rigidity and strength) as compared to rounded particle assembly. Soil **angularity** can be determined by comparing particle shape (under a microscope, if needed) with the sample shapes shown in Figure 2.2.

For smaller particle assemblies, such as in clay minerals, particle shapes are much flatter and sometimes flaky as shown in Figure 2.3 as an example. In those smaller

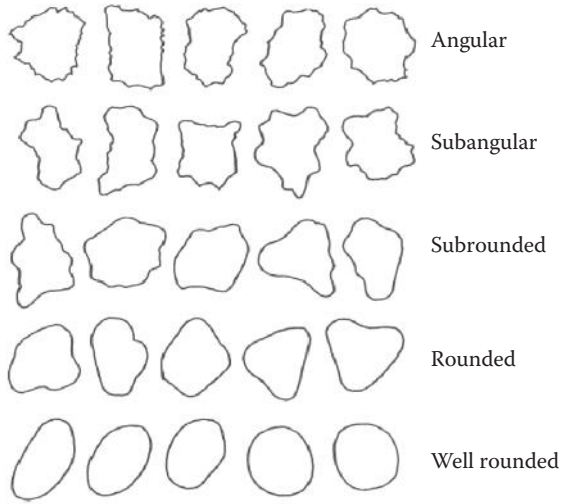


FIGURE 2.2 Soil's angularity. (Müller, G., 1967, *Methods in Sedimentary Petrology*, Hafner.)

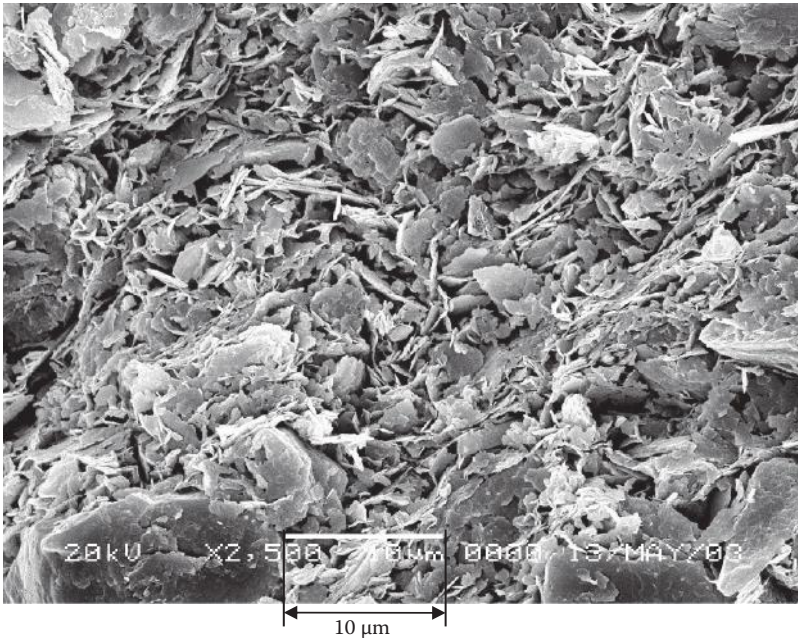


FIGURE 2.3 Scanned electron microscope (SEM) picture of clay particle assembly (Hai-Phong, Vietnam, clay: 50% kaolinite and 50% illite). (Watabe, Y., Tanaka, M., and Takemura, J., 2004, *Proceedings of the 2nd International Conference on Site Characterization*, Porto, 1765–1772.)

particle soils, short-range interparticle forces play an important role to determine the behavior of soils. The details of clay minerals will be discussed in Chapter 3.

2.4 DEFINITIONS OF TERMS WITH THREE-PHASE DIAGRAM

Soil is an assemblage of particles, and thus there are separate volumes of solid and void. The voids are occupied with air and water. To define many key parameters in soil mechanics, soil assemblage with spaces of solid (grain) and void (air and water) is modeled into a **three-phase diagram**, as shown in Figure 2.4. Volume side and weight side are drawn in three phases (solid, water, and air), and individual components are designated as $V_s, V_w, V_a,$ and $W_s, W_w, W_a.$ The total volume and the total weight are also designated as V and $W,$ respectively. The volume of void V_v is the volume of water V_w plus the volume of air $V_a.$ Note that the weight of air W_a is assigned as zero since it is negligible relative to other weights of the element. The following definitions are made based on the three-phase diagram:

$$\text{Porosity: } n = \frac{\text{volume of void}}{\text{total volume}} = \frac{V_v}{V} = \frac{V_a + V_w}{V} \tag{2.1}$$

$$\text{Void ratio: } e = \frac{\text{volume of void}}{\text{volume of solid}} = \frac{V_v}{V_s} \tag{2.2}$$

Referring to Figure 2.5 and applying the definitions of n and e to Figure 2.5(a) and (b), respectively, the following relationships are obtained:

$$n = \frac{e}{1 + e} \quad \text{or} \quad e = \frac{n}{1 - n} \tag{2.3}$$

Void ratio ranges from around 0.3 for **glacial tills** (highly compacted mixtures of gravel to clay under glacial load) to 1.4 for very soft clay. For **organic clay**, the value could go to around 3.0 or more as a special case. According to

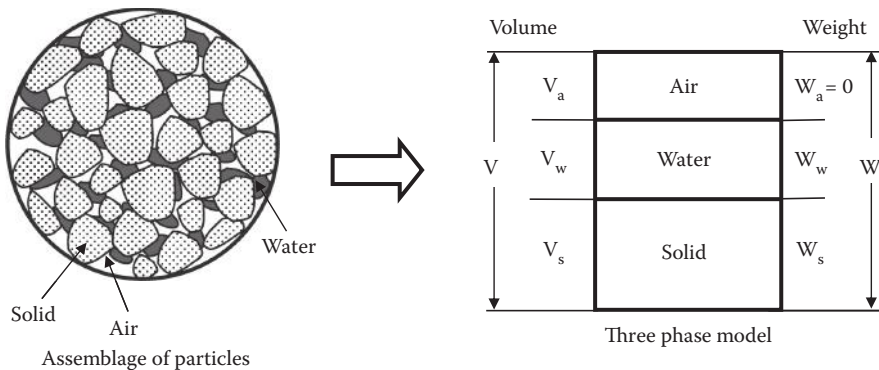


FIGURE 2.4 Three-phase diagram of soil.

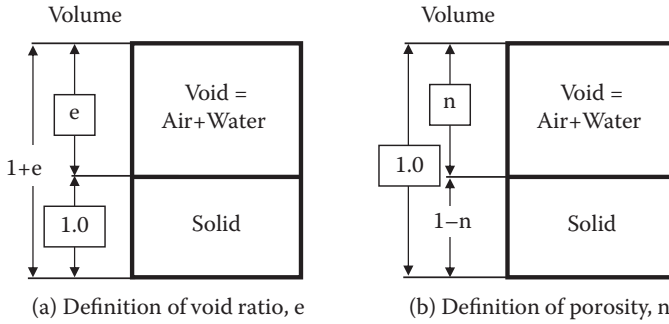


FIGURE 2.5 Relationship between porosity, n and void ratio, e .

Equation (2.3), corresponding (to $e = 0.3$ to 1.4) porosity (n) values are about 0.23 to 0.58 , respectively.

$$\text{Water content: } w = \frac{\text{weight of water}}{\text{weight of solid}} = \frac{W_w}{W_s} (\times 100\%) \quad (2.4)$$

The water content for completely **dry soil** is 0% and normally up to several tens of percentages for **fully saturated** soils. However, the value may go up to more than 200% for highly open-structured clay formed under a marine environment and for **organic soils** as unusual cases.

$$\text{Degree of saturation: } S = \frac{\text{volume of water}}{\text{volume of void}} = \frac{V_w}{V_v} (\times 100\%) \quad (2.5)$$

The S value changes from 0% for completely **dry soil** conditions to 100% for **fully saturated** soil. The soils with $0 < S < 100\%$ are called **partially saturated** soils. Note that on many occasions, “saturated” soils are interpreted as “fully saturated” without spelling out “fully.”

$$\text{Specific gravity: } G_s = \frac{\text{unit weight of solid}}{\text{unit weight of water}} = \frac{W_s/V_s}{\gamma_w} \quad (2.6)$$

where γ_w is the unit weight of water and is 9.81 kN/m^3 or 62.4 lb/ft^3 . Most soils have a rather narrow range of G_s values: 2.65 to 2.70 . This implies that solid particle is about 2.65 to 2.70 times heavier than the weight of water for the same volume. If a specific gravity test was not performed during the initial evaluation of geotechnical engineering problems, assuming G_s as a value between 2.65 or 2.70 would not produce a major error in the results.

There are several definitions of **unit weight** of soil.

$$\text{Total unit weight: } \gamma_t = \frac{\text{total weight}}{\text{total volume}} = \frac{W}{V} = \frac{W_s + W_w}{V_s + V_w + V_a} \quad (2.7)$$

$$\text{Dry unit weight: } \gamma_d = \frac{\text{weight of solid}}{\text{total volume}} = \frac{W_s}{V} \quad (2.8)$$

Note that γ_d is *not necessarily physical dry unit weight of soils: rather, it is treated as a case by mathematically removing water while maintaining constant total volume V without shrinkage*, which takes place during the physical drying process. From Exercise 2.1, the following relation is obtained:

$$\gamma_t = \frac{(1+w)G_s}{1+e} \gamma_w = \frac{G_s + Se}{1+e} \gamma_w \quad (2.9)$$

The dry unit weight γ_d can be obtained by substituting $S = 0$ in the last term in Equation (2.9). Thus, for mathematically dried soil, the following equation is obtained:

$$\gamma_t = (1+w) \frac{G_s \gamma_w}{1+e} = (1+w) \gamma_d \quad \text{or} \quad \gamma_d = \frac{G_s \gamma_w}{1+e} = \frac{\gamma_t}{1+w} \quad (2.10)$$

This relationship is conveniently used to compare the effectiveness of compaction by the dry unit weight in **compaction test** analysis (Chapter 5). *The total unit weight does not measure the effectiveness of compaction directly since the weight of water in void will increase the total unit weight, regardless of increase in void (less compaction) or decrease in void (more compaction).*

The last important definition is the **submerged unit weight** γ' (or **buoyant unit weight**), which is the soil's unit weight under water:

$$\gamma' = \gamma_t - \gamma_w = \frac{G_s + Se}{1+e} \gamma_w - \gamma_w = \frac{G_s - 1 - e(1-S)}{1+e} \gamma_w \quad (\text{for partially saturated}) \quad (2.11)$$

In the last-term expression in Equation (2.11), the degree of saturation S could be other than 1.0 (fully saturated) even though soils are under water. It is possible shortly after soils were submerged in water. However, eventually, soils under the water table will be fully saturated ($S = 1.0$), and thus it becomes

$$\gamma' = \gamma_t - \gamma_w = \frac{G_s + e}{1+e} \gamma_w - \gamma_w = \frac{G_s - 1}{1+e} \gamma_w \quad (\text{for fully saturated}) \quad (2.12)$$

As will be discussed in detail in Chapter 7 (effective stress), stresses in a soil mass due to its own weight are given by the total unit weight γ_t times the thickness of the soil when soil is above the ground water table, while it is given by submerged unit weight γ' times the thickness when soil is below the ground water table. A range of γ_t is about 15 to 20 kN/m³ (or 90 to 130 lb/ft³). Since γ_w is 9.81 kN/m³ (or 62.4 lb/ft³), γ' becomes about a half of γ_t . This is a significant reduction in the stress induced in soil mass.

Exercise 2.1

Using the three-phase diagram for a general soil, derive a formula to determine γ_t from the known values of S , e , w , and G_s .

SOLUTION

Referring to Figure 2.6, first assume that $W_s = 1$, then $W_w = w$.

$$\text{From the definition, } G_s = \frac{W_s}{V_s} / \gamma_w, \text{ thus } V_s = \frac{W_s}{G_s \gamma_w} = \frac{1}{G_s \gamma_w} \quad (2.13)$$

$$\text{From the definition, } \gamma_w = \frac{W_w}{V_w}, \text{ thus } V_w = \frac{W_w}{\gamma_w} = \frac{w}{\gamma_w} \quad (2.14)$$

$$\text{From the definition, } S = \frac{V_w}{V_a + V_w}, \text{ thus } V_a = \frac{(1-S)V_w}{S} = \frac{(1-S)w}{S \gamma_w} \quad (2.15)$$

$$\text{From the definition, } e = \frac{V_a + V_w}{V_s} = \frac{V_a + V_w}{1/G_s \gamma_w}, \text{ thus } V_a + V_w = \frac{e}{G_s \gamma_w} \quad (2.16)$$

From Equations (2.14) through (2.16),

$$S = \frac{V_w}{V_a + V_w} = \frac{w / \gamma_w}{e / G_s \gamma_w} = \frac{w G_s}{e}, \text{ then } S e = w G_s \quad (2.17)$$

Now, using Equations (2.13) through (2.17) and applying the definition of γ_t

$$\begin{aligned} \gamma_t &= \frac{W_s + W_w}{V_a + V_w + V_s} = \frac{1 + w}{e / G_s \gamma_w + 1 / G_s \gamma_w} = \frac{(1 + w) G_s}{1 + e} \gamma_w \\ &= \frac{G_s + w G_s}{1 + e} \gamma_w = \frac{G_s + S e}{1 + e} \gamma_w \end{aligned} \quad (2.18)$$

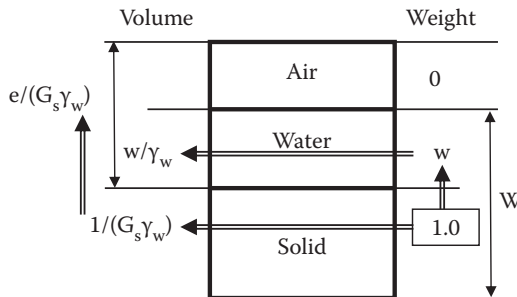


FIGURE 2.6 Exercise 2.1.

In Exercise 2.1, the weight of solid W_s was first assumed as 1.0, and then other components in the three phases were computed. **Any one component can be assumed with any value such as 100 or 1000** since all definitions of w , S , e , n , γ_t , etc. are the ratios of components, and thus the same results can be obtained. Note that G_s and γ_w **always work as key bridge values to connect the weight side and the volume side** as shown in Exercise 2.1. Exercise 2.2 demonstrates that the two different initial assumptions yield the same solutions.

Exercise 2.2

For a given soil, $w = 25\%$ and $\gamma_t = 18.5 \text{ kN/m}^3$ are measured. Determine void ratio e and degree of saturation S . Assume that G_s is 2.70.

SOLUTION (A)

First assume $W_s = 100 \text{ kN}$ as shown in Figure 2.7(a). Then, $W_w = 100 \times 0.25 = 25 \text{ kN}$.

Calculate $V_s = W_s / G_s \gamma_w = 100 / (2.7 \times 9.81) = 3.775 \text{ m}^3$.

Calculate $V_w = W_w / \gamma_w = 25 / 9.81 = 2.548 \text{ m}^3$.

Since $\gamma_t = 18.5 \text{ kN/m}^3 = (W_s + W_w) / (V_s + V_w + V_a) = (100 + 25) / (3.775 + 2.548 + V_a)$; thus, $V_a = 0.434 \text{ m}^3$.

Now, all components in the three phases are obtained as shown in Figure 2.7(a) and

$e = (V_w + V_a) / V_s = (2.548 + 0.434) / 3.775 = \mathbf{0.790}$. ←

$S = V_w / (V_w + V_a) = 2.548 / (2.548 + 0.434) = 0.854 = \mathbf{85.4\%}$. ←

SOLUTION (B)

First, assume $V = 10 \text{ m}^3$ as seen in Figure 2.7(b).

From $W_s + W_w = W_s + wW_s = (1 + w)W_s = V\gamma_t = 10 \times 18.5 = 185 \text{ kN}$;

$W_s = 185 / (1 + 0.25) = 148 \text{ kN}$, and $W_w = 185 - 148 = 37 \text{ kN}$.

Using G_s as a bridge value, $V_s = W_s / (G_s \gamma_w) = 148 / (2.7 \times 9.81) = 5.588 \text{ m}^3$.

Using γ_w as a bridge value, $V_w = W_w / \gamma_w = 37 / 9.81 = 3.772 \text{ m}^3$.

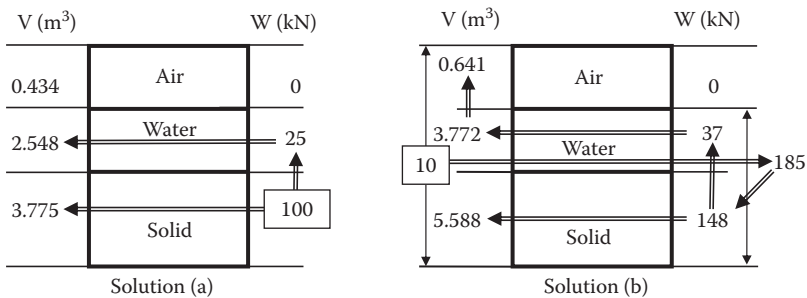


FIGURE 2.7 Exercise 2.2.

Thus, $V_a = V - (V_s + V_w) = 10 - (5.588 - 3.772) = 0.641 \text{ m}^3$.

Now, all components in the three phases are obtained as shown in Figure 2.7(b) and

$$e = (V_w + V_a)/V_s = (3.772 + 0.641)/5.588 = \mathbf{0.789}. \leftarrow$$

$$S = V_w/(V_w + V_a) = 3.772/(3.772 + 0.641) = 0.855 = \mathbf{85.5\%}. \leftarrow$$

In Solutions (a) and (b) of Exercise 2.2, the same results were obtained even though all components had different values. First assumed values ($W_s = 100 \text{ kN}$ or $V = 10 \text{ m}^3$ in the example) are arbitrary so that any convenient number can be assumed.

The three-phase diagram is also a convenient tool to handle many real-world problems that relate soil weight, water content, and volume. Exercise 2.3 shows one such problem.

Exercise 2.3

In a fill section of a construction site, 1500 m^3 of moist compacted soils is required. The design water content of the fill is 15%, and the design unit weight of the compacted soil is 18.5 kN/m^3 . Necessary soil is brought from a borrow site, with the soil having 12% natural water content, 17.5 kN/m^3 wet unit weight, and $G_s = 2.65$. How much (in cubic meters) of the borrow material is required to fill the construction fill section? And how heavy is it?

SOLUTION

Draw three-phase diagrams of the fill site and the borrow site in Figure 2.8(a) and (b), respectively.

First, for the fill site in Figure 2.8(a), $V = 1500 \text{ m}^3$, so that $W_s + W_w = V\gamma_t = 1500 \times 18.5 = 27750 \text{ kN}$.

$W_s + W_w = (1 + w)W_s = 27750 \text{ kN}$, so that $W_s = 27750/(1 + 0.15) = 24130 \text{ kN}$.

This much solid weight of the soil is required at the fill site.

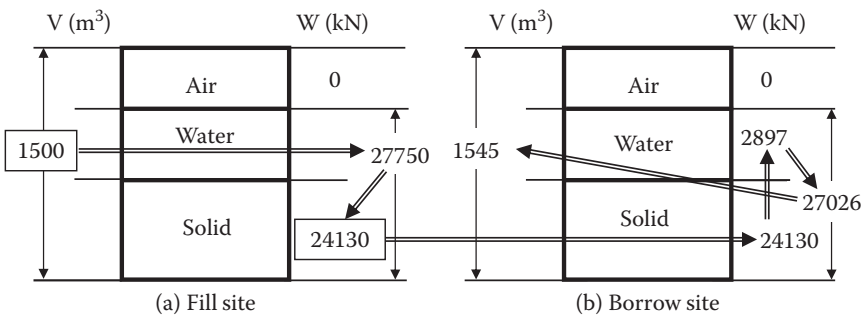


FIGURE 2.8 Exercise 2.3.

At the borrow site, the same solid weight 24130 kN is needed as shown in Figure 2.8(b).

Thus, $W_w = wW_s = 0.12 \times 24130 = 2896$ kN, and $W_s + W_w = 24130 + 2896 = 27026$ kN. ←

Since $\gamma_t = (W_s + W_w)/V = 17.5$ kN/m³, $V = 27026/17.5 = 1545$ m³. ←

Thus, 1545 m³ of the borrow material is needed for the project carrying a total weight of 27026 kN.

2.5 PARTICLE SIZE AND GRADATION

Particle size plays a dominant role in distinguishing soil types. Commonly used names of soil such as **gravel**, **sand**, **silt**, and **clay** are based on their grain sizes. Figure 2.9 shows those names with ranges of grain size. The boundary particle sizes are slightly different depending on the standards. 2.0 mm in **AASHTO** or 4.75 mm in **USCS (Unified Soil Classification System)** and in the **ASTM Soil Classification System** are the boundary particle sizes between gravel and sand. 75 μm (0.075 mm) is the boundary between sand and silt in both standards, and 5 μm is the one between silt and clay in AASHTO. In USCS (and also in ASTM), materials that are finer than 75 μm are called “**fine**.” Note that in some other standards, such as British Soil Classification (**BS8004 1986**), 2 μm is used as the boundary between silt and clay.

In order to separate grain sizes of soil assembly, a set of sieves is used for larger grain sizes. In particular, the boundary of 75 μm grain size is important; 75 μm is the opening size of a No. 200 sieve, which is practically the smallest size of sieves. Particles that are smaller than No. 200 sieve (**minus No. 200 material**) cannot be mechanically sieved easily due to developed static electricity on the surface of particles. If water is poured on dry minus No. 200 material, particles are easily suspended in the water and the water gets dirty. That is a good indication of an existence of minus No. 200 or “fine” material in it. Gravel and sand are called **cohesionless (granular) soils**, and clay is called **cohesive soils**. Silt is a transitional material between granular soils and cohesive soils. These two soil groups have distinguished differences in engineering behavior. Granular soils’ resistance upon shearing mostly comes from their surface friction and interlocking mechanisms. On the other hand, cohesive soils’ resistance comes from short-range particle-to-particle interactive forces, as will be discussed in Chapter 3. The former is less compressive than the latter, and the former has much higher capacity of water flow through it than the latter, etc.

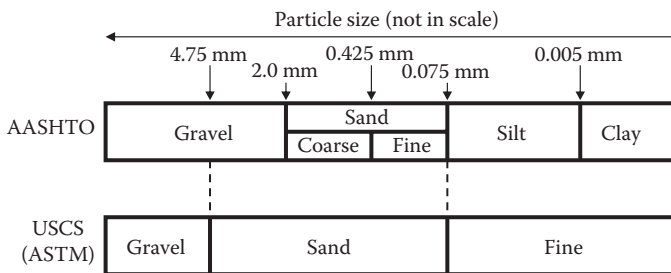


FIGURE 2.9 Soil names with grain sizes.

To identify grain size characteristics of soils, a **grain size distribution curve** is developed. First, **sieve analysis** is conducted. A variety of sieves with different openings are stacked, with the largest opening sieve on the top and smaller ones on the lower sections. The smallest (usually a No. 200 sieve) is placed at the second from the bottom and a pan with no opening at the bottom. Table 2.1 shows **US standard sieve numbers** and their corresponding openings.

Note that the sieve number is designated as the number of meshes in a 1 in. (25.4 mm) square spacing. For example, a No. 4 sieve's opening is calculated from 1 in. (25.4 mm) divided by 4 minus 4 wire thicknesses. An oven-dried specimen (ASTM D 422 and D 6913) with a known weight is placed on the top of the sieve stack, and a lid is placed on the top of the sieve. The whole stack of sieves with lid is vibrated vertically and horizontally until no more weight change in each sieve occurs (less than 1% change in 1 min shaking). A typical shaking period is approximately 8 to 10 min. After the shaking, weights of soils retained on each sieve are carefully measured on a balance to confirm that the initial weight and the summation of weights on each sieve after sieving are nearly equal.

Table 2.2 shows an example computation of the sieve analysis. The values in Column C of Table 2.2 are measured during the experiment, and the rest are completed by a spreadsheet setup as seen at the bottom of the table. The percentage finer implies that the percentage of the soil passes the corresponding sieve or that the percentage of soil weight is finer than the corresponding sieve opening. For example, in F(6) block data of Table 2.2, 63.6 imply that 63.6% of the soil passes through a No. 100 sieve, or 63.6% of soil is finer than 0.15 mm.

When a relatively large percentage passing through a No. 200 sieve (e.g., more than 10% as a guideline) is obtained from the sieve analysis, a **hydrometer analysis** is conducted. A **hydrometer** is a float with a bulb in the middle, as seen in Figure 2.10. Since larger (heavier) particles settle more quickly than the finer (lighter) ones in the suspension, the density of the suspension reduces with time. A hydrometer reading at the surface of the suspension reflects this density change around the bulb section. In theory, it assumes that the soil particles are spheres and that individual particles settle in the water solution with a certain velocity, which is a function of particle diameter and the time passing after the agitation.

TABLE 2.1
US Standard Sieve Numbers and Openings

US Standard Sieve No.	Opening (mm)
4	4.75
10	2.00
20	0.85
40	0.425
60	0.25
100	0.15
140	0.106
200	0.075

TABLE 2.2
Example Computation of Sieve Analysis

i	A US Standard Sieve No.	B Opening (mm)	C Weight Retained (gf)	D Weight Retained (%)	E Cumulative Retained (%)	F Percentage Finer
1	4	4.75	0	0.0	0.0	100
2	10	2.00	16.8	3.1	3.1	96.9
3	20	0.85	37.8	7.0	10.1	89.9
4	40	0.425	45.9	8.4	18.5	81.5
5	60	0.25	44.4	8.2	26.7	73.3
6	100	0.15	52.5	9.7	36.4	63.6
7	140	0.106	50.7	9.3	45.7	54.3
8	200	0.075	39.0	7.2	52.9	47.1
9	Pan		255.6	47.1	100	0
10		summation	542.7	100		

Notes: Column D(i) = C(i)/C(10) × 100.
 Column E(1) = D(1) and E(i) = E(i - 1) + D(i).
 Column F(i) = 100 - E(i).

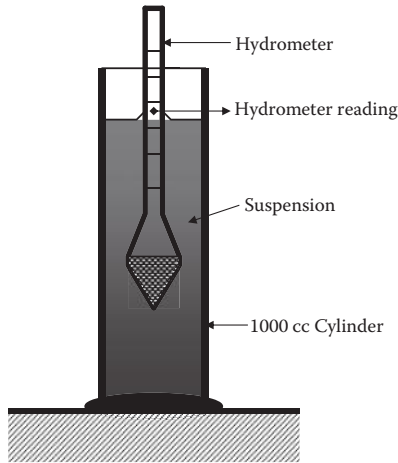


FIGURE 2.10 Hydrometer test setup.

Hydrometer analysis (**ASTM D 422**) is performed for the material collected in the pan (minus No. 200 material) during the sieve analysis. Take exactly 50 gram force (gf) of oven-dried, well-pulverized soil in a mixing beaker and thoroughly mix the soil with 125 cc of **Calgon solution** (or another **deflocculation agent**). Note that the purpose of the deflocculation agent is to change the chemical environment of the solution so that its clay structures start dispersing. This makes the lumped particles break down to individual particles for accurate particle size measurement. Detailed discussions on clay structures (flocculated or dispersed) will be given in Chapter 3.

Using distilled water, transfer the soil–water slurry completely into a 1000 cc hydrometer cylinder exactly to its 1000 cc mark. By using the palm of the hand over the open end of the cylinder (or with a rubber stopper), the cylinder is then turned upside down and back for 1 min for a full agitation of the suspension to get a fully mixed uniform suspension. At the end of 1 min agitation, place it carefully, but quickly, on a flat table. Set the time as zero when the cylinder is placed on the table and then insert the hydrometer immediately into the suspension. Read the hydrometer at 0.25, 0.5, 1, and 2 min while it is immersed in the suspension. Then the hydrometer is removed and immersed back at each reading of 4, 8, 16, and 30 min, 1, 2, 4, 8, and 24 h from the initial time. Record the exact times and the corresponding hydrometer readings.

This determines the relationship between the particle size and the corresponding percentage of weight settled. Refer to the details of hydrometer test procedure, theory, and computation in soil testing manuals such as ASTM D 422.

Columns A and B in Table 2.3 show a set of data obtained from a hydrometer test. Test data from the sieve analysis and hydrometer test are then combined (**combined grain size analysis**). The relationship between the sieve opening and the percentage finer are plotted in semi-log scale to generate the **grain size distribution curve** as shown in Figure 2.11. The data in Table 2.2 are plotted as Curve 1. The hydrometer test result from Columns A and B of Table 2.3 is also plotted as Curve 2. The latter data were obtained on the minus No. 200 material and are an enlarged curve of the minus No. 200 section of Curve 1. Thus, the vertical values of Curve 2 are proportionally reduced by multiplying by $F_{200(\text{Curve 1})}/F_{200(\text{Curve 2})}$ as shown in Figure 2.11, where $F_{200(\text{Curve 1})}$ is the percentage finer with the No. 200 sieve from the sieve analysis and $F_{200(\text{Curve 2})}$ is that from the hydrometer test. Column C in Table 2.3 shows those

TABLE 2.3
Example of Hydrometer Test Result

A	B	C
Particle Dia., D (mm)	Percentage Finer	Modified % Finer
0.066	84.5	45.7
0.045	74.3	40.2
0.036	68.3	37.0
0.025	58.2	31.5
0.015	48.4	26.2
0.011	42.3	22.9
0.007	34.6	18.7
0.005	28.1	15.2
0.004	24.3	13.2
0.003	20.1	10.9
0.0018	16.2	8.8
0.0012	12.3	6.7

Note: Column C = Column B $\times F_{200(\text{Curve 1})}/F_{200(\text{Curve 2})}$.

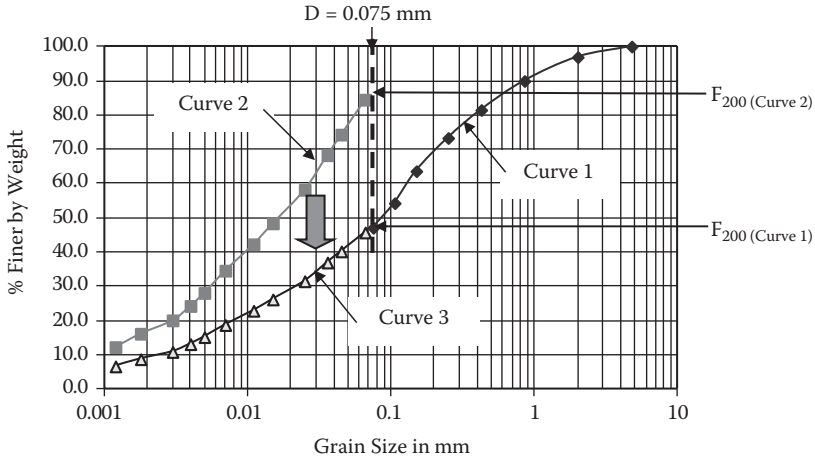


FIGURE 2.11 Combined grain size analysis.

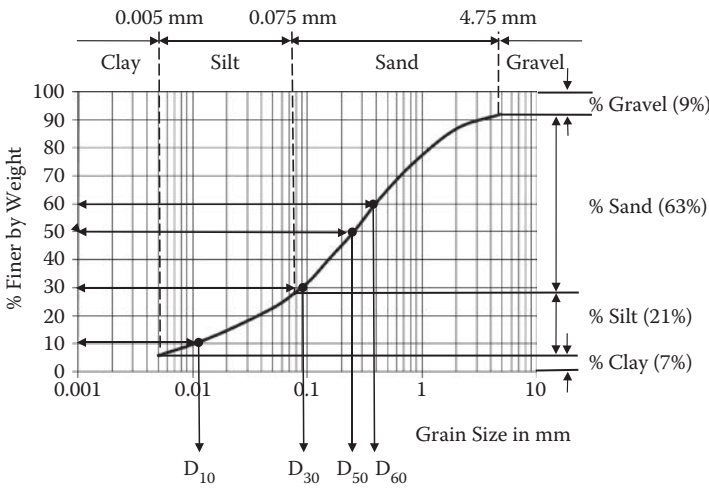


FIGURE 2.12 Grain size distribution curve.

modified percentage finer used in this combined analysis. The modified Curve 3 is considered as an extension of Curve 1 to cover the finer section ($D < 75 \mu\text{m}$) to complete the entire domain of the grain size distribution curve.

Referring to a grain size distribution curve in Figure 2.12, since the boundary grain sizes defining gravel, sand, silt, and clay were given earlier in Figure 2.9, the percentages of individual constituents of soil can be obtained as seen in the figure. In this example, 9% gravel, 63% sand, 21% silt, and 7% clay are obtained. Also, in the grain size distribution curve, several key grain sizes are utilized: D_{10} , D_{30} , D_{50} , and D_{60} , which are the grain sizes corresponding to 10%, 30%, 50%, and 60% finer by weight, respectively.

D_{50} is called the **mean grain size**, and D_{10} is called the **effective grain size**. The latter is the grain size at a finer portion of the soil assembly and is rather influential with such water flow characteristics as permeability (Chapter 6), capillary rise (Chapter 7), etc.; thus, it is called the effective grain size.

The **coefficient of uniformity** C_u is defined as

$$C_u = D_{60} / D_{10} \quad (2.19)$$

Figure 2.13 shows a variety of grain size distribution curves. Curves 1, 2, and 3 have different C_u values (2.0, 4.7, and 13, respectively). Curve 1 soil is a **uniformly graded** (or **poorly graded**) soil, while Curve 3 soil is a **well-graded soil**. In the Unified Soil Classification System, C_u less than 4 for gravels or C_u less than 6 for sands is classified as **uniformly graded soils**, and those with higher than 4 for gravels or higher than 6 for sands are classified as **well-graded soils**.

Coefficient of gradation C_g is defined from the gradation curve as

$$C_g = \frac{D_{30} / D_{10}}{D_{60} / D_{30}} = \frac{(D_{30})^2}{D_{60} D_{10}} \quad (2.20)$$

For smooth gradation curves, the range of C_g values is between 1 and 3. Curve 4 in Figure 2.13 shows a rather low C_g value (= 0.29) in comparison with other soils (e.g., $C_g = 1.33$ for Curve 2). Soils with $1 < C_g < 3$ are considered **well-graded soils** as long as $C_u > 4$ for gravels or $C_u > 6$ for sands, according to the USCS. On the other hand, soils with $C_g > 3$ or $C_g < 1$ are called **gap-graded soils**.

The coefficient of uniformity and the coefficient of gradation affect the soil-packing arrangement. Well-graded soils make more stable packing since finer particles fill voids made by larger particle assemblages. On the other hand, uniformly graded soils make rather ordinary arrangement of packing and thus less interlocking mechanisms. Soil classification systems use C_u and C_g as key parameters in their methods.

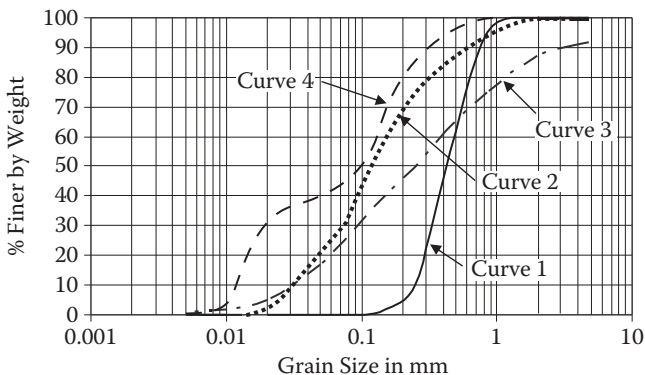


FIGURE 2.13 Various grain size distribution curves.

2.6 SUMMARY

In this chapter, starting with the origin of soils, soil formation processes were studied. Soil was then modeled by the three phases (solid, water, and air), and key definitions were made on unit weight (γ), void ratio (e), porosity (n), water content (w), degree of saturation (S), specific gravity (G_s), etc., based on the three-phase diagram. These terms are used throughout the book hereafter. Some of their interrelationships are derived for convenient uses later. It was also demonstrated that the three-phase diagram is a convenient tool to solve field problems with volume and weight determinations in the exercises. At the end, grain shape, size, and distribution were discussed. From a grain size distribution curve, several key parameters can be obtained, such as the effective grain size (D_{10}), the mean grain size (D_{50}), the coefficient of uniformity (C_u), and the coefficient of gradation (C_g). Those parameters will be used in soil classification practices in Chapter 4 and also will be correlated with many engineering properties of soils such as in compaction (Chapter 5), permeability (Chapter 6), etc.

REFERENCES

- ASTM (2005), Standard Test Method for Particle-Size Distribution (Gradation) of Soils Using Sieve Analysis, *Annual Book of ASTM Standards*, vol. 04.09, Designations D 6913–04.
- ASTM (2012), Standard Test Method for Particle-Size Analysis of Soils, *Annual Book of ASTM Standards*, vol. 04.08, Designations D 422–63.
- BS8004 (1986), *Code of Practice for Site Investigation*, British Standard Institution, London.
- Müller, G. (1967), *Methods in Sedimentary Petrology*, Hafner, New York and London.
- Watabe, Y., Tanaka, M., and Takemura, J. (2004), Evaluation of in-situ K_0 for Ariake, Bangkok and Hai-Phong clays, *Proceedings of the 2nd International Conference on Site Characterization*, Porto, 1765–1772.

Problems

- 2.1 For a given soil, derive the following relation by drawing the three-phase diagram:

$$\gamma_t = G_s \gamma_w (1 - n) (1 + w)$$

- 2.2 For a given soil, derive the following relation by drawing the three-phase diagram:

$$\gamma_t = G_s \gamma_w (1 - n) + n S \gamma_w$$

- 2.3 For an organic soil, the void ratio e is found to be 10.0, and G_s is 2.35. If this soil is fully saturated, find:
- Total unit weight of the soil γ_t
 - Water content w
 - Does this soil sink in water?

- 2.4 For a given soil, the void ratio e , water content w , and specific gravity G_s are found to be 0.50, 15%, and 2.65, respectively. Find:
- Total unit weight of the soil γ_t
 - Degree of saturation S
 - Dry unit weight γ_d if the water in the void is removed
- 2.5 For a given soil, $G_s = 2.70$, $\gamma_t = 19.0 \text{ kN/m}^3$, and $w = 12.5\%$ were measured. Determine:
- Degree of saturation S
 - Dry unit weight of the soil γ_d
 - Submerged unit weight of the soil γ'
 - Total unit weight of the soil γ_t if the air void is filled with water
- 2.6 The dry unit weight of a soil is found to be 15.8 kN/m^3 and its porosity $n = 0.40$. Determine:
- The total unit weight of the soil γ_t when the soil's degree of saturation S is increased to 50%
 - The total unit weight of the soil γ_t when the soil is fully saturated
 - The specific gravity G_s of this soil
- 2.7 Soil collected from the site is found to have $\gamma_t = 18.5 \text{ kN/m}^3$, $w = 8.6\%$, and $G_s = 2.67$. After a heavy rainfall overnight, 10% increase in the degree of saturation S was observed. Determine:
- The degree of saturation S of the soil before the rainfall
 - The void ratio e of the soil before the rainfall
 - The water content w after 10% increase in S
 - The total unit weight γ_t after 10% increase in S
- 2.8 In a construction site, 100 m^3 of the volume is excavated. γ_t , G_s , and w of the excavated soil are 18.5 kN/m^3 , 2.68, and 8.2%, respectively.
- How heavy is the whole excavated soil?
 - What is the porosity of the soil?
 - If the excavated soil is dried out to 5% water content at the site, how heavy does it become?
- 2.9 The following table shows a data set from a sieve analysis.
- Complete the rest of the table using a spreadsheet as in Table 2.2.
 - Plot the grain size distribution curve.
 - Determine D_{10} , D_{30} , D_{50} , and D_{60} .
 - Compute C_u and C_g .
 - Report the percentage gravel, sand, silt, and clay according to AASHTO.

US Sieve No.	Opening (mm)	Weight Retained (gf)
4	4.75	135.9
10	2.0	97.5

Continued

US Sieve No.	Opening (mm)	Weight Retained (gf)
20	0.85	108
40	0.425	67.8
60	0.25	41.4
100	0.15	15
140	0.106	0
200	0.075	0
Pan		0

2.10 The following table shows a data set from a sieve analysis.

- Complete the rest of the table using a spreadsheet as in Table 2.2.
- Plot the grain size distribution curve.
- Determine D_{10} , D_{30} , D_{50} , and D_{60} .
- Compute C_u and C_g .
- Report the percentage gravel, sand, silt, and clay according to AASHTO.

US Sieve No.	Opening (mm)	Weight Retained (gf)
4	4.75	16.8
10	2.0	38.4
20	0.85	54.9
40	0.425	67.8
60	0.25	101.7
100	0.15	94.2
140	0.106	77.4
200	0.075	61.8
Pan		70.5

2.11 The following table shows a data set from a sieve analysis.

- Complete the rest of the table using a spreadsheet as in Table 2.2.
- Plot the grain size distribution curve.
- Determine D_{10} , D_{30} , D_{50} , and D_{60} .
- Compute C_u and C_g .
- Report the percentage gravel, sand, silt, and clay according to AASHTO.

US Sieve No.	Opening (mm)	Weight Retained (gf)
4	4.75	0
10	2.0	0
20	0.85	6.9
40	0.425	71.7
60	0.25	109.2

Continued

US Sieve No.	Opening (mm)	Weight Retained (gf)
100	0.15	126.9
140	0.106	147.6
200	0.075	115.8
Pan		110.7

2.12 The following table shows a data set from a sieve analysis.

- Complete the rest of the table using a spreadsheet as in Table 2.2.
- Plot the grain size distribution curve.
- Determine D_{10} , D_{30} , D_{50} , and D_{60} .
- Compute C_u and C_g .
- Report the percentage gravel, sand, silt, and clay according to AASHTO.

US Sieve No.	Opening (mm)	Weight Retained (gf)
4	4.75	15.6
10	2.0	35.4
20	0.85	121.8
40	0.425	102.3
60	0.25	82.8
100	0.15	50.4
140	0.106	37.8
200	0.075	30.6
Pan		56.7

2.13 The following table shows the sieve analysis data on the left and hydrometer test data on the right for the minus No. 200 sieve material for a given soil.

- Plot grain size distribution curves for both tests individually on a graph.
- Combine two curves into a combined grain size distribution curve for the soil.

Sieve Analysis			Hydrometer Analysis	
US Sieve No.	Opening (mm)	Weight Retained (gf)	Particle Diameter (mm)	Percentage Finer
4	4.75	0	0.072	78.2
10	2.0	0	0.046	58.2
20	0.85	0	0.034	50.4
40	0.425	13.5	0.026	42.8
60	0.25	45.3	0.017	38.1

Continued

Sieve Analysis			Hydrometer Analysis	
US Sieve No.	Opening (mm)	Weight Retained (gf)	Particle Diameter (mm)	Percentage Finer
100	0.15	75.4	0.012	35.4
140	0.106	147.6	0.007	30.2
200	0.075	168.2	0.004	25.7
Pan		230.5	0.0032	22.9
			0.0024	20.9
			0.0017	18.2
			0.0012	14.5

2.14 The following table shows the sieve analysis data on the left and a hydrometer test data on the right for the minus No. 200 sieve material for a given soil.

- Plot grain size distribution curves for both tests individually on a graph.
- Combine two curves into a combined grain size distribution curve for the soil.

Sieve Analysis			Hydrometer Analysis	
US Sieve No.	Opening (mm)	Weight Retained (gf)	Particle Diameter (mm)	Percentage Finer
4	4.75	0	0.071	67.8
10	2.0	0	0.05	57.2
20	0.85	11.2	0.03	48.2
100	0.15	81.3	0.011	37.1
140	0.106	189.3	0.0072	35.2
200	0.075	152.1	0.0046	31.5
Pan		280.3	0.0035	30.2
			0.0025	29.1
			0.0016	27.2
			0.0012	26.2

Chapter 3 Soil Classification

4.1 INTRODUCTION

Soils are all different, depending on their origins, compositions, locations, geological histories, and many other factors. Two soils may be quite different, even though they were obtained from nearby boring holes on the same construction site. And thus, in-situ and laboratory tests on soil specimens are critically important to obtain their index parameters and engineering characteristics. However, it is more convenient for engineers when soils are categorized into several groups with similar engineering behaviors. Engineers can understand approximate engineering characteristics of those grouped soils without actual laboratory or field tests. This process is called **soil classification**, and it helps engineers in the preliminary design stage of geotechnical engineering problems.

Most soil classification standards use soil indices such as Atterberg limits (liquid limit, plastic limit), soil gradation information (D_{10} , D_{50} , C_u , C_g), etc. In current geotechnical engineering practice, two standards are widely used in the United States: the **Unified Soil Classification System (USCS)** and **AASHTO** (American Association of State Highway and Transportation Officials) methods.

4.2 UNIFIED SOIL CLASSIFICATION SYSTEM (USCS)

First developed by Arthur Casagrande for wartime airfield construction in 1942, the system was modified and adopted for regular use by the Army Corps of Engineers and then by the Bureau of Reclamation in 1952 as the **Unified Soil Classification System (Casagrande 1948)**. Currently, it is adapted in ASTM (designation D 2487) and periodically updated. This is the most widely used classification system by geotechnical engineers.

The system uses six simple major symbols and four modifiers as in the following:

Major symbols:

- G** Gravel
- S** Sand
- M** Silt (Swedish word = *mjåla*)
- C** Clay
- O** Organic
- Pt** Peat

Modifiers:

- W** Well graded (for gravel and sand)
- P** Poorly graded (for gravel and sand)
- H** High plasticity (for silt, clay, and organic soils)
- L** Low plasticity (for silt, clay, and organic soils)

Classified group names are combinations of these characters—for example, GP for poorly graded gravel, SW for well-graded sand, CH for high-plasticity clay, SM for silty sand, etc. And thus, GW, GP, GM, and GC are possible group names for gravelly soil; SW, SP, SM, and SC are for sandy soils; MH and ML are for silty soils; CH and CL are for clayey soils; OH and OL are for organic soils; and Pt stands alone for peat. Dual naming is also possible for several boundary soils, such as GW–GM (well-graded gravel with silt), GC–GM (silty clayey gravel), SW–SM (well-graded sand with silt), etc.

This system uses LL, PL, and PI (= LL – PL), and soils’ gradation information. First, from a grain size distribution curve, the percentages of each component (gravel [$d \geq 4.75$ mm], sand [$4.75 \text{ mm} > d > 0.075$ mm], and fine [$d \leq 0.075$ mm]) are identified as shown in Figure 4.1.

From this, the values of F_{200} , R_{200} , F_4 , and R_4 are obtained as

- F_{200} : % finer than No. 200 sieve (0.075 mm) = percentage of fine content
- R_{200} : percentage retained on No. 200 sieve (0.075 mm) = percentage of sand and gravel content
- F_4 : percentage finer than No. 4 sieve (4.75 mm) = percentage of sand and fine content
- R_4 : percentage retained on No. 4 sieve (4.75 mm) = percentage of gravel content

Note that in USCS, clay and silt are categorized as **fine**.

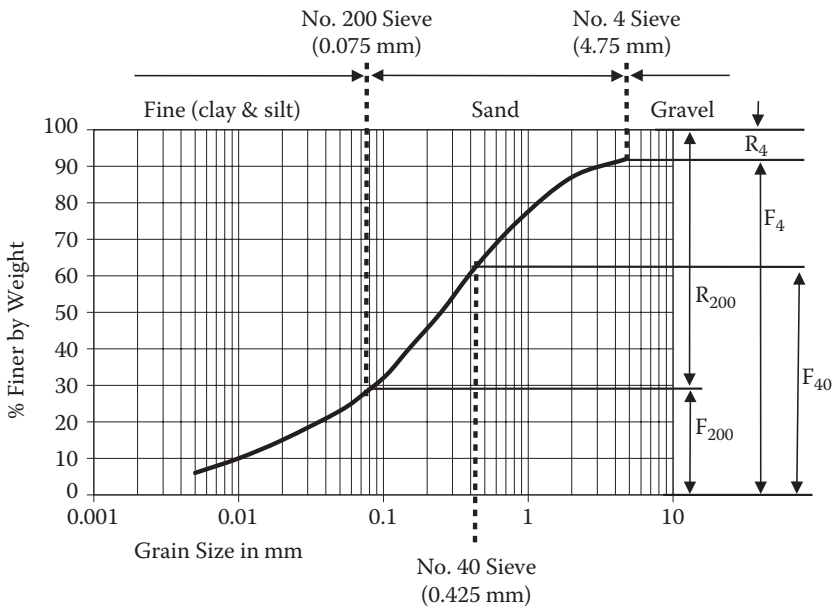


FIGURE 4.1 Definitions of F_{200} , R_{200} , F_4 , and R_4 .

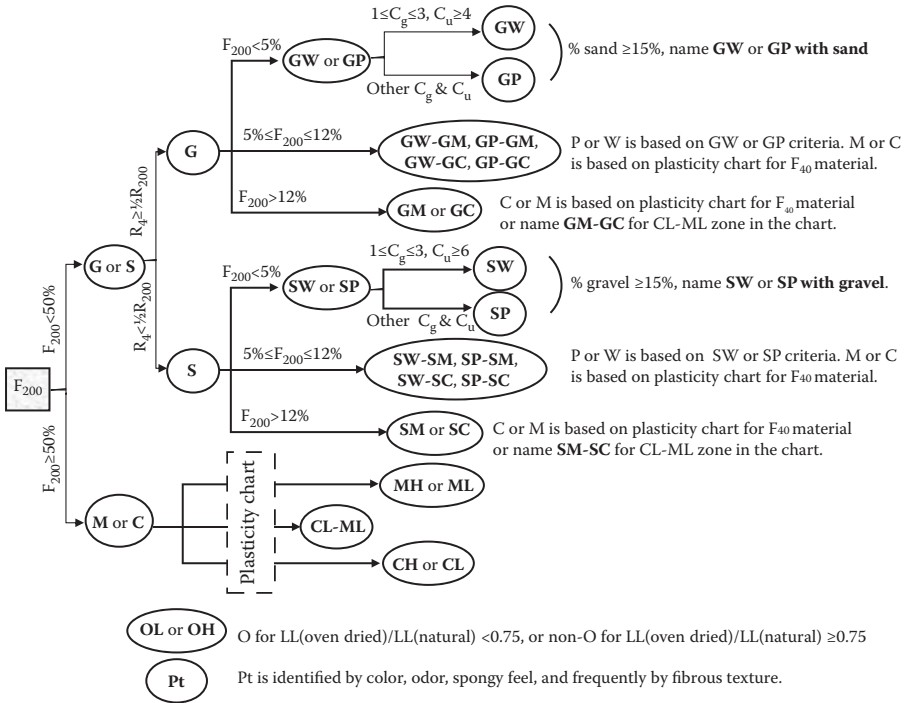


FIGURE 4.2 Flow chart for USCS.

Next, the coefficient of uniformity $C_u (= D_{60}/D_{10})$ and the coefficient of gradation $C_g (= (D_{30})^2/(D_{60} \times D_{10}))$ are calculated.

Based on these values, the classification procedure is summarized in a flow chart in Figure 4.2. The chart starts from F_{200} information as far left in the following steps.

4.2.1 FOR G OR S

1. If $F_{200} < 50\%$ (or $R_{200} \geq 50\%$, i.e., gravel and sand content is more than 50%), then soil is **G** or **S**.
2. Then, if $R_4 \geq \frac{1}{2}F_{200}$ (gravel content \geq sand content), it is **G**, or if $R_4 < \frac{1}{2}F_{200}$ (gravel content < sand content), it is **S**.
3. In the next step, F_{200} (fine content) is checked for **G** and **S**. If $F_{200} < 5\%$, naming fine content is ignored and soils will be **GW**, **GP**, **SW**, or **SP**. If $F_{200} > 12\%$, soils will be **GM**, **GC**, **SM**, or **SC**. When $5\% \leq F_{200} \leq 12\%$, double naming comes in as **GW-GM**, **GW-GC**, **GP-GM**, and **GP-GC** for gravel or **SW-SM**, **SW-SC**, **SP-SM**, and **SP-SC** for sand. In such double naming cases, modifier **M** or **C** is determined based on a **plasticity chart** (see Section 4.2.2) for F_{40} materials.

4. In the final step for gravel and sand, C_u and $C_{g\%}$ values are evaluated for modifiers **W** or **P**. For gravel, $C_u \geq 4$ and $1 \leq C_{g\%} \leq 3$ are conditions for **W** and the other values of those are for **P**. For sand, the condition for **W** is $C_u \geq 6$ and $1 \leq C_{g\%} \leq 3$ and the other values are for **P**.
5. For **GW** and **GP** soils, if percentage of sand content is at or more than 15%, it is named as **GW (or GP) with sand**.
6. Similarly, for **SW** and **SP** soils, if percentage of gravel content is at or more than 15%, it is named as **SW (or SP) with gravel**.

4.2.2 FOR C, M, O, OR Pt

1. Going back to the F_{200} value in Figure 4.2, if $F_{200} \geq 50\%$ (i.e., fine contents are at or more than 50%), then soil is either **M** or **C** (or possibly **O** or **Pt**).
2. To classify **M** or **C**, **plasticity chart** (Figure 4.3) is used. It utilizes **LL** and **PI** ($= LL - PL$) values. **LL** and **PL** tests should be performed on the F_{40} specimen (soil passed No. 40 sieve—0.425 mm), and the **LL** and **PI** data point of the soil tested is plotted on the plasticity chart to identify soil type (**CH**, **CL**, **MH**, **ML**, or **CL-ML**) by the zone on which the data point falls.
3. On the plasticity chart, most natural soils fall below the “**U**” line and around the “**A**” line or **CL-ML** zone. It should be noted that $LL = 50$ is the boundary **LL** for high-plastic ($LL > 50$) or low-plastic ($LL < 50$) soils.
4. The **M** or **C** classification method by the plasticity chart is also used in subgroup names in gravel and sand category soils with their fine contents between 5% and 12%. These are **GM**, **GC**, **SM**, **SC**, and dual named soils **GW-GM**, **GW-GC**, **GP-GM**, and **GP-GC** for gravel or **SW-SM**, **SW-SC**, **SP-SM**, and **SP-SC** for sand. Note that, for those gravels and sands, only the F_{40} specimen is used for **LL** and **PL** tests as mentioned before.

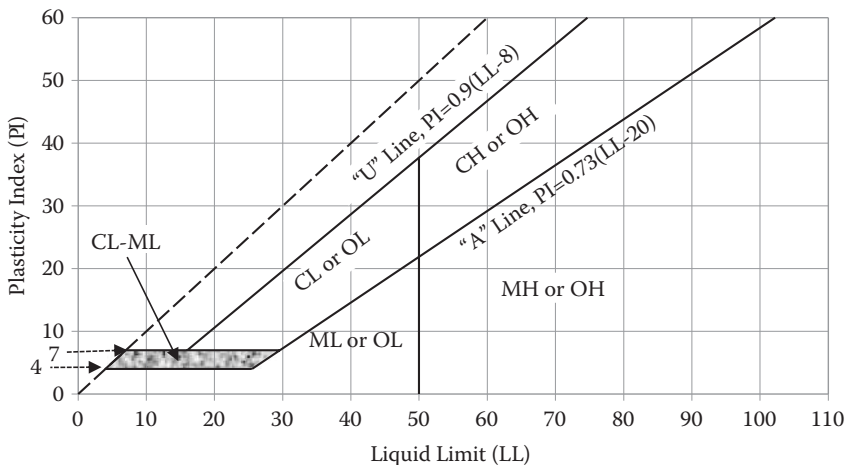


FIGURE 4.3 Plasticity chart for USCS.

5. **Pt (peat)** should be identified by its color, odor, spongy feeling, and, frequently, by its fibrous texture by testing engineers. **O (organic soil)** can be identified by observing the change in LL values from natural soil to oven-dried (burns some organic) soil. If $LL(\text{oven dried})/LL(\text{natural}) < 0.75$, it is classified as O. If the ratio is ≥ 0.75 , it is non-organic.

Since USCS uses simple symbols with their meanings, it is easy to understand the nature of soils from classified group names. Also, this requires only LL and PL tests and sieve analysis. A hydrometer test is not required since the silt and clay are treated as **fine** and it uses the plasticity chart to identify the clay and silt. USCS and its modified versions are the most widely used soil classification system around the world today. Engineers provided useful general guidelines for those classified soils in different applications, such as roadway construction, compaction practice, etc. Table 4.1 shows such an example for roads and airfields.

4.3 AASHTO CLASSIFICATION SYSTEM

AASHTO soil classification was developed in the late 1920s by the US Bureau of Public Roads (now the Federal Highway Administration) for road constructions. The current version, which was revised in 1945, is used for extended applications in road bases, sub-bases, subgrades, and embankment constructions (*AASHTO 1995*). As a reference, the terminology of road construction materials such as **base**, **sub-base**, **subgrade**, etc. is shown in Figure 4.4 for typical rigid (concrete) and flexible (asphalt cement) pavement systems.

The AASHTO method uses Atterberg limits (LL and PL), and information on grain size distribution curve (F_{10} , F_{40} , and F_{200}), which are the percentage passing on No. 10 sieve, No. 40 sieve, and No. 200 sieve, respectively. The procedure uses an elimination process of columns in Table 4.2, from the upper left corner (F_{10}) toward downward and right. If the condition on the row is not satisfied, the entire column is eliminated and it is never referred back. After the last row check for PI, one or possibly more than one column may survive this elimination process.

If more than one column survived, the first column from the left is selected as a group or subgroup name. The group names are A-1 through A-7, with some subgroups. In general, the left-side group is better than the right side one for use as roadway construction materials.

In addition, **group index (GI)**, as defined in the following, should be calculated and reported in the AASHTO system:

$$GI = (F_{200} - 35) [0.2 + 0.005(LL - 40)] + 0.01(F_{200} - 15)(PI - 10) \quad (4.1)$$

There are some rules in Equation (4.1):

1. When GI is calculated as negative values, report $GI = 0$.
2. GI is reported in rounded integer numbers. For example, $GI = 4.4$ should be reported as 4 and $GI = 4.5$ should be reported as 5.

TABLE 4.1
USCS Group and Its Relation to Various Engineering Properties

Symbols	Value as Sub-base or Subgrade	Value as Base Course	Potential Frost Action	Compressibility and Expansion	Drainage Characteristics	Dry Unit Weight pcf	Field CBR	Subgrade Modulus K, pci ^d
GW	Excellent	Good	None to very slight	Almost none	Excellent	125–140	60–80	>300
GP	Good to excellent	Poor to fair	None to very slight	Almost none	Excellent	110–130	25–60	>300
GM	d ^a Good to excellent	Fair to good	Slight to medium	Very slight	Fair to good	130–145	40–80	>300
GC	u ^b Good	Poor	Slight to medium	Slight	Poor to impervious	120–140	20–40	200–300
SW	Fair to good	Poor	Slight to medium	Slight	Poor to impervious	120–140	20–40	200–300
SP	Good	Poor	None to very slight	Almost none	Excellent	110–130	20–40	200–300
SM	Fair to good	Poor to not suitable	None to very slight	Almost none	Excellent	100–120	10–25	200–300
SC	Good	Poor	Slight to high	Very slight	Fair to poor	120–135	20–40	200–300
ML	Fair to good	Not suitable	Slight to high	Slight to medium	Poor to impervious	105–130	10–20	200–300
CL	Fair to good	Not suitable	Slight to high	Slight to medium	Poor to impervious	105–130	10–20	200–300
OL	Fair to poor	Not suitable	Medium to very high	Slight to medium	Fair to poor	100–125	5–15	100–200
MH	Fair to poor	Not suitable	Medium to high	Medium	Impervious	100–125	5–15	100–200
CH	Poor	Not suitable	Medium to high	Medium to high	Poor	90–105	4–8	100–200
OH	Poor	Not suitable	Medium to very high	High	Fair to poor	80–100	4–8	100–200
Pt	Poor to very poor	Not suitable	Medium	High	Impervious	90–110	3–5	50–100
	Poor to very poor	Not suitable	Medium	High	Impervious	80–105	3–5	50–100
	Not suitable	Not suitable	Slight	Very high	Fair to poor	—	—	—

Source: After *US Air Force Engineering Support Agency/Civil Engineering Squad (AFCESA/CES), 1997*, Criteria and Guidance for C-17 Contingency and Training on Semi-Prepared Airfields, *Engineering Technical Letter*, 97–99.

^a d for LL ≤ 28 and PI ≤ 6.

^b u for LL > 28.

^c 1 pcf (lb/ft³) = 0.1572 kN/m³.

^d 1 pci (lb/in.²) = 271.43 kN/m².

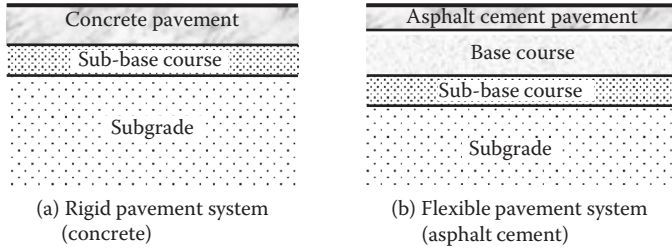


FIGURE 4.4 Typical road pavement systems.

3. For A-2-6 and A-2-7 subgroups, use only the second term of Equation (4.1) and assign the first term always as zero, that is,

$$GI = 0.01(F_{200} - 15)(PI - 10) \quad (4.2)$$

The standard says that under average conditions of good drainage and thorough compaction processes, the supporting value of a material as subgrade may be assumed as an inverse ratio to the group index; that is, a group index of 0 indicates a “good” subgrade material and a group index of 20 or greater indicates a “very poor” subgrade material.

Exercise 4.1

A soil gradation curve is shown in Figure 4.5. Classify the soil (a) by USCS, and (b) by AASHTO classification methods. LL = 46% and PL = 35% were obtained for F_{40} material of the specimen.

SOLUTION

From the gradation curve, the following values can be read.

Percentage passing No. 4 (4.75 mm) = 92%
 Percentage passing No. 10 (2.0 mm) = 87%
 Percentage passing No. 40 (0.425 mm) = 63%
 Percentage passing No. 200 (0.075 mm) = 28%
 $F_{200} = 28\%$, and thus $R_{200} = 72\%$
 $F_4 = 92\%$, and thus $R_4 = 8\%$
 $D_{10} = 0.01$ mm
 $D_{30} = 0.090$ mm
 $D_{60} = 0.39$ mm
 $C_u = D_{60}/D_{10} = 0.39/0.01 = 39$
 $C_g = (D_{30})^2/(D_{60} \times D_{10}) = (0.090)^2/(0.01 \times 0.39) = 2.08$

and

LL = 46
 PI = 46 – 35 = 11

Dr Mohamed Abd Eltatief

TABLE 4.2
Classification of Soils and Soil Aggregate Mixtures

General Classification	Granular Materials (35% or Less Passing 0.075 mm)			Silt-Clay Materials (More than 35% Passing 0.075 mm)		
	A-1	A-2	A-3	A-4	A-5	A-6
Group Classification	A-1-a A-1-b	A-2-4 A-2-5 A-2-6 A-2-7	A-3	A-4	A-5	A-6 A-7-5 ^a A-7-6 ^b
Sieve analysis, percentage passing						
No. 10 (2.00 mm)	—	—	—	—	—	—
No. 40 (0.425 mm)	50 max 30 max	—	51 min	—	—	—
No. 200 (0.075 mm)	15 max 25 max	35 max 35 max	10 max	36 min	36 min	36 min
For F ₄₀ materials						
Liquid limit	—	40 max	—	40 max	41 min	40 max 41 min
Plasticity index	6 max	10 max 11 min	N.P.	10 max 11 min	10 max 11 min	11 min 11 min
Usual types of significant constituent materials	Stone fragments, gravel, and sand	Silty or clayey gravel and sand	Fine sand	Silty soils	Silty soils	Clayey soils
General ratings as subgrade	Excellent to good			Fair to poor		

Source: After **AAASHTO, 1995**.

^a For A-7-5, $PI \leq LL - 30$.

^b For A-7-6, $PI > LL - 30$.

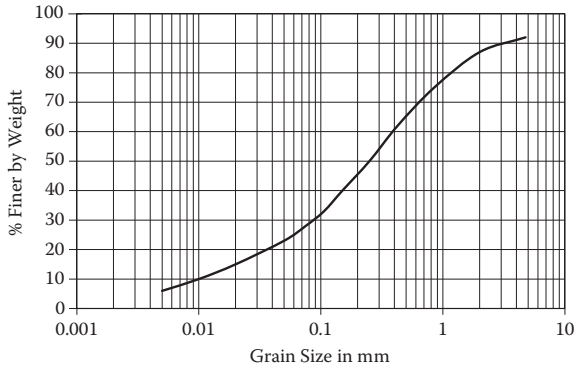


FIGURE 4.5 Gradation curve for Exercise 4.1.

(A) USCS METHOD

By using the flow chart in Figure 4.2, since F_{200} (28%) < 50%, it should be G or S.

R_4 (= 8%) < $\frac{1}{2}R_{200}$ (= 72%) = 36%, and thus it should be S.

F_{200} (= 28%) > 12% and it should be SM or SC.

LL (= 46) and PL (= 11) fall in the region of ML or OL in the plasticity chart (Figure 4.3).

Thus, the soil is classified as **SM (silty sand)**. ←

(B) AASHTO METHOD

By using the elimination process in Table 4.2, from top left,

(Percentage passing No. 10 = 87%) eliminates A-1-a

(Percentage passing No. 40 = 63%) eliminates A-1-b, but A-3 survives

(Percentage passing No. 200 = 28%) eliminates A-3, A-4, A-5, A-6, and A-7

LL = 46 eliminates A-2-4 and A-2-6

PI = 11 eliminates A-2-5

And thus the survived subgroup is **A-2-7 (silty or clayey gravel and sand)**. ←

Since it is A-2-7, Equation (4.2) is used for group index computation.

$GI = 0.01(F_{200} - 15)(PI - 10) = 0.01(28 - 15)(11 - 10) = 0.13 \rightarrow 0$ (rounded integer). **GI = 0**

And thus, this soil is classified as **A-2-7 (GI = 0)**. ←

4.4 SUMMARY

Widely used soil classification systems, namely, USCS and AASHTO methods, were presented in this chapter. Classification systems provide general guidelines of soil types based on the results of rather simple sieve analysis and Atterberg limits tests. Based on the accumulated data for many years, many convenient relationships between classified soil groups and many engineering properties have been prepared. Table 4.1 is such an example. Geotechnical engineers could use those properties for the primary phase of engineering design and analysis if

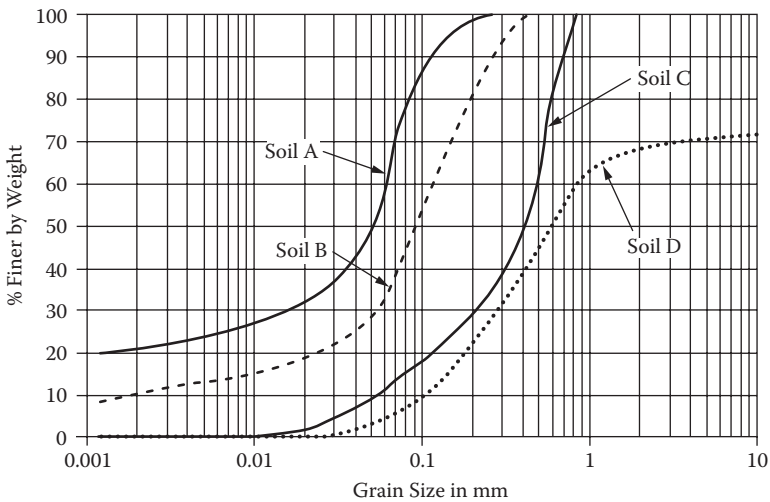
needed. For detailed and later design phases, however, undisturbed specimens should be tested in the field (in-situ test), or sampled and tested in laboratories to obtain more reliable soils' properties and engineering design values.

REFERENCES

AASHTO (1995), *Standard Specifications for Transportation Materials and Methods of Sampling and Testing*, 17th ed., Part I Specifications, Designation M 145.
 Casagrande, A. (1948), Classification and Identification of Soils, *Transactions of ASCE*, vol. 113, 901–991.
 US Air Force Engineering Support Agency/Civil Engineering Squad (AFCESA/CES) (1997), Criteria and Guidance for C-17 Contingency and Training on Semi-Prepared Airfields, *Engineering Technical Letter*, 97–99.

Problems

4.1–4.4 The following figure shows grain size distribution curves for soils A, B, C, and D with their LL and PL values.

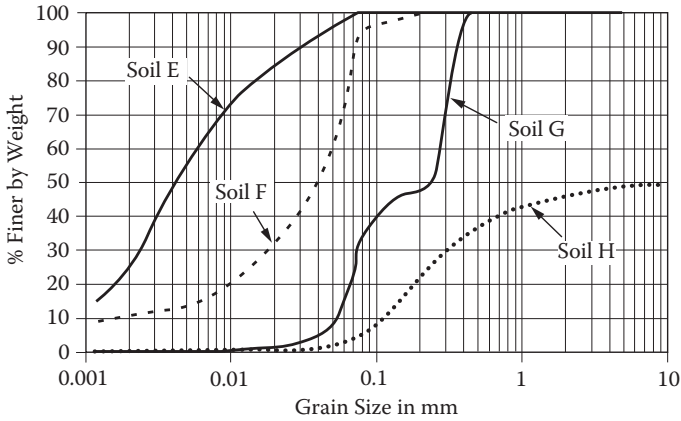


For each soil,

- (a) Classify the soil according to USCS.
- (b) Classify the soil according to AASHTO including GI computation.
- (c) Discuss the suitability of the soil as subgrade material.

Problem	Soil	LL	PL
4.1	A	55	25
4.2	B	45	26
4.3	C	25	19
4.4	D	42	33

4.5–4.8 The following figure shows grain size distribution curves for soils E, F, G, and H with their LL and PL values.



For each soil,

- (a) Classify the soil according to USCS.
- (b) Classify the soil according to AASHTO including GI computation.
- (c) Discuss the suitability of the soil as subgrade material.

Problem	Soil	LL	PL
4.5	E	55	27
4.6	F	43	22
4.7	G	46	28
4.8	H	41	32

Dr Mohamed Abd Elatif

Chapter 4 Soil Compaction

5.1 INTRODUCTION

The soil's footing supporting capacity (bearing capacity), settlement, shear strength, etc. all depend on how well the soil is compacted. Compaction increases the soil's strength and decreases compressibility and permeability. It may also control characteristics of swelling and shrinkage and frost susceptibility. Compaction is a physical process to decrease the voids of soil by static or dynamic loading. For example, granular soils are easily compacted by vibration, while saturated cohesive soils cannot be well compacted by dynamic loads because of viscous resistance of pore water pressure to transient loadings.

Compaction characteristics are first discussed based on the laboratory compaction test. Then compaction specification in the field, field compaction techniques, and field inspection methods are discussed. Related subjects, such as relative density and California Bearing Ratio (CBR), are also presented in this chapter.

5.2 RELATIVE DENSITY

It is important to know the level of compaction that has been achieved on earth works or on existing earth. Soil's unit weight value (either dry or wet condition) alone cannot tell its compaction level since the ranges of unit weight vary depending on the type of soil. For example, well-graded, gravelly soil may have unit weight ranging from 18 to 20 kN/m³ (or 115 to 127 lb/ft³). On the other hand, for soils with more cohesive materials, the range may be 15 to 18 kN/m³ (or 96 to 115 lb/ft³). In order to indicate the level of compaction relative to the densest and the loosest compaction level for a given specific soil, mostly for granular soils, **relative density** (D_r) is introduced and is defined in the following equation:

$$D_r = \frac{e_{\max} - e}{e_{\max} - e_{\min}} \times (100\%) \quad (5.1)$$

where e_{\max} , e_{\min} , and e are the **maximum**, **minimum**, and in-situ soil's **void ratios**, respectively. When the in-situ soil's void ratio is in its loosest ($e = e_{\max}$) state, then, $D_r = 0\%$. If it is in its densest ($e = e_{\min}$) state, $D_r = 100\%$. D_r values of most in-situ soils are between 0% and 100%. Table 5.1 shows some guidelines of relationships among the relative densities, level of denseness, SPT (standard penetration test) N_{60} value, and the soil's effective angle of internal friction ϕ' (Chapter 11). As will be discussed in Chapter 13, the **N_{60} value** is the modified SPT blow count adjusted to 60% hammer drop energy and is related to many practical design values in foundation engineering practice.

e_{min} and e_{max} are determined in the laboratory as follows (ASTM D 4253) for dry granular soils. As shown in Figure 5.1, dry granular soil is poured gently (without any vibration) into a rigid mold through a funnel. The funnel is moved up in a spiral motion to distribute grains evenly over the entire cross-section of the mold, and the drop heights of particles are maintained at about 25.4 mm (1 in.) till the top of the mold. The top surface is leveled by a straight edge to coincide exactly with the level of the top edge of the mold. The specimen in the mold is weighed, and the unit

TABLE 5.1
Relative Density with Soil Parameters

Relative Density, D_r (%)	Level of Denseness	Standard Penetration Resistance, N_{60} ^a	Effective Friction Angle ϕ' (Degree) ^b
<20	Very loose	<4	<29
20–40	Loose	4–10	29–30
40–60	Medium	10–30	30–36
60–80	Dense	30–50	36–41
>80	Very dense	>50	>41

Source: After *US Army Corps of Engineers, 1992, Engineer Manual, EM 1110-1-1905.*

^a Terzaghi, K. and Peck, R. B., 1967, *Soil Mechanics in Engineering Practice*, 2nd ed., John Wiley & Sons, New York.

^b Peck, R. B., Hanson, W. E., and Thornburn, T. H., 1974, *Foundation Engineering*, 2nd ed., John Wiley & Sons, New York.

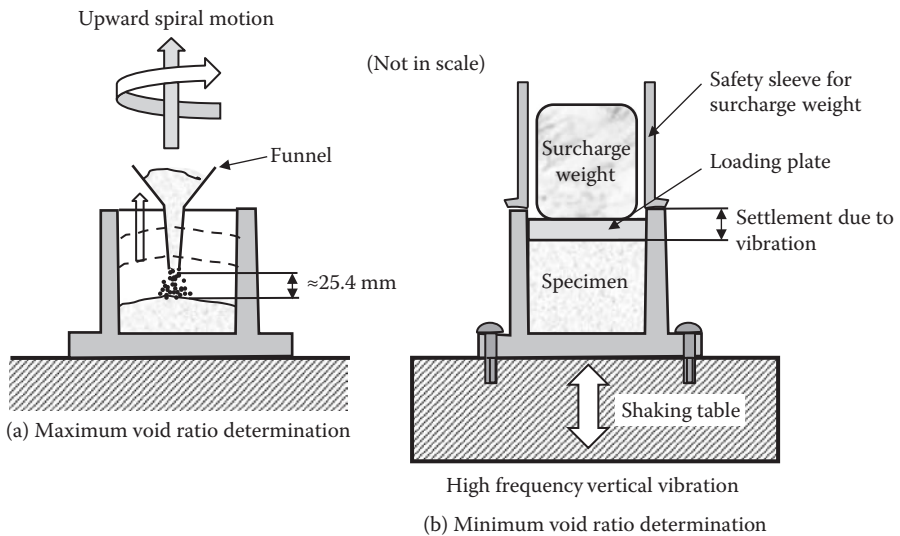


FIGURE 5.1 Maximum and minimum void ratio determination.

weight of the loosest specimen, γ_{\min} , is calculated from the weight of the soil and inside volume of the mold.

From Equation 2.9 (by substituting $S = 0$ for dry soil),

$$\gamma_{\min} = \frac{G_s \gamma_w}{1 + e_{\max}}, \quad \text{and thus} \quad e_{\max} = \frac{G_s \gamma_w}{\gamma_{\min}} - 1 \quad (5.2)$$

To determine e_{\min} , the loosest specimen, after the γ_{\min} determination test, is used. A surcharged weight, which applies 13.8 kN/m² (2 psi) on the specimen, is placed on the top of the specimen in the mold. The whole mold is firmly bolted on a shaking table and shaken vertically for 8 min with 60 Hz (cycle/second) and 0.33 mm peak-to-peak displacement, or for 12 min with 50 Hz and 0.48 mm peak-to-peak displacement. After shaking, the settlement of the specimen is carefully measured by a dial gauge, and the volume of the densified specimen is computed. The value of γ_{\max} is calculated as the weight of soil divided by its volume in the mold. The void ratio e_{\min} is then obtained as

$$e_{\min} = \frac{G_s \gamma_w}{\gamma_{\max}} - 1 \quad (5.3)$$

After several experimental trials, the mean values are reported as γ_{\min} (or e_{\max}) and γ_{\max} (or e_{\min}).

By substituting Equations (5.2) and (5.3) into Equation (5.1), the relative density D_r can also be defined as

$$D_r = \frac{\gamma - \gamma_{\min}}{\gamma_{\max} - \gamma_{\min}} \cdot \frac{\gamma_{\max}}{\gamma} (\times 100\%) \quad (5.4)$$

5.3 LABORATORY COMPACTION TEST

In the laboratory, specimens with different water contents are compacted with the same level of compaction energy. Water contents versus compacted soil's dry densities are then plotted to determine the optimum compaction effort. In the early 1930s, **Proctor (1933)** developed a standard compaction procedure during earth dam construction projects. This method is called the **Proctor method**, and its original version and some modified versions are currently used in ASTM (D 698 and D 1557) and AASHTO (T 99 and T 180).

5.3.1 STANDARD PROCTOR TEST PROCEDURE

The standard Proctor method follows the following steps:

1. Mix dry soil thoroughly with water to prepare a uniform specimen with designed water content.
2. Pour the loose specimen at a little over one-third depth into a standard-size mold 101.6 mm (4 in.) in diameter and 116.43 mm (4.584 in.) high without an extension collar, with 944 cm³ (1/30 ft³) volume as seen in Figure 5.2(a).

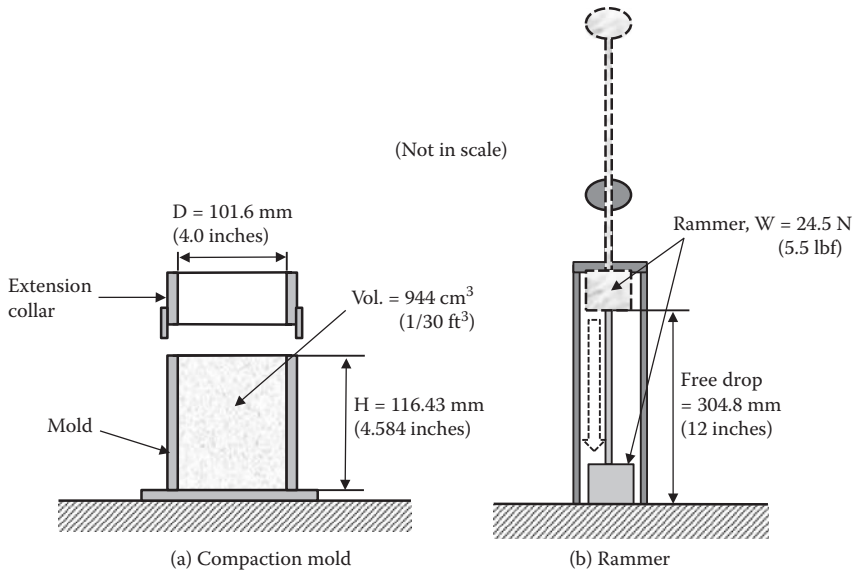


FIGURE 5.2 Standard Proctor compaction device.

3. Compact the specimen by 25 free drops of a rammer ($W = 24.5 \text{ N}$ (2.5 kilogram force [kgf] or 5.5 pound force [lbf]) from 304.8 mm (12 in.) high as seen in Figure 5.2(b). The mold should be placed on hard ground to avoid possible compaction energy loss.
4. Repeat steps 1 to 3 for the second and third layers to fill the mold with soil slightly above the top level of the mold. For the third-layer compaction, an extension collar is attached.
5. Remove the extension collar and trim the specimen surface by a straight edge to get exactly 944 cm^3 ($1/30 \text{ ft}^3$) volume of the specimen.
6. Weigh the whole mold with soil in it to obtain the wet weight of the specimen.
7. Extrude the specimen from the mold and obtain a representative soil specimen in a container for water content determination.
8. Repeat steps 1 to 7 for several different water contents. In general, soils from the previous experiment could be reused for the next test by breaking them down to particles and remixing with additional water.

5.3.2 COMPACTION CURVE

After the experiment, a set of total (moist) unit weight (γ_t) and water content (w) are measured. The compaction effectiveness, however, is compared in terms of increased **dry unit weight** (γ_d) of the specimen, instead of total unit weight (γ_t). Equation (2.10) in Chapter 2 (reappearing as Equation 5.5 in this chapter) is used to explain this:

$$\gamma_t = (1 + w) \frac{G_s \gamma_w}{1 + e} = (1 + w) \gamma_d \quad \text{or} \quad \gamma_d = \frac{G_s \gamma_w}{1 + e} = \frac{\gamma_t}{1 + w} \quad (5.5)$$

As seen in the first term of the γ_t expression, increasing w increases γ_t for a same value of void ratio e , which is a measure of compaction effectiveness. **Thus, γ_t cannot be used to evaluate the effectiveness of compaction. The γ_d expression in Equation (5.5) shows a direct relationship between “ e ” and “ γ_d .” Accordingly, $\gamma_d = \gamma_t/(1 + w)$ in Equation (5.5) is used in the compaction analysis.** Note that the obtained γ_d is for a mathematically dried-out specimen (the weight of water was removed in its three-phase diagram by keeping the same volume for the void) without any shrinkage, which occurs in the physical drying process. Accordingly, γ_d and w relations are plotted. Table 5.2 shows an example computation of test data, and the results are plotted in Figure 5.3.

TABLE 5.2
Example Computation of Compaction
Test Data

A	B	C
Water Content w (%)	Total (Moist) Unit Weight γ_t (kN/m ³)	Dry Unit Weight γ_d (kN/m ³) (= $\gamma_t/(1 + w)$)
2.3	15.80	15.45
4.5	17.27	16.53
6.7	19.13	17.93
8.5	20.41	18.81
10.8	21.41	19.32
13.1	21.73	19.22
15	21.48	18.68

Note: $C(i) = B(i)/(1 + A(i)/100)$.

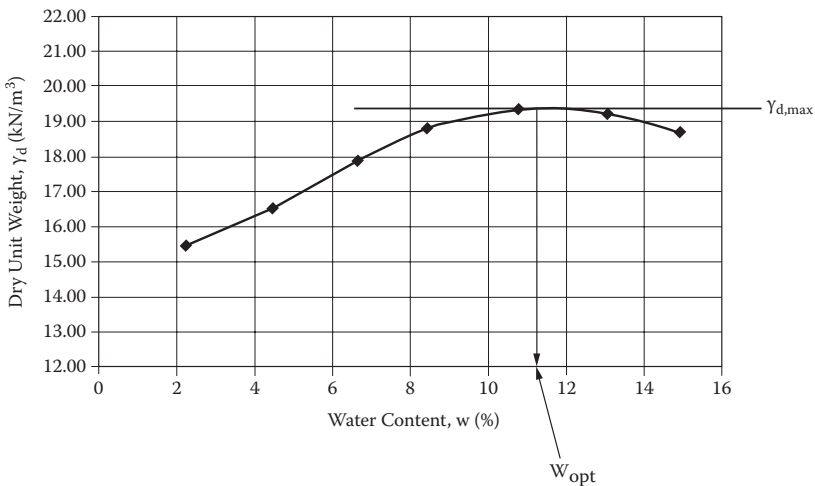


FIGURE 5.3 Example compaction curve.

In Figure 5.3, γ_d increases with increase of w in the beginning, reaches the peak value ($\gamma_{d,max} = 19.3 \text{ kN/m}^3$) at $w \approx 11.3\%$, and drops thereafter. The peak γ_d is defined as the **maximum dry unit weight**, $\gamma_{d,max}$, and the corresponding water content is called the **optimum water content**, w_{opt} . In the beginning, the addition of water works as a lubricant between particles to reduce the void and then to increase the dry unit weight. However, when the void is highly saturated with water, water starts to work as a cushion against compaction energy and does not work anymore to increase the soil's dry unit weight, but rather to decrease it with increased water content due to reduced actual compaction energy to the soil's skeleton. Thus, there is an optimum amount of water to transmit the most compaction energy to soil grain structures. During compaction experiments, the optimum water content can be felt by pushing the surface of compacted soil with a thumb. Until the optimum water content has been reached, the surface is hard to push. After the optimum level is passed, the surface becomes soft and spongy.

5.3.3 ZERO AIR VOID CURVE

From Equation (5.5) and using $Se = G_s w$ relation in Equation (2.17) in Chapter 2, γ_d can be rewritten as

$$\gamma_d = \frac{G_s \gamma_w}{1 + e} = \frac{G_s \gamma_w}{1 + \frac{G_s w}{S}} \tag{5.6}$$

Equation (5.6) shows a unique relationship between γ_d and w for a fixed S (degree of saturation) value and for a given G_s value. Figure 5.4 plots a group of

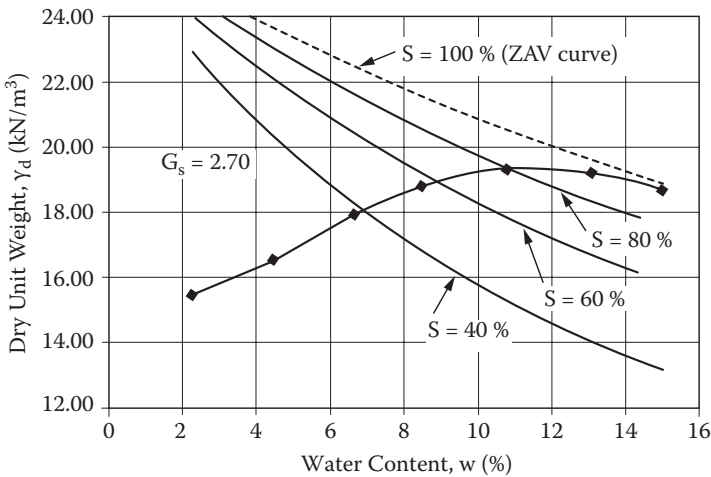


FIGURE 5.4 Compaction curve with various S (degree of saturation) values.

curves from Equation (5.6) for various S values (40%, 60%, 80%, and 100%) with $G_s = 2.7$.

It can be seen that, when w increases, S increases. At $\gamma_{d,max}$, S reaches more than 90%, and S approaches nearly 100% (full saturation) when the water content passes w_{opt} . The $S = 100\%$ curve is called the **zero air void (ZAV) curve**, and compaction curves approach the ZAV curve at high water content as seen. Therefore, this curve is often used as a guideline to construct a proper compaction curve for a high water content zone.

5.3.4 COMPACTION ENERGY

The Proctor test is a standard test with compaction energy of

$$\begin{aligned}
 E &= \Sigma[W \text{ (weight of rammer)} \times h \text{ (height of drop)}/\text{volume of specimen}] \\
 &= 24.5 \text{ N} \times 0.3048 \text{ m} \times 3 \text{ (layers)} \times 25 \text{ (drops)}/944 \times 10^{-6} \text{ m}^3 \quad (5.7) \\
 &= 594 \text{ kN-m/m}^3 \rightarrow 600 \text{ kN-m/m}^3
 \end{aligned}$$

Several other modified versions of compaction energy are obtained by changing the mold size, the weight of the rammer, the drop height, the number of drops, and the number of layers. Table 5.3 summarizes some of these modified versions.

When the compaction energy is increased, $\gamma_{d,max}$ increases. Since the ZAV curve confines the upper limit of the compaction curve, the corresponding w_{opt} decreases slightly as seen in an example in Figure 5.5. This observation suggests that, when a higher dry unit weight is required in the field, the field compaction

TABLE 5.3
Various Compaction Energies in Laboratory Tests

Tests	Mold Size (D × H) (mm)	Mold Volume (cm ³)	Weight of Rammer (N)	Drop Height (m)	No. of Drops/ Layer	No. of Layers	Total Energy/ Vol. (kN-m/m ³)
Standard Proctor	101.6 × 116.4	944	24.5	0.3048	25	3	593 ≈ 600
ASTM, D 698, Method C	152.4 × 116.4	2124	24.5	0.3048	56	3	591 ≈ 600
ASTM, D 1557, AASHTO, T 180, Method A	101.6 × 116.4	944	44.5	0.4572	25	5	2694 ≈ 2700
ASTM, D 1557, Method C, and AASHTO, T 180, Method D	152.4 × 116.4	2124	44.5	0.4572	56	5	2682 ≈ 2700

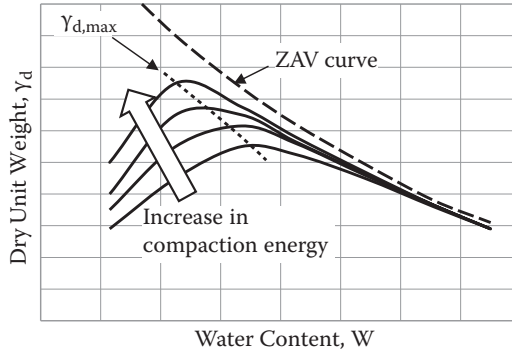


FIGURE 5.5 Compaction curves with various compaction energies.

energy shall be increased and, at the same time, water content shall be adjusted to have a slightly lower value to obtain the maximum effect of the increased compaction energy.

5.4 SPECIFICATION OF COMPACTION IN THE FIELD

After the compaction curve for a given soil is obtained from laboratory tests, the specification of compaction in the field is made. **Relative Compaction (R.C.)** is defined as

$$\text{R.C.} = \frac{\gamma_{d,\text{field}}}{\gamma_{d,\text{max}}} (\times 100\%) \quad (5.8)$$

where $\gamma_{d,\text{field}}$ is the specified dry unit weight, that shall be achieved in the field, and $\gamma_{d,\text{max}}$ is the maximum dry unit weight obtained from the laboratory compaction test. Since $\gamma_{d,\text{max}}$ varies depending on the compaction energy level or test method such as standard Proctor, etc., it shall be noted that R.C. could be more than 100% if the compaction energy in the laboratory was low. This implies that if a higher R.C. value (>100%) is required in the field, higher field compaction energy than the laboratory energy level is required to achieve the specified requirement. Table 5.4 provides a guide for tentative R.C. requirements for various types of soils in USCS and the importance of earthworks.

As can be seen in Table 5.4, the poorer the type of soil or the higher the importance of the earthwork is, the higher are the required R.C. values. It is noted again that those R.C. values are based on the standard Proctor test, so, if other standards with different energy levels are used, the required R.C. values may change.

It is cautioned that the relative compaction R.C. in Equation (5.8) and the relative density D_r in Equation (5.1) (or Equation 5.4) shall not be mixed up since γ_{max} in Equation (5.8) is obtained from the compaction test at its optimum water content, while γ_{max} in Equation (5.4) for D_r is obtained from a standard maximum unit weight test for a dry granular specimen. These values are not necessarily the same.

TABLE 5.4
Tentative Requirements for Compaction Based on USCS

Soil Group in USCS	Required R.C.: % of Standard Proctor Maximum		
	Class 1	Class 2	Class 3
GW	97	94	90
GP	97	94	90
GM	98	94	90
GC	98	94	90
SW	97	95	91
SP	98	95	91
SM	98	95	91
SC	99	96	92
ML	100	96	92
CL	100	96	92
OL	—	96	93
MH	—	97	93
CH	—	—	93
OH	—	97	93

Source: After **Sowers, G. F., 1979, Introductory Soil Mechanics and Foundations: Geotechnical Engineering**, 4th ed., Macmillan, New York.

Class 1: Upper 3 m of fills supporting one- or two-story buildings.
 Upper 1 m of subgrade under pavement.
 Upper 0.3 m of subgrade under floors.

Class 2: Deeper parts of fills under buildings.
 Deeper parts (to 10 m) of fills under pavements, floors.
 Earth dams.

Class 3: All other fills requiring some degree of strength or incompressibility.

Exercise 5.1

A soil sample is tested by the standard Proctor test, and the compaction curve obtained is shown in Figure 5.3. The specification says that the in-situ soil shall be compacted with 95% of R.C. and above the maximum dry unit weight from the standard Proctor test. Determine the range of field water content to achieve the preceding specification.

SOLUTION

From Figure 5.6, $\gamma_{d,max} = 19.3 \text{ kN/m}^3$, and thus $\gamma_{d,field} = 0.95 \times 19.3 \text{ kN/m}^3 = 18.3 \text{ kN/m}^3$.

In Figure 5.6, $\gamma_{d,max} = 19.3 \text{ kN/m}^3$ and $\gamma_{d,field} = 18.3 \text{ kN/m}^3$ lines are drawn, and the corresponding range of water content, which satisfies $\gamma_{d,field} = 18.3 \text{ kN/m}^3$ and above, is obtained as 7.5% to 16.0%.

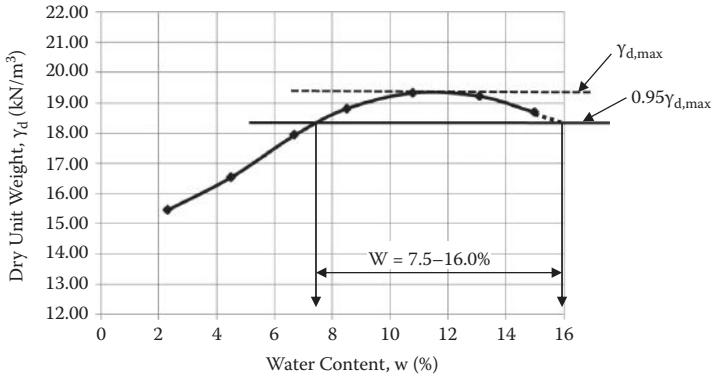


FIGURE 5.6 Solution to Exercise 5.1.

5.5 FIELD COMPACTION METHODS

5.5.1 COMPACTION EQUIPMENT

After the compaction specification is given at the site, contractors are required to achieve its specified dry unit weight as the minimum in the field with proper equipment. For small jobs such as filling excavated small trenches, hand-operated vibratory tampers (Figure 5.7a) may be used. For larger job sites, several different types of heavy-duty compaction rollers are available. The commonly used compaction equipment is as follows:

Pneumatic rubber tire rollers (Figure 5.7b) can be used for both sandy soils and clayey soils. Soils are compacted with both tire pressure and kneading action.

Sheep's-foot rollers (Figure 5.7c) have unique wheel surfaces that can effectively compact the clayey soils and the deeper part of soils in earlier passes.

Smooth-wheel (drum) rollers (Figure 5.7d) are mostly used for sandy and clayey soils for the finishing and smoothing process. The compaction pressure is not as high as in the former two rollers, and therefore it is not used to compact thicker layers.

Vibratory wheels are usually part of all the foregoing rollers; they are especially effective in compacting granular soils.

Table 5.5 provides some guidelines on the types of soils and applicable compaction equipments.

There are several key parameters that influence field compaction in addition to the level of compaction energy and controlling the water content as close as possible to its optimum water content. They are as follows:

Number of passes: In general construction practice, several or more passes of rollers are required to obtain a specified dry unit weight. The more passes applied, the higher the dry unit weight is obtained. Figure 5.8 plots



(a) Hand-operated vibratory tamper



(b) Pneumatic rubber tire roller



(c) Sheep's-foot roller



(d) Smooth wheel drum roller

FIGURE 5.7 Field compaction equipments.

TABLE 5.5
Soil Compaction Characteristics and Recommended Equipment

General Soil Description	USCS Group	Compaction Characteristics	Recommended Equipments
Sand and sand–gravel mixture (no silt and clay)	SW, SP, GW, GP	Good	Vibratory drum roller, vibratory rubber tire, or pneumatic tire equipment
Sand or gravel–sand with silt	SM, GM	Good	Vibratory drum roller, vibratory rubber tire, or pneumatic tire equipment
Sand or sand–gravel with clay	SC, GC	Good to fair	Pneumatic tire, vibratory rubber tire, or vibratory sheep's-foot equipment
Silt	ML	Good to poor	Pneumatic tire, vibratory rubber tire, or vibratory sheep's-foot equipment
Silt	MH	Fair to poor	Pneumatic tire, vibratory rubber tire, or vibratory sheep's-foot, sheep's-foot-type equipment
Clay	CL	Good to fair	Pneumatic tire, sheep's-foot, vibratory rubber tire, or vibratory sheep's-foot equipment
Organic soil	CH OL, OH, Pt	Fair to poor Not recommended for structural earth fill	

Source: After McCarthy, D., 2008, *Essentials of Soil Mechanics and Foundations: Basic Geotechnics*, 7th ed., Pearson/Prentice Hall, Upper Saddle River, NJ.

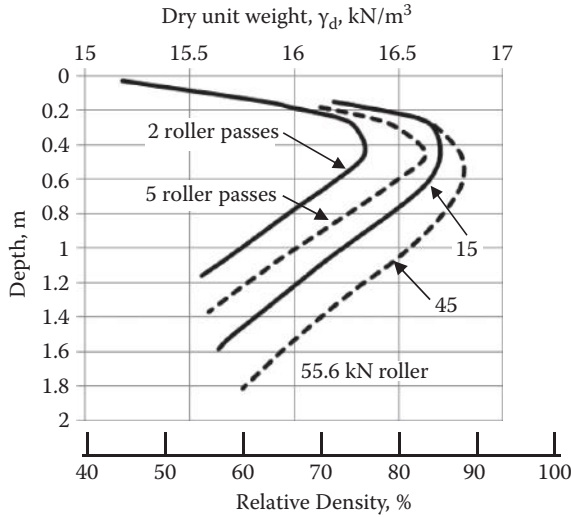


FIGURE 5.8 Effect of field compaction with depth and number of passes. (After D'Appolonia, D. J. et al., 1969, *Journal of Soil Mechanics and Foundation Division*, vol. 95, no. SM1, 263–284.)

γ_d versus depth with various numbers of passes from 2 to 45 of a single 2.44 m (8 ft) lift fill by 55.6 kN (12.5 kips) smooth roller (*D'Appolonia, Whitman, and D'Appolonia 1969*). After five passes, a large increase is needed in the number of passes to achieve significant increase in compaction. In general, it is considered that more than 10 to 15 rolls may not be effective and not be an economical way to compact fills.

Amount of lift: The amount of lift is also significant. Figure 5.8 shows that only the upper section at 0.3–0.5 m (1–1.5 ft) deep is effectively compacted. The lift should be small enough to get the maximum compaction effect over the entire depth, but not be too small, since the very top portion of the layer also cannot be well compacted because of particle segregation upon vibration application. In general applications, a loose lift is limited to about 0.5 m (20 in.).

5.5.2 DYNAMIC COMPACTION

Recently, this simple yet effective compaction technique was introduced. The **dynamic compaction** method involves dropping a heavy weight repeatedly on the ground at regularly spaced intervals, as seen in Figure 5.9. The weight is typically between 80 and 360 kN, and the drop height changes from 10 to 30 m. The impact of the free drop of weight creates stress waves that densify the soil to a relatively large depth. The method is effectively used for sandy soils but is also applied to silt and clay soils. This is a rather economical way to compact the site if such operations with vibration and noise can be tolerated.



FIGURE 5.9 Dynamic compaction. (Photo courtesy of Terra Systems, Inc.)

5.6 FIELD DENSITY DETERMINATIONS

The final important step in compaction is field monitoring and inspection of compaction practice. After the completion of compaction, it is not easy to tell whether the site is properly compacted or not according to the specification, so monitoring during compaction practice and inspection after compaction are needed.

A preliminary check can be made by an inspector who can observe penetration of a probe (typically 13 mm diameter steel rod) pushed by the inspector's own weight to find any abnormal spots over the entire site. However, in most cases, the field dry unit weight is measured after the completion of compaction. There are several methods available, such as the sand cone method (ASTM D 1556 and AASHTO T 191), rubber balloon method (ASTM D 2167 and AASHTO T 205), nuclear density method (ASTM D 2922 and AASHTO T 238), etc. The **sand cone method** is widely used and is described next.

5.6.1 SAND CONE METHOD

As shown in Figure 5.10, this method uses free-flowing sand to fill a field-excavated hole to measure its volume. Then, the field total unit weight as well as the dry

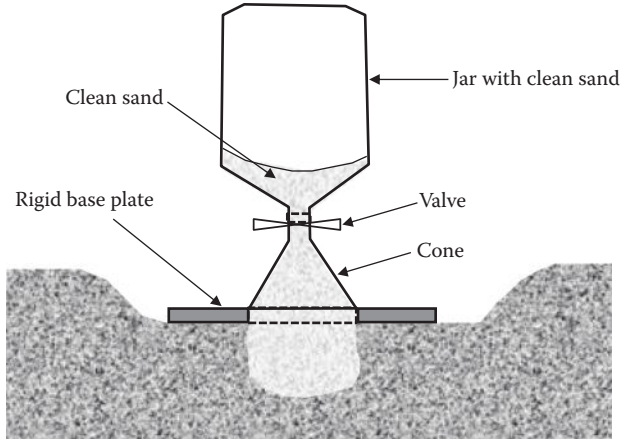


FIGURE 5.10 Sand cone method.

unit weight is calculated with the measured moist weight and water content of the excavated soil. To fill the excavated hole, uniformly graded, dry, clean sand with gradation between 2 mm (passing No. 10 sieve) and 0.25 mm (retaining No. 60 sieve) is used. Calibration is made to determine the sand's dry unit weight, $\gamma_{d,sand}$, with free drop in the laboratory prior to the field measurement. The field procedure involves the following steps:

1. Prior to field work, $\gamma_{d,sand}$ shall be calibrated. Several jars with identification numbers for each are filled with the sand, and their total weights are recorded.
2. At a site selected for field density determination, the surface of the ground is flattened and leveled by the edge of the rigid base plate. The surface level is typically located at a certain depth since the compacted top surface does not necessarily represent the true compaction result of the soil layer.
3. Through the circular opening at the center of the base plate, the ground is carefully excavated by using a spoon, and all soil from the excavated hole shall be collected in a plastic bag.
4. The jar filled with sand is placed upside down so that the top of the cone engages into the inner edge of the opening in the base plate. At this stage, the valve in the jar is kept closed.
5. After the jar with the cone is securely placed on the base plate, the valve is carefully opened to allow free flow of sand into the excavated hole and the cone section of the device.
6. After the observation of completion of sand flow into the space, the valve is carefully closed. The total weight of the jar and the remaining sand is measured later. This weight is subtracted from the original weight of the jar of sand, and then the weight that filled the space of the hole and cone is obtained as $W_{sand,cone+hole}$.

7. The field moist specimens from the excavated holes are weighted as $W_{t, \text{hole}}$ and water contents are determined as w . These measurements could be done in the field, by using a balance and a quick-drying microwave oven, or in the laboratory.

The calculation is as follows:

$$\gamma_{d, \text{sand}} = W_{\text{sand, cone + hole}} / V_{\text{hole + cone}} \quad (5.9)$$

and thus,

$$V_{\text{hole + cone}} = W_{\text{sand, cone + hole}} / \gamma_{d, \text{sand}} \quad (5.10)$$

$$V_{\text{hole}} = V_{\text{hole + cone}} - V_{\text{cone}} \quad (5.11)$$

$$\gamma_{t, \text{hole}} = W_{t, \text{hole}} / V_{\text{hole}} \quad (5.12)$$

$$\gamma_{d, \text{hole}} = \gamma_{t, \text{hole}} / (1 + w) = \gamma_d \quad (5.13)$$

In the foregoing computation of γ_d , the volume of the cone V_{cone} and $\gamma_{d, \text{sand}}$ shall be calibrated with the sand used. The values of $W_{\text{sand, cone + hole}}$, $W_{t, \text{hole}}$, and w are field-measured properties.

The measured γ_d value is compared with the specified $\gamma_{d, \text{field}}$. When the measured values do not satisfy the requirement, the field inspector shall direct the contractor to recompact the site.

Exercise 5.2

The following data are obtained from a field sand cone test. Determine $\gamma_{d, \text{field}}$ and the relative compaction R.C. The $\gamma_{d, \text{max}}$ value from the standard Proctor test for the soil was 18.8 kN/m^3 .

$$\gamma_{d, \text{sand}} = 15.5 \text{ kN/m}^3 \text{ (calibrated dry unit weight of sand)}$$

$$W_{\text{sand, cone}} = 1.539 \text{ kgf (weight of sand to fill the cone section only)}$$

$$\text{Weight of jar + cone + sand (before the test)} = 7.394 \text{ kgf}$$

$$\text{Weight of jar + cone + sand (after the test)} = 2.812 \text{ kgf}$$

$$W_{t, \text{hole}} = 3.512 \text{ kgf (moist weight of soil obtained from the hole)}$$

$$w = 10.6\% \text{ (water content of in-situ soil after laboratory determination)}$$

SOLUTION

$$V_{\text{cone}} = W_{\text{sand,cone}}/\gamma_{d,\text{sand}} = 1.539 \times 9.81 \times 10^{-3}/15.5 = 0.974 \times 10^{-3} \text{ m}^3$$

$$W_{\text{sand,cone+hole}} = 7.394 - 2.812 = 4.582 \text{ kgf}$$

$$V_{\text{sand,cone+hole}} = W_{\text{sand,cone+hole}}/\gamma_{d,\text{sand}} = 4.582 \times 9.81 \times 10^{-3}/15.5 = 2.900 \times 10^{-3} \text{ m}^3$$

$$V_{\text{sand,hole}} = V_{\text{sand,cone+hole}} - V_{\text{cone}} = 2.900 \times 10^{-3} - 0.974 \times 10^{-3} \text{ m}^3 = 1.926 \times 10^{-3} \text{ m}^3$$

$$\gamma_{t,\text{hole}} = \gamma_t = W_{t,\text{hole}}/V_{\text{sand,hole}} = 3.512 \times 9.81 \times 10^{-3}/1.926 \times 10^{-3} = 17.89 \text{ kN/m}^3$$

$$\gamma_d = \gamma_t/(1 + w) = 17.89/(1 + 0.106) = \mathbf{16.18 \text{ kN/m}^3} \leftarrow$$

$$\text{R.C.} = \gamma_d/\gamma_{d,\text{max}} = 16.18/18.8 = 0.860 = \mathbf{86.0\%} \leftarrow$$

5.6.2 OTHER FIELD DENSITY METHODS

Regarding other popular field density determination methods, the **rubber balloon method** uses a similar principle as the sand cone method. Instead of dry, clean sand, it uses the rubber balloon to fill the excavated hole, and the hole is replaced with water to measure the volume.

In recent years, the **nuclear density method** (ASTM D 2922) became a popular method to determine the field density and the water content. It uses gamma radiation for density determination. It measures the scatter of radiation, which is proportional to the density, while the scatter of alpha particles detects water content. Both need prior calibrations to establish empirical correlations. This quick and nondestructive test is handy, but it requires specially trained technicians and careful handling of low-level radioactive materials.

5.7 CALIFORNIA BEARING RATIO TEST

CBR is a penetration test for evaluating the strength of road subgrade and base course materials. This was originally developed by the California Department of Transportation and became standards in ASTM (D 1883) and AASHTO (T 193). As seen in Figure 5.11, the test uses 152.4 mm (6 in.) diameter molds, and 24.4 N (5.5 lbf) or 44.5 N (10 lbf) rammers are used as in standard or modified Proctor tests. The specimen is compacted in the mold to have a specified dry unit weight, which simulates the field situation. To achieve selected conditions, a proper compaction energy level is chosen by adjusting the numbers of drops and layers and the drop height. Water content could be at its optimum or as desired. A selected surcharge load, which simulates the field situation, is applied on top of the specimen with metal discs, and the specimen could be soaked or unsoaked in water to simulate the in-situ condition. The vertical load then is applied on the surface of the specimen by the penetration piston. The penetration value and load are recorded.

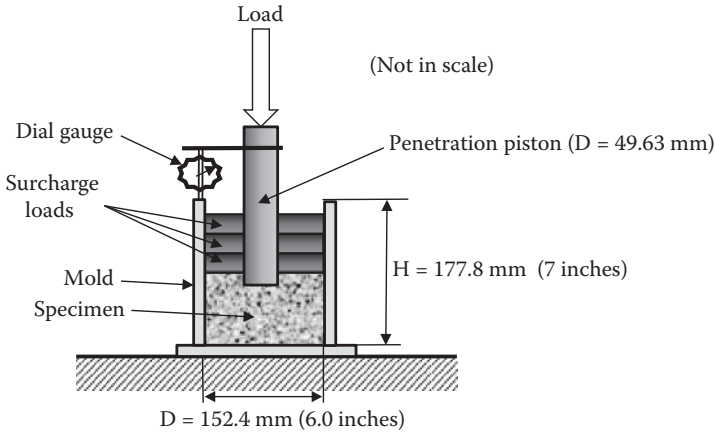


FIGURE 5.11 California Bearing Ratio (CBR) test device.

The CBR value is defined as

$$CBR = (P/P_s) \times 100 \tag{5.14}$$

where P is the load intensity on the penetrometer at 2.54 mm (0.1 in.) penetration in the specimen, and P_s is the load intensity on the penetrometer at 2.54 mm (0.1 in.) penetration in the standard crushed stone. The value of P is obtained from the load-penetration curve of the test, and P_s is given as 6900 kN/m². As seen in the definition, CBR is the percentage of load level at 2.54 mm penetration on a given soil to that of the best available material (crushed stone). The standard material for this test is crushed California limestone for which the CBR value is 100. The harder the surface is, the higher the CBR rating will be.

The CBR rating was developed for measuring the load-bearing capacity of soils used for building roads, and it can also be used for measuring the load-bearing capacity of unimproved airstrips or for soils in underpaved airstrips. CBR is incorporated in many pavement design charts, and it is also related to other useful engineering properties. Table 4.1 in Chapter 4 includes typical CBR values with USCS group names.

5.8 SUMMARY

Compaction is a very important practice for the proper placement of fills. Without proper compactions, ground surfaces may settle in the future and cause many problems. Laboratory and field compaction methods were presented in this chapter. Proper utilization of laboratory compaction results in field practice with an adequate compaction machine was studied. Also, the importance of in-situ density inspection was emphasized. The CBR method, which is popularly used in the pavement design practice, was also presented.

REFERENCES

- D'Appolonia, D. J., Whitman, R. V., and D'Appolonia, E. D. (1969), Sand compaction with vibratory rollers, *Journal of Soil Mechanics and Foundation Division*, vol. 95, no. SM1, 263–284.
- McCarthy, D. (2008), *Essentials of Soil Mechanics and Foundations: Basic Geotechnics*, 7th ed., Pearson/Prentice Hall, Upper Saddle River, NJ.
- Peck, R. B., Hanson, W. E., and Thornburn, T. H. (1974), *Foundation Engineering*, 2nd ed., John Wiley & Sons, New York.
- Proctor, R. R. (1933), Fundamental principles of soil mechanics, *Engineering News Record*, vol. 111, nos. 9, 10, 12, and 13.
- Sowers, G. F. (1979), *Introductory Soil Mechanics and Foundations: Geotechnical Engineering*, 4th ed., Macmillan, New York.
- Terzaghi, K., and Peck, R. B. (1967), *Soil Mechanics in Engineering Practice*, 2nd ed., John Wiley & Sons, New York.
- US Army Corps of Engineers (1992), *Engineer Manual*, EM 1110-1-1905.

Problems

- 5.1 At a borrow site, sandy soil was excavated. The soil had $\gamma_t = 19.3 \text{ kN/m}^3$, $w = 12.3\%$, and $G_s = 2.66$. The soil was dried, the maximum and minimum void ratio tests were performed, and $e_{\max} = 0.564$ and $e_{\min} = 0.497$ were obtained. Determine the relative density of the soil at the borrow site.
- 5.2 The soil in Problem 5.1 is used to fill an earthwork, and 75% of relative density is required in the field compaction with 10% water content. Determine the required total unit weight of the soil γ_t for this earthwork.
- 5.3 For a given soil with $G_s = 2.65$, plot the γ_d versus w relations for $S = 40\%$, 60%, 80%, and 100% for a range of $w = 0\%$ to 20%.
- 5.4 The standard Proctor test was performed for a soil with $G_s = 2.66$, and the results are as follows:

Water Content, %	Moist Weight in Mold, gf
5.6	1420
7.9	1683
10.8	1932
13.3	1964
14.8	1830
16.2	1630

- (a) Plot the γ_d versus w relation.
- (b) Determine $\gamma_{d,\max}$ and w_{opt} .
- (c) Calculate S and e at the maximum dry unit weight point.

- (d) What is γ_t at w_{opt} ?
- (e) What is the range of water content if the relative compaction (R.C.) is required to be 90% of the standard Proctor $\gamma_{d,max}$?

5.5 The modified Proctor test (ASTM D 698, method C) was performed for a soil with $G_s = 2.70$, and the results are as follows:

Water Content, %	Moist Weight in Mold, gf
6.5	3250
9.3	3826
12.6	4293
14.9	4362
17.2	4035
18.6	3685

- (a) Plot the γ_d versus w relation.
- (b) Determine $\gamma_{d,max}$ and w_{opt} .
- (c) Calculate S and e at the maximum dry unit weight point.
- (d) What is γ_t at w_{opt} ?
- (e) What is the range of water content if the relative compaction (R.C.) is required to be 95% of the modified Proctor $\gamma_{d,max}$?

5.6 The following table shows a standard Proctor test result. Soil type was SW according to USCS. The soil is used for filling a small trench that was excavated in a parking lot. Determine the required γ_d and the range of water content for this job. Use Table 5.4 as a guideline.

Water Content, %	Dry Unit Weight γ_d , kN/m ³
3.5	14.3
6.2	16.8
9.2	18.6
12.5	18.7
15.3	17.6
18.6	14.6

5.7 The same soil as in Problem 5.6 is used for the upper section of subgrade under a pavement. Determine the required γ_d and the range of water content for this job. Use Table 5.4 as a guideline.

5.8 A planned fill site requires 2500 m³ of fill material with $\gamma_d = 18.5$ kN/m³ and $w = 14\%$. The material is brought from a borrow site that has $\gamma_t = 19.5$ kN/m³ with $w = 16.5\%$ and $G_s = 2.70$.

- (a) How much of the borrow material (in cubic meters) is required to fill the site?
 - (b) How heavy is it?
- (Hint: use the three-phase diagrams for the fill site and the borrow site.)

- 5.9 The field sand cone test was performed, and the following data were obtained. Determine the field dry unit weight of the soil:

$$\gamma_{\text{sand}} = \text{calibrated unit weight of sand} = 16.2 \text{ kN/m}^3$$

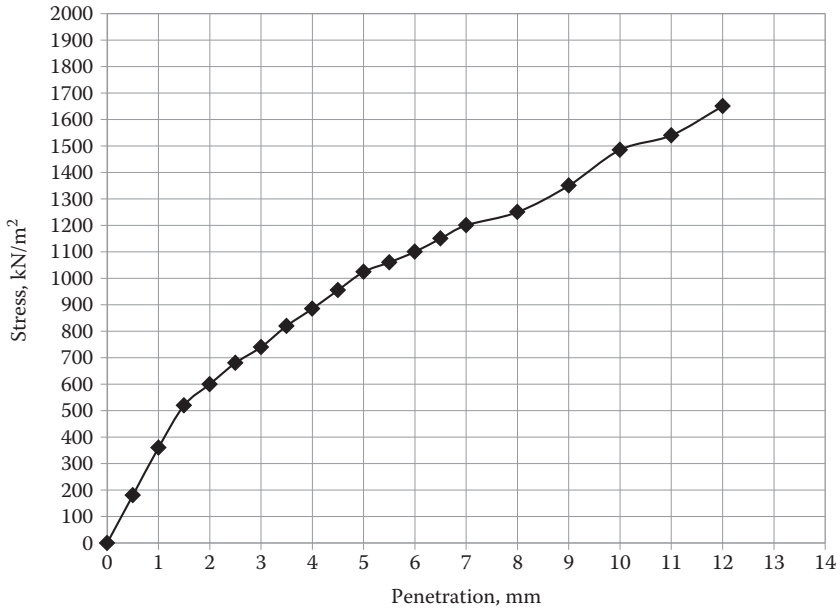
$$V_{\text{cone}} = \text{calibrated volume of cone} = 0.974 \times 10^{-3} \text{ m}^3$$

$$W_{\text{wet soil}} = \text{wet soil obtained from the hole} = 3.425 \text{ kgf}$$

$$W_{\text{sand to fill cone + hole}} = 4.621 \text{ kgf}$$

$$W_{\text{dry,soil}} = \text{oven-dried soil obtained from the hole} = 3.017 \text{ kgf}$$

- 5.10 The CBR penetration data (stress versus penetration) for a given soil is as follows:



- (a) Determine the CBR value.
 (b) Evaluate the suitability of the soil as pavement subgrade material.

Dr Mohamed Abd Elatif

Chapter 5 Effective Stress

7.1 INTRODUCTION

Terzaghi (1925) developed the **effective stress concept**, which became a key concept in modern soil mechanics. Effective stress in soil contributes to its strength and volume change. It also influences the capillary rise, seepage force due to water flow, quicksand (sand boiling), and heaving at the bottom of the excavation. These are discussed in this chapter.

7.2 TOTAL STRESS VERSUS EFFECTIVE STRESS

Soil is an assemblage of particles so that the soil's skeleton (particle connected structure) is a major body to resist against external forces. This is seen in Figure 7.1, in which two-headed arrow vectors indicate interparticle forces at contact points, including normal contact forces as well as shear contact forces. In a dry situation, interparticle forces are in equilibrium with the external forces as seen. However, if the soil is saturated or partially saturated, pore water pressure develops, and it also resists against some part of the external forces.

Figure 7.2 models the interparticle and pore water pressure resistances against the external stress σ . The model consists of a water-filled cylinder with a frictionless piston that is supported by a spring. In the piston, there is a small hole to allow drainage. The spring represents the skeleton's resistance, filled water represents the pore water of the soils, and a small hole in the piston reflects the permeability of the soil. Terzaghi defined **effective stress** σ' as

$$\sigma' = \sigma - u \quad (7.1)$$

where σ is the applied **total stress** and u is the **pore water pressure**. In the model, the applied stress is carried partially by σ_{spring} and partially by the pore water pressure u . Thus, the skeleton's stress is closely related to the effective stress. When the volume change occurs (the spring is compressed), σ_{spring} (effective stress) develops, or vice versa, as demonstrated in the model. **Total stress does not contribute to the volume change of soils; rather, the effective stress is the one to cause the volume change of soils.**

7.3 EFFECTIVE STRESS COMPUTATIONS IN SOIL MASS

In-situ soil at a certain depth is subjected to an **overburden stress**, which generally determines the current formation of the soil's skeleton. At a greater depth beneath the ground surface, the soil is more compacted because of its higher

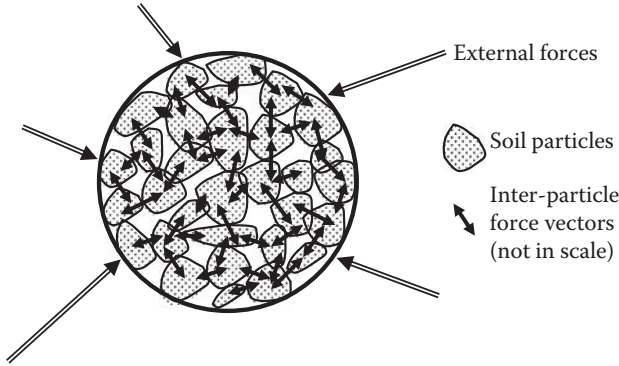


FIGURE 7.1 Interparticle stresses in particle assemblage.

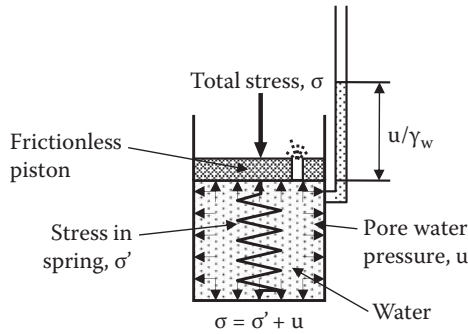


FIGURE 7.2 Terzaghi's effective stress model.

overburden stress. *According to the effective stress concept, the stress that determines the current form of the skeleton is the effective stress.* In the following text, effective overburden stress computations are demonstrated for various situations.

7.3.1 DRY SOIL LAYERS

Figure 7.3 shows several layers of dry soil deposit. The total vertical (overburden) stress at Point A is the weight of a soil column of 1×1 area above Point A, and thus,

$$\sigma = H_1\gamma_1 + H_2\gamma_2 + H_3\gamma_3 = \Sigma(H_i\gamma_i) \tag{7.2}$$

The vertical stress distribution σ with the depth is plotted alongside. In this case, $u = 0$, and thus $\sigma' = \sigma$ throughout the depth.

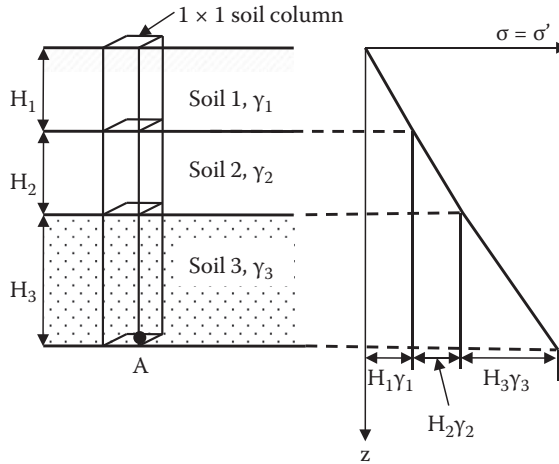


FIGURE 7.3 Effective stress computation for dry soil layers.

7.3.2 SOIL LAYERS WITH STEADY WATER TABLE

Figure 7.4 plots a situation with a steady groundwater table. The water table is midway of soil layer 2. In this case, the total vertical stress σ at Point A is calculated first as the weight of a 1×1 soil column as before, and then the hydrostatic water pressure u is computed. Finally, the effective vertical stress σ' is computed as follows:

$$\sigma = H_1\gamma_1 + H_2\gamma_2 + H_3\gamma_3 + H_4\gamma_4 = \Sigma(H_i\gamma_i) \tag{7.3}$$

$$u = (H_3 + H_4)\gamma_w \tag{7.4}$$

$$\begin{aligned} \sigma' &= \sigma - u = [H_1\gamma_1 + H_2\gamma_2 + H_3\gamma_3 + H_4\gamma_4] - [(H_3 + H_4)\gamma_w] \\ &= H_1\gamma_1 + H_2\gamma_2 + H_3(\gamma_3 - \gamma_w) + H_4(\gamma_4 - \gamma_w) \\ &= \Sigma(H_i\gamma_i)_{\text{above W.T.}} + \Sigma[(H_j(\gamma_j - \gamma_w))]_{\text{below W.T.}} \end{aligned} \tag{7.5}$$

where i and j denote the values for above the water table and below the water table, respectively. These individual distribution curves are also plotted in Figure 7.4. The effective stress distribution curve and Equation (7.5) suggest that **σ' can be directly calculated by a summation of soil layer thickness multiplied by the unit weight for all layers by assigning the total unit weight γ_t for soils above the water table and submerged unit weight γ' ($= \gamma_t - \gamma_w$) for soils below the water table.**

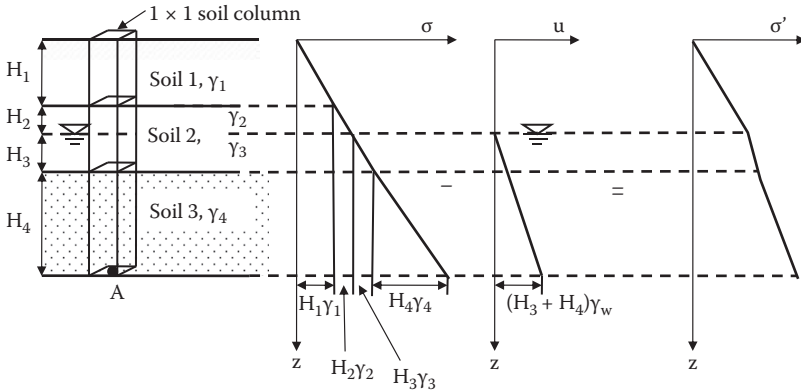


FIGURE 7.4 Effective stress computation for dry and wet soil layers.

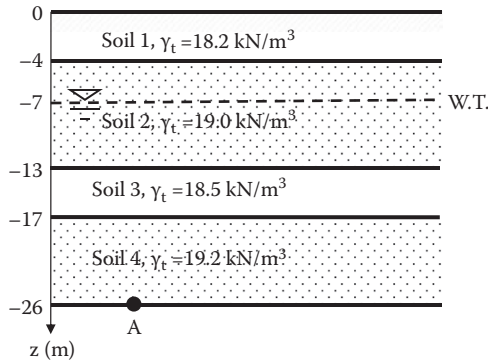


FIGURE 7.5 Exercise 7.1 problem.

Exercise 7.1

Figure 7.5 shows soil conditions and water table elevation. Calculate the effective overburden stress at Point A, (a) by computing σ and u individually, and (b) by directly using γ_t above the water table and γ' below the water table.

SOLUTION

Assume that for Soil 2, γ_t values above and below the water table are the same.

Method (a): individual computations of σ and u , then σ' :

$$\sigma_A = \Sigma(H_i\gamma_i) = 4 \times 18.2 + 9 \times 19.0 + 4 \times 18.5 + 9 \times 19.2 = 490.6 \text{ kPa}$$

$$u_A = (6 + 4 + 9) \times 9.81 = 186.4 \text{ kPa}$$

$$\sigma'_A = \sigma_A - u_A = 490.6 - 186.4 = \mathbf{304.2 \text{ kPa} \leftarrow}$$

Method (b): direct computation of σ' :

$$\sigma'_A = \Sigma(H_i\gamma_i) + \Sigma(H_j\gamma'_j) = 4 \times 18.2 + 3 \times 19.0 + 6 \times (19.0 - 9.81) + 4 \times (18.5 - 9.81) + 9 \times (19.2 - 9.81) = \mathbf{304.2 \text{ kPa} \leftarrow}$$

Both solutions yielded the same results.

7.3.3 TOTALLY SUBMERGED SOIL LAYERS

To calculate the effective stress σ' for soils under lakes or at ocean bottom, the same principles as before are used—that is, use of γ' for soils under the water table. Since all soils are under the water table, it is simply

$$\sigma' = \Sigma(H_j\gamma'_j)_{\text{below W.T.}} \tag{7.6}$$

Figure 7.6 shows the distributions of σ , u , and σ' for this case. It can be seen that σ' is not affected by the depth of water H_w .

Exercise 7.2

Calculate σ , u , and σ' on a soil element at 2 m depth from an ocean-bottom surface under 300 m deep water. The soil's unit weight is 17.5 kN/m³. Does this high water pressure compress soil?

SOLUTION

$$\sigma = H_w\gamma_w + H_{\text{soil}}\gamma_{\text{soil}} = 300 \times 9.81 + 2 \times 17.5 = 2978 \text{ kPa}$$

$$u = H_w\gamma_w = (300 + 2) \times 9.81 = 2963 \text{ kPa}$$

$$\sigma' = \sigma - u = 2978 - 2963 = 15 \text{ kPa}$$

Total stress and pore water pressure are very high, but the effective stress is very low. Since the formation of the soil's skeleton is controlled by interparticle stress

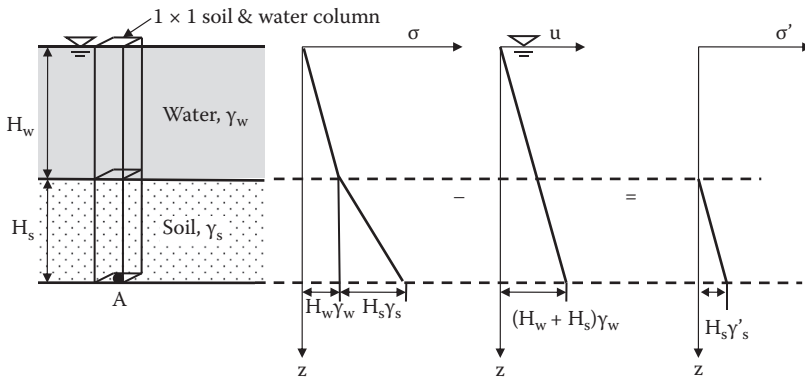


FIGURE 7.6 Effective stress computation for underwater soil layers.

(effective stress), soils at the near surface of the ocean bottom are not compressed much because of the rather small effective overburden stress.

The foregoing exercise demonstrates that very soft soils exist at deep ocean bottoms even though those soils are subjected to extremely high water pressures. **High water pressure acts hydrostatically all around the surface of grain particles, and thus it does not contribute to increase in interparticle stresses.**

7.4 EFFECTIVE STRESS CHANGE DUE TO WATER TABLE CHANGE

When the water table changes, the effective overburden stress changes since the effective stress computation uses either γ_t or γ' depending on the water table elevation, as seen in the previous section. In particular, when the water table drops, the effective stress increases. The effective stress increase implies higher grain skeleton stress, and thus it causes volume decrease or settlement. In modern history, many urban industrial cities pumped up underground water for industrial uses and lowered the water table elevation permanently. The consequence was ground surface settlement in many cities around the world.

Exercise 7.3

For the site shown in Figure 7.7 (the same site as in Exercise 7.1), water table elevation was at -7 m originally, and lowered 6 m to -13 m due to heavy industrial water use. Calculate the change of the effective overburden stress at Point A. What is a consequence of the lowering water table elevation?

SOLUTION

Assume that, for Soil 2, γ_t values above and below the water table are the same. Before lowering the water table using the result in Exercise 7.1:

$$\begin{aligned}\sigma'_A &= \Sigma(H_i\gamma_i) + \Sigma(H_j\gamma'_j) = 4 \times 18.2 + 3 \times 19.0 + 6 \times (19.0 - 9.81) \\ &+ 4 \times (18.5 - 9.81) + 9 \times (19.2 - 9.81) = 304.2 \text{ kPa}\end{aligned}$$

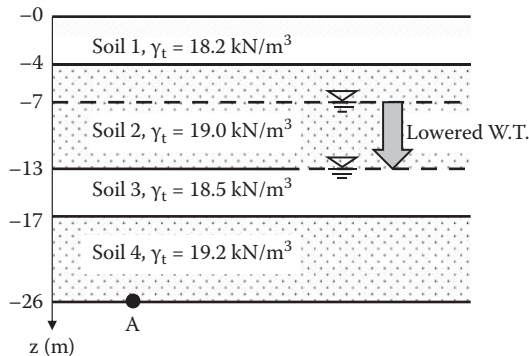


FIGURE 7.7 Exercise 7.3 problem.

After lowering the water table to -13 m:

$$\sigma'_A = \Sigma(H_i\gamma_i) + \Sigma(H_i\gamma'_i) = 4 \times 18.2 + 9 \times 19.0 + 4 \times (18.5 - 9.81) + 9 \times (19.2 - 9.81) = 363.1 \text{ kPa}$$

Thus, the change in σ' , $\Delta\sigma' = 363.1 - 304.2 = 58.9 \text{ kPa increase} \leftarrow$
 This increase in effective stress would cause ground settlement in the near future. \leftarrow

In contrast to cases with lowering water table elevation, its rise causes a reduction in effective stress. In such cases, some swell is possible, but it may not be as severe as in the case of settlement. One potential problem of this case is that underground structures, including buried pipes as well as massive underground structures, may be pushed upward due to increased buoyancy forces applied to such structures. In recent years, it was reported that the underground sections of the Tokyo railway station have been affected by increased upward pressure. This is due to the restriction of groundwater use in that area. As a result, in recent years, the water table has been rising gradually.

7.5 CAPILLARY RISE AND EFFECTIVE STRESS

Soils above the water table are, in general, not completely dry due to **capillary rise**, as seen in Figure 7.8(a). The capillary zone affected by the rise depends on the size of the void opening of the soils. The smaller the void spacing is, the higher the rise is. Small void spaces in soil assemblages work as capillary tubes. **Hazen (1930)** empirically gave an approximate maximum height of capillary rise, $h_{\text{capillary}}$, as a function of soil properties D_{10} and e as

$$h_{\text{capillary}} \text{ (in mm)} = \frac{C}{eD_{10}} \tag{7.7}$$

where e is the void ratio, D_{10} is the effective grain size in millimeters, and C is a constant with a range of 10 to 50. Table 7.1 shows general values of these for different soils. As can be seen in the table, capillary rise is quite high for finer soils.

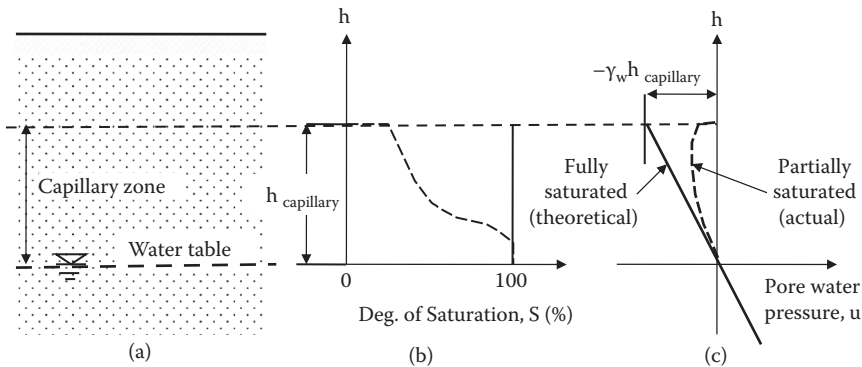


FIGURE 7.8 Capillary rise.

TABLE 7.1
Approximate Capillary Rise in Different Soils

Soil Type	Loose	Dense
Coarse sand	0.03–0.12 m	0.04–0.15 m
Medium sand	0.12–0.50 m	0.35–1.10 m
Fine sand	0.30–2.0 m	0.40–3.5 m
Silt	1.5–10 m	2.5–12 m
Clay		≥10 m

Source: *Hansbo, S., 1975, Jordmaterillara*, Almqvist & Wiksell Forlag AB, Stockholm, 218 pp.

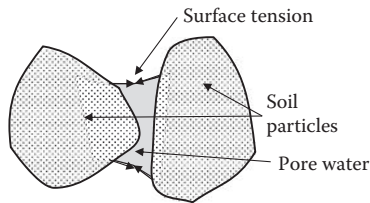


FIGURE 7.9 Surface tension between particles.

Capillary rise is due to suction created by the **surface tension** of water films around particles, as seen in Figure 7.9. This suction works as attractive particle-to-particle stress and creates negative pore water pressure. Thus, it increases the effective stress, according to Equation (7.1). Theoretically, the pore water pressure u in the capillary zone is $-\gamma_w h$ for fully saturated soils, as seen in Figure 7.8(c). However, the degree of saturation S in the capillary zone changes from nearly fully saturated condition (100%) at the water table level to very low at the highest rise, as shown in Figure 7.8(b). Therefore, the actual u is smaller than the theoretical one. It is approximated by

$$u = -\left(\frac{S}{100}\right) \gamma_w h_{\text{capillary}} \quad (7.8)$$

where S is expressed in percentage and $h_{\text{capillary}}$ is the height from the water table level. Actual pore water pressure u in the capillary rise zone is also shown in Figure 7.8(c) with a dotted line.

Because of negative pore water pressure in the capillary zone, the effective stress in that zone needs to be modified from the one without capillary rise consideration, as demonstrated in Exercise 7.4.

Exercise 7.4

For the soil condition given in Figure 7.10, calculate and plot σ , u , and σ' distributions with the depth considering the capillary rise. Assume that the average degree of saturation S in the capillary zone is 50%.

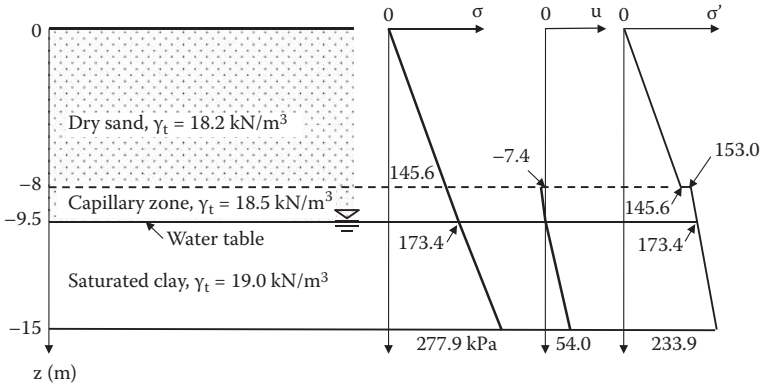


FIGURE 7.10 Effective stress computation with capillary tension.

SOLUTION

At $z = -8$ m without capillary rise,

$$\sigma = 8 \times 18.2 = 145.6 \text{ kPa}$$

$$u = 0$$

$$\sigma' = 145.6 \text{ kPa}$$

At $z = -8$ m with capillary rise,

$$\sigma = 8 \times 18.2 = 145.6 \text{ kPa}$$

$$u = -(S/100)\gamma_w h_{\text{capillary}} = -0.5 \times 9.81 \times 1.5 = -7.4 \text{ kPa}$$

$$\sigma' = 145.6 - (-7.4) = 153.0 \text{ kPa}$$

At $z = -9.5$ m,

$$\sigma = 8 \times 18.2 + 1.5 \times 18.5 = 173.4 \text{ kPa}$$

$$u = 0$$

$$\sigma' = 173.4 \text{ kPa}$$

At $z = -15$ m,

$$\sigma = 8 \times 18.2 + 1.5 \times 18.5 + 5.5 \times 19.0 = 277.9 \text{ kPa}$$

$$u = 5.5 \times 9.81 = 54.0 \text{ kPa}$$

$$\sigma' = 277.9 - 54.0 = 223.9 \text{ kPa}$$

The preceding distributions are plotted in Figure 7.10.

Note that, in Exercise 7.4, there is a discontinuity in the effective stress distribution curve at the top of capillary zone. This occurred due to using an average S value for the capillary zone in the exercise. In reality, that portion of the curve should change smoothly when the variation of S is properly accounted, though the precise estimation of S is not an easy task.

7.6 EFFECTIVE STRESS WITH WATER FLOW

When water flows through pores of soil mass, it drags the particles. The dragging action creates frictional force on the particle surface toward the direction of water flow, as seen in Figure 7.11. These frictional forces act on particles' surface work as **seepage force** and change the effective stress.

In Figure 7.12, a cylinder filled with soil is subjected to upward water flow due to the head difference at both ends of the soil column. Alongside, water pressure through the soil column is plotted. Point E is the water pressure from the supply side of water, and Point F is the discharge side of the system.

The pressure line AEB is a hydrostatic water pressure based on the left side of the water supply, while CFD is a hydrostatic water pressure from the right side of the water supply, both of which are parallel and have a slope of $1/\gamma_w$. On these two lines, only EB and CF sections with solid lines are real pressures, and AE and FD sections with dotted lines are just extensions of the EB and CF lines, respectively.

Since the water pressure is continuous through the soil column and the upward seepage force changes proportionally with hydraulic pressure loss through the specimen, the water pressure changes linearly with the depth in this case; thus, Points F and E are connected with the solid straight line. Therefore, the real water pressure distribution of the system becomes CFEB in the figure.

Referring to Figure 7.12, at the bottom of the soil column (at Point E), the total water pressure is $\gamma_w(\Delta h + H_1 + H)$, which is higher than the hydrostatic pressure $\gamma_w(H_1 + H)$ without water flow (i.e., at Point D). This extra water pressure of $\gamma_w\Delta h$ is called upward **seepage pressure**. This is created by dragging the force of upward water flow through the soil. The triangle FDE in the figure is the upward seepage pressure due to upward water flow. At arbitrary depth z , the seepage pressure is

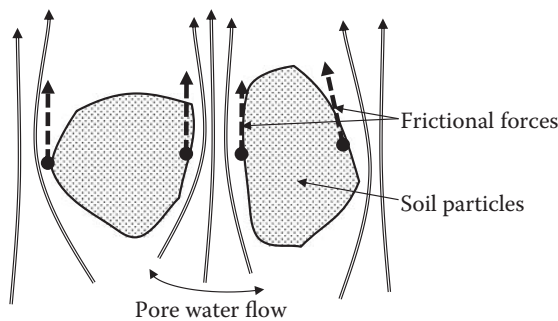


FIGURE 7.11 Upward seepage force.

Dr Mohamed Adb Elatif

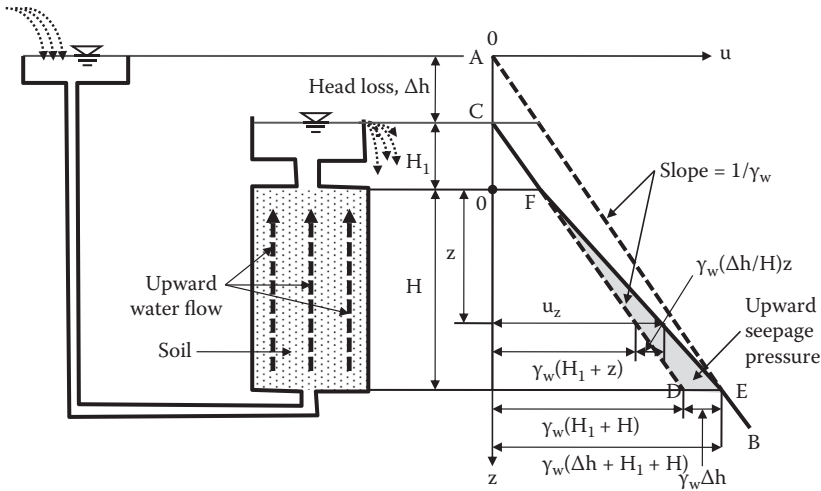


FIGURE 7.12 Water pressure with upward seepage flow.

calculated as $\gamma_w(\Delta h/H)z$ by using a proportionality in the triangle FDE. Thus, water pressure u_z at depth z is

$$u_z = \gamma_w(H_1 + z) + \gamma_w(\Delta h/H)z \tag{7.9}$$

And the total stress σ_z at depth z is

$$\sigma_z = \gamma_w H_1 + \gamma_t z = \gamma_w H_1 + (\gamma' + \gamma_w)z = \gamma_w(H_1 + z) + \gamma'z \tag{7.10}$$

And thus, the effective stress σ'_z at depth z is

$$\sigma'_z = \sigma_z - u_z = [\gamma_w(H_1 + z) + \gamma'z] - [\gamma_w(H_1 + z) + \gamma_w(\Delta h/H)z] = \gamma'z - \gamma_w(\Delta h/H)z \tag{7.11}$$

Now, the condition of $\sigma'_z = 0$ in Equation (7.11) yields

$$\frac{\Delta h}{H} = \frac{\gamma'}{\gamma_w} = i_c \tag{7.12}$$

where i_c is called **critical hydraulic gradient**. Equation (7.12) implies that, when the $\Delta h/H$ ratio is equal to or higher than the i_c value (γ'/γ_w), the effective stress is zero or negative. The effective stress is the interparticle stress, and thus zero or negative interparticle stress implies separation of the particles. This condition causes **quick-sand** (or **sand boiling**) of granular soils and **heave** of cohesive soils.

The i_c value is approximately 1.0 since γ' ($= \gamma_t - \gamma_w$) is nearly equal to γ_w for many soils (e.g., $\gamma_t = 18 \sim 20 \text{ kN/m}^3$, and $\gamma_w = 9.81 \text{ kN/m}^3$). Thus, **when the total head loss Δh exceeds approximately the length of the specimen H , these critical conditions would prevail.**

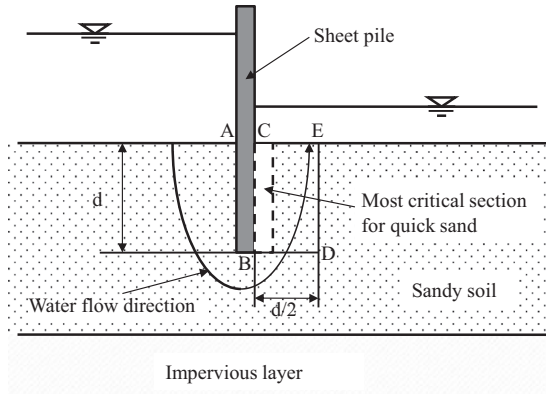


FIGURE 7.13 Critical section for quicksand on cut-off sheet pile.

7.7 QUICKSAND (SAND BOILING)

Quicksand or sand boiling is best demonstrated in the case of the cut-off sheet pile situation as shown in Figure 7.13. In the figure, water flows from left to right due to the total head difference. Soils near the BC section of the sheet pile are subjected to upward seepage pressure and potentially possess the quicksand condition. The **factor of safety for the quicksand condition** can be measured by

$$F.S. = \frac{i_c}{i_{B \rightarrow C}} \tag{7.13}$$

where i_c is the critical hydraulic gradient defined in Equation (7.12), and $i_{B \rightarrow C}$ is the hydraulic gradient from Point B to Point C computed by

$$i_{B \rightarrow C} = \frac{\Delta h_{B \rightarrow C}}{\overline{BC}} = \frac{h_B - h_C}{\overline{BC}} \tag{7.14}$$

where h_B and h_C are total hydraulic heads at Points B and at C, respectively, and \overline{BC} is the length of the water flow from Point B to Point C. The heads h_B and h_C can be read from equipotential lines of the flow net as discussed in Chapter 6. The zone along BC is the most critical section for the quick condition, since H in Equation (7.12) is the smallest for the same interval of equipotential lines (Δh_i) in this sheet pile problem.

Terzaghi (1922) suggested evaluating the factor of safety against quicksand for the section of $d \times d/2$ (area BCED) based on his experimental observation, which is seen in Figure 7.13. The average head loss from B–D to C–E can be computed from $\Delta h_{B-D \rightarrow C-E} = h_{B-D} - h_{C-E}$, and the flow distance B–D to C–E is approximately d (depth of sheet pile) in this case.

Exercise 7.5

For a given flow net of water flow around a sheet pile in Figure 7.14, compute the factor of safety against the quicksand condition, (a) at the most critical section

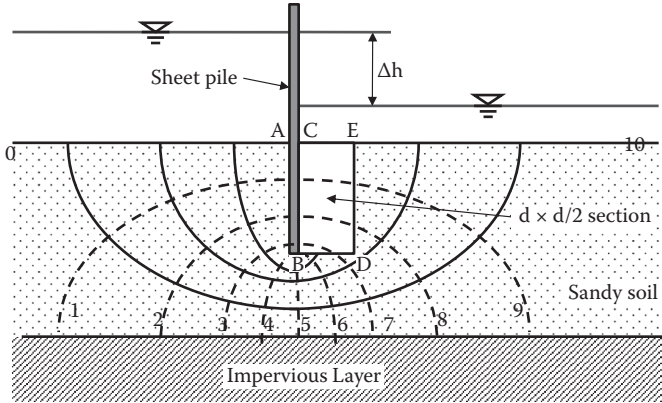


FIGURE 7.14 Exercise 7.5 problem.

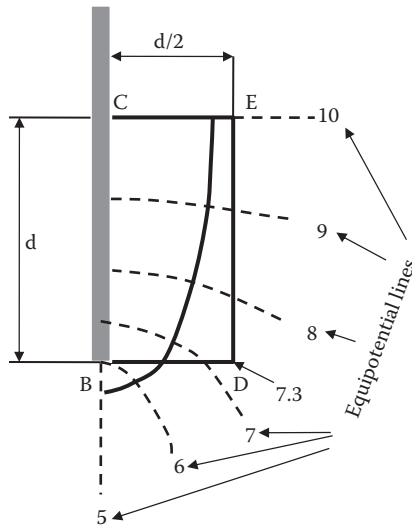


FIGURE 7.15 Enlarged picture of Terzaghi’s quicksand computation zone.

along the downstream face of the sheet pile, and (b) by Terzaghi’s method. In the figure, the total head loss $\Delta h = 7$ m, the sheet pile depth $d = 10$ m, and $\gamma_t = 19.0$ kN/m³.

SOLUTION

Figure 7.15 is an enlarged drawing of the right side of the sheet pile where the quicksand condition is most likely to occur.

$$N_d = 10.0$$

$$\Delta h_i = \Delta h / n_d = 7 / 10 = 0.7 \text{ m}$$

- (a) Along the face BC, there are five equipotential drops from Point B to Point C, and thus,

$$i_{B \rightarrow C} = \Delta h_i \times (10 - 5)/d = 0.7 \times 5/10 = 0.35$$

$$i_c = \gamma'/\gamma_w = (19 - 9.81)/9.81 = 0.937$$

Thus, F.S. = $i_c/i_{B \rightarrow C} = 0.937/0.35 = 2.68$ (>1.0 , safe against the quicksand condition).

- (b) For the BDEC section ($d \times d/2$ section) by Terzaghi:
Referring to Figure 7.15, Point D is on nearly the 7.3rd equipotential line. From Points B to C, there are five equipotential drops, and from Points D to E, there are 2.7 equipotential drops; thus, the average equipotential drops from line B–D to line C–E is approximately $(5 + 2.7)/2 = 3.85$. Therefore, the average total head drop from B–D to C–E is

$$\Delta h_{B-D \rightarrow C-E} = \Delta h_i \times 3.85 = 0.7 \times 3.85 = 2.695 \text{ m.}$$

$$i_{B-D \rightarrow C-E} = \Delta h_{B-D \rightarrow C-E}/d = 2.695/10 = 0.270$$

Thus, F.S. = $i_c/i_{B-D \rightarrow C-E} = 0.937/0.270 = 3.47$ (>1.0 , safe against the quicksand condition).

In Exercise 7.5, it should be noted that Terzaghi's $d \times d/2$ section method provides a higher factor of safety than the critical section along the downstream face of the sheet pile.

7.8 HEAVE OF CLAY DUE TO EXCAVATION

When ground excavation is done on clay soil to a certain depth, the heave at the bottom of the excavation will pose a potential danger at construction sites. This bottom heave is due to reduction in effective stress. Depending on excavation procedures, there are two categories of the heave problem: **dry excavation** and **wet excavation**.

7.8.1 DRY EXCAVATION

When excavation is done rather quickly or the water in the excavated pit is continuously pumped out, **dry excavation** may prevail. In this case, during excavation, the bottom of the excavated pit is rather dry. A typical situation is shown in Figure 7.16. The top clay layer is under an artesian water pressure. That is, due to a nearby lake or river, the water table in the clay layer is steady, and a permeable layer underneath the clay layer is connected to the water supply from the lake or river. The effective stress at the bottom of a clay layer, Point A, is computed as

$$\sigma = (H_{\text{clay}} - H_{\text{ex}})\gamma_{\text{clay}} \quad (7.15)$$

$$u = h_w \gamma_w \quad (7.16)$$

$$\sigma' = \sigma - u = (H_{\text{clay}} - H_{\text{ex}})\gamma_{\text{clay}} - h_w \gamma_w \quad (7.17)$$

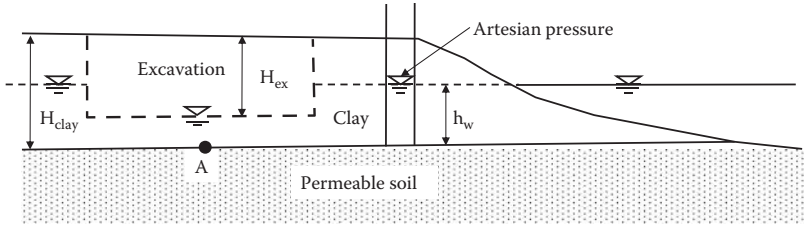


FIGURE 7.16 Heave of clay (dry excavation).

Note that the pore water pressure at Point A is not $(H_{\text{clay}} - H_{\text{ex}})\gamma_w$, but rather $h_w\gamma_w$ due to the artesian water pressure. In Equation (7.17), $\sigma' > 0$ is the condition for safe excavation without heave. If the $\sigma' < 0$ condition would prevail, the bottom of the excavation would heave, and the excavation site would be prone to disaster.

Exercise 7.6

As in Figure 7.16, the excavated pit is kept dry by continuous pumping of water. The clay layer thickness H_{clay} is 15 m, and the artesian pressure height h_w is 10 m. $\gamma_{\text{clay}} = 18.0 \text{ kN/m}^3$. Determine the maximum excavation depth H_{ex} without heave.

SOLUTION

At the bottom of the clay layer, Point A, from Equation (7.17), the effective stress σ' is

$$\sigma' = \sigma - u = (H_{\text{clay}} - H_{\text{ex}})\gamma_{\text{clay}} - h_w\gamma_w = (15 - H_{\text{ex}}) \times 18.0 - 10 \times 9.81 > 0$$

By solving this equation for H_{ex} , $H_{\text{ex}} < 9.55 \text{ m}$, and thus the maximum safe excavation depth without bottom heave is **9.55 m**. ←

7.8.2 WET EXCAVATION

When an excavation process is rather slow, water seeps out and fills the excavation site. This situation is called **wet excavation**. Figure 7.17 shows such a situation, in which h_{ex} is the water depth in the excavated pit. In this case, the effective stress at Point A is

$$\sigma = h_{\text{ex}}\gamma_w + (H_{\text{clay}} - H_{\text{ex}})\gamma_{\text{clay}} \tag{7.18}$$

$$u = h_w\gamma_w \tag{7.19}$$

$$\sigma' = \sigma - u = h_{\text{ex}}\gamma_w + (H_{\text{clay}} - H_{\text{ex}})\gamma_{\text{clay}} - h_w\gamma_w \tag{7.20}$$

Note again that the pore water pressure at Point A still remains the same as in the dry excavation case due to the artesian water pressure. In Equation (7.20), $\sigma' > 0$ is the condition for safe excavation without heave.

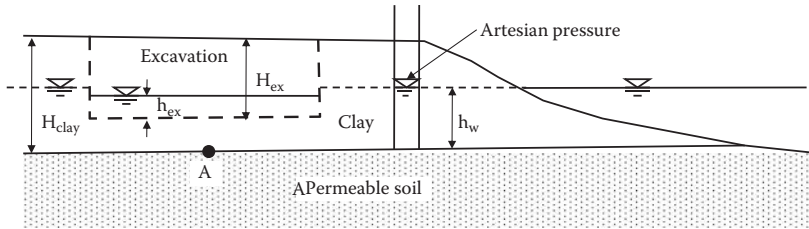


FIGURE 7.17 Heave of clay (wet excavation).

Exercise 7.7

Referring to Figure 7.17, H_{clay} is 15 m, h_w is 10 m, and $\gamma_{\text{clay}} = 18.0 \text{ kN/m}^3$. First, wet excavation was done to 10 m ($H_{\text{ex}} = 10 \text{ m}$), and the water level in the pit was 5 m ($h_{\text{ex}} = 5 \text{ m}$).

- Check the excavation safety for this wet excavation.
- If (a) is safe against heaving, by how much could the water level in the pit be lowered by pumping without heave?

SOLUTION

- From Equation (7.20), the effective stress at Point A is

$$\begin{aligned}\sigma' &= \sigma - u = h_{\text{ex}}\gamma_w + (H_{\text{clay}} - H_{\text{ex}})\gamma_{\text{clay}} - h_w\gamma_w \\ &= 5 \times 9.81 + (15 - 10) \times 18 - 10 \times 9.81 = \mathbf{+40.95 \text{ kPa}}\end{aligned}$$

The σ' value is a positive number, so the pit is safe against bottom heave. ←

- When the water level in the pit is further lowered with Δh_{ex} ($h_{\text{ex}} - \Delta h_{\text{ex}}$) is substituted in h_{ex} in Equation (7.20) to compute the effective stress at Point A, and it becomes

$$\begin{aligned}\sigma' &= \sigma - u = (h_{\text{ex}} - \Delta h_{\text{ex}})\gamma_w + (H_{\text{clay}} - H_{\text{ex}})\gamma_{\text{clay}} - h_w\gamma_w \\ &= (5 - \Delta h_{\text{ex}}) \times 9.81 + (15 - 10) \times 18 - 10 \times 9.81\end{aligned}$$

By solving $\sigma' > 0$ in the equation, the limit of Δh_{ex} value is obtained as

$$\Delta h_{\text{ex}} < \mathbf{4.15 \text{ m}} \text{ (or water depth in the pit} = 5.0 - 4.15 = \mathbf{0.85 \text{ m}}) \leftarrow$$

This is the lowest depth to which water could be pumped out without heave at the bottom.

By comparing Exercises 7.6 (dry excavation) and 7.7 (wet excavation), it can be observed that wet excavation can go a little deeper (i.e., 9.55 m in dry excavation versus 10 m with 0.85 m water depth in the pit). It should be noted that computation of the critical excavation depth is a purely theoretical one, so the actual safe excavation

depth would be smaller than that of the computed value considering water level fluctuation, uncertainty in soil properties, etc.

7.9 SUMMARY

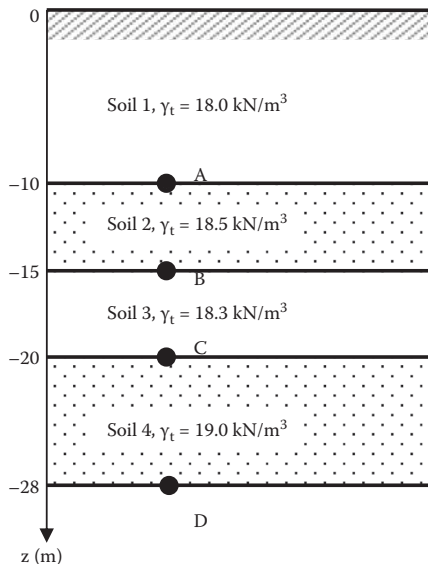
The effective stress concept is the most important contribution made by Terzaghi. The effective vertical stress is the one used to determine the current soil formation. It controls volume change (Chapter 9) and strength of soils (Chapter 11). Capillary rise, seepage force, quicksand or sand boiling, and heaving of clay at the bottom of the excavation are all related to effective stress. *Understanding the concept and computation technique of effective stress is critically important in the modern soil mechanics of practice.*

REFERENCES

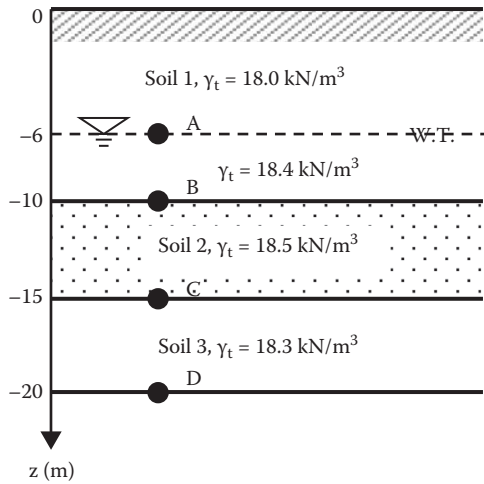
Hansbo, S. (1975), *Jordmateriallara*, Almqvist & Wiksell Forlag AB, Stockholm.
 Hazen, A. (1930), Water supply, in *American Civil Engineering Handbook*, Wiley, New York.
 Terzaghi, K. (1922), Der Groundbruch an Stauwerken und seine Verhutung, *Die Wasserkraft*, vol. 17, 445–449.
 ———. (1925), *Erdbaumechanik*, Franz Deuticke.

Problems

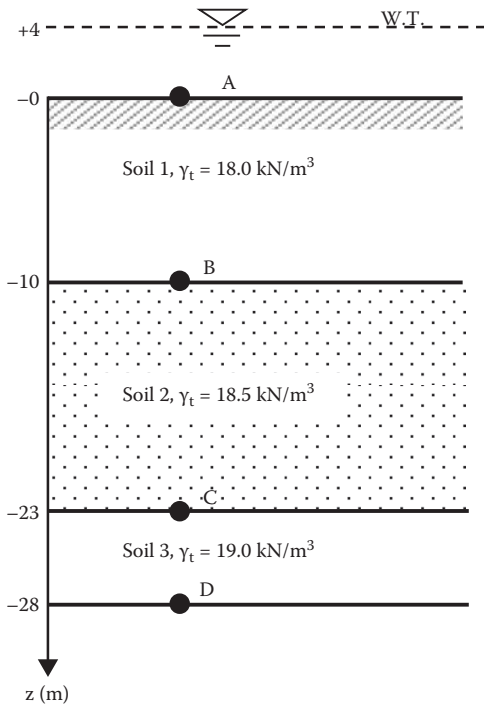
- 7.1 Define effective stress and explain its importance in soil mechanics.
- 7.2 Compute the total vertical stress σ , pore water pressure u , and then the effective vertical stress σ' at Points A, B, C, and D in the soil profile shown in the following figure. Plot those with the depth z .



- 7.3 Compute the total vertical stress σ , pore water pressure u , and then the effective vertical stress σ' at Points A, B, C, and D in the soil profile shown in the following figure. Plot those with the depth z .



- 7.4 Compute the total vertical stress σ , pore water pressure u , and then the effective vertical stress σ' at Points A, B, C, and D in the soil profile shown in the following figure. Plot those with the depth z .



Chapter 6 Stress Increments in Soil Mass

8.1 INTRODUCTION

Chapter 7 studied computational techniques of current effective vertical stress in soil mass and that stress defines the current structure of soils in most cases. Soils are stable under the existing effective overburden stresses. However, when additional loads are placed on the ground surface, such as by footings, traffic loads, etc., those additional loads increase the stresses in the soil mass. These extra stresses are major sources of the settlement of soils. In this chapter, equations for the vertical stress increments in soil mass due to various types of load on the ground surface are discussed. They will be used in settlement computation in Chapter 9.

8.2 2:1 APPROXIMATE SLOPE METHOD

Figure 8.1 shows a $B \times L$ rectangular footing on ground surface with a vertical force P , which is applied on the center of the footing. Vertical stress $\sigma_{v,0}$ on the ground surface is $P/(B \times L)$. This $\sigma_{v,0}$ is redistributed over a wider loading area with increasing depth z . A slope with 2 in vertical to 1 in horizontal defines spread loading areas within the soil mass. Stress is spread over an area of $(B + z) \times (L + z)$ at depth z . Accordingly, the vertical stress increment $\Delta\sigma_v$ at depth z can be calculated from

$$\Delta\sigma_v = \frac{P}{(B + z)(L + z)} \quad (8.1)$$

Vertical stress decreases with increasing depth z with increased distributed area as seen in Figure 8.1. In this method, it is assumed that the stress is uniformly distributed over $(B + z) \times (L + z)$ area and it suddenly becomes zero beyond the zone defined by 2:1 slope. The real stress distribution is, however, a smooth change over the area under the footing with its maximum value directly under the center of the footing. This is a simple approximate method, and thus it could be used as a rough estimation of stress increment computation.

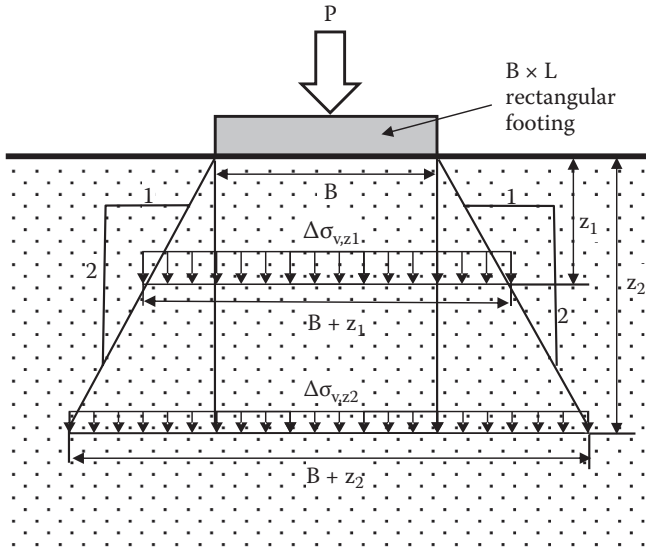


FIGURE 8.1 Vertical stress increment by approximate 2:1 slope method.

Exercise 8.1

A 5 kN point load is applied at the center of 1 m × 1 m square footing on the ground surface. Compute and plot the magnitudes of a vertical stress increment under the center of the footing at the depths 2, 4, 6, 8, and 10 m from the ground surface. Use the 2:1 approximate slope method.

SOLUTION

$P = 5 \text{ kN}$, $B = L = 1 \text{ m}$, and a spreadsheet (Table 8.1) is prepared to obtain vertical stress distribution $\Delta\sigma_v$ with depth z using Equation (8.1). The result is plotted in Figure 8.2.

TABLE 8.1
 $\Delta\sigma_v$ by 2:1 Slope Method

A	B
$z \text{ (m)}$	$\Delta\sigma_v \text{ (kN/m}^2\text{)}$
0	5.00
2	0.56
4	0.20
6	0.10
8	0.06
10	0.04

Note: $B_i = P/(1 + A_i)(1 + A_i)$.

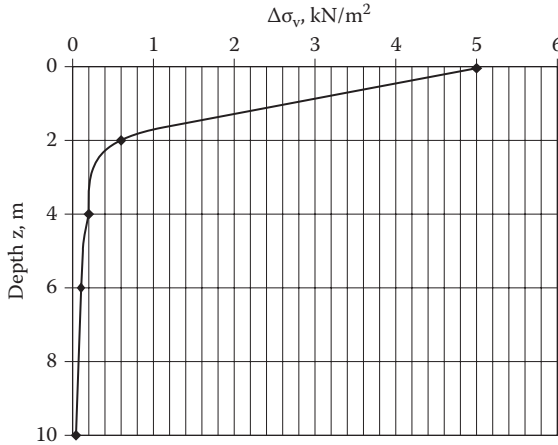


FIGURE 8.2 Δσ_v distribution (Exercise 8.1).

8.3 VERTICAL STRESS INCREMENT DUE TO A POINT LOAD

Boussinesq (1885) developed an elastic solution for stresses in an isotropic homogeneous elastic media due to a point load on the ground surface, as shown in Figure 8.3.

The vertical stress increment under radius *r* from the loading point is given by

$$\Delta\sigma_v = \frac{3}{2\pi} \frac{P}{z^2} \cos^5 \theta = \frac{3Pz^3}{2\pi(r^2 + z^2)^{5/2}} = \frac{P}{z^2} \frac{3}{2\pi} \frac{1}{\left[1 + \left(\frac{r}{z}\right)^2\right]^{5/2}} = \frac{P}{z^2} I_1 \quad (8.2)$$

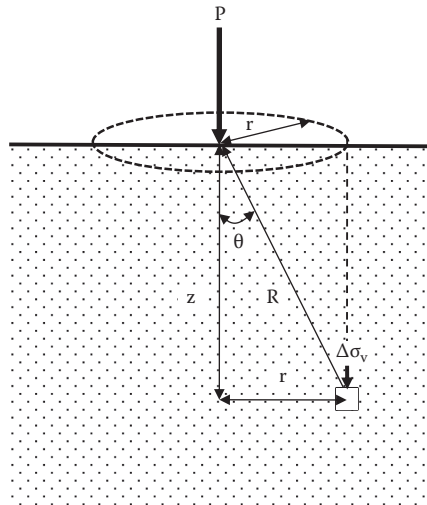


FIGURE 8.3 Boussinesq's point load problem.

$$I_1 = \frac{3}{2\pi} \frac{1}{\left[1 + \left(\frac{r}{z}\right)^2\right]^{5/2}} \tag{8.3}$$

where I_1 is called the **influence factor** for stress increment computation, and R , r , z , and θ are defined in Figure 8.3. I_1 is a sole function of the r/z ratio, and the values are tabulated in Table 8.2 and plotted in Figure 8.4.

TABLE 8.2
Influence Factor I_1 by Equation (8.3) (Boussinesq's Point Load)

r/z	I_1	r/z	I_1	r/z	I_1
0	0.4775	0.32	0.3742	0.85	0.1226
0.02	0.4770	0.34	0.3632	0.9	0.1083
0.04	0.4756	0.36	0.3521	0.95	0.0956
0.06	0.4732	0.38	0.3408	1	0.0844
0.08	0.4699	0.4	0.3295	1.2	0.0513
0.1	0.4657	0.42	0.3181	1.4	0.0317
0.12	0.4607	0.44	0.3068	1.6	0.0200
0.14	0.4548	0.46	0.2955	1.8	0.0129
0.16	0.4482	0.48	0.2843	2	0.0085
0.18	0.4409	0.5	0.2733	2.2	0.0058
0.2	0.4329	0.55	0.2466	2.4	0.0040
0.22	0.4243	0.6	0.2214	2.6	0.0028
0.24	0.4151	0.65	0.1978	2.8	0.0021
0.26	0.4054	0.7	0.1762	3	0.0015
0.28	0.3954	0.75	0.1565	4	0.0004
0.3	0.3849	0.8	0.1386	5	0.0001

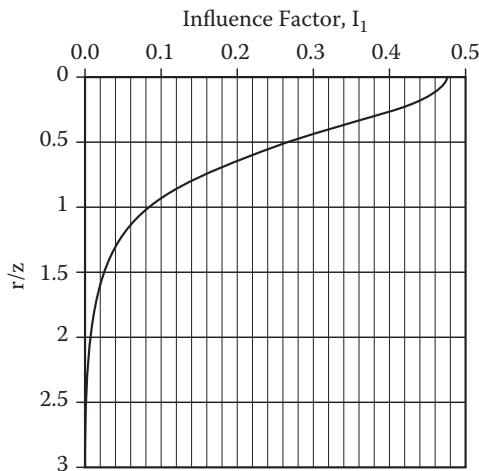


FIGURE 8.4 Influence factor, I_1 versus r/z (point load).

Exercise 8.2

A 5 kN point load is applied on the ground surface. Compute and plot the magnitudes of vertical stress increment (a) under the point load at the depth z from 0 to 10 m below the ground surface, and (b) under 1.0 m off from the load application point at the same depth as above. Use Boussinesq’s method.

SOLUTION

- (a) $r/z = 0$ and $I_1 = 0.4775$ is obtained from Equation (8.3) or Table 8.2.
- (b) $r = 1$ m, and thus r/z varies with depth.

A spreadsheet (Table 8.3) is created for the computation, and the results are plotted in Figure 8.5. Note that Equation (8.2) gives the infinite $\Delta\sigma_v$ value

TABLE 8.3
 $\Delta\sigma_v$ Computation under a Point Load

(a) $r = 0$ m					(b) $r = 1$ m				
A	B	C	D	E	A	B	C	D	E
$z, \text{ m}$	$r, \text{ m}$	r/z	I_1	$\Delta\sigma_v$	$z, \text{ m}$	$r, \text{ m}$	r/z	I_1	$\Delta\sigma_v$
0	0	0	0.4775	∞	0	1	∞	0	0
0.3	0	0	0.4775	26.53	0.3	1	3.33	0.0009	0.05
0.5	0	0	0.4775	9.55	0.5	1	2.00	0.0085	0.17
1	0	0	0.4775	2.39	1	1	1.00	0.0844	0.42
2	0	0	0.4775	0.60	2	1	0.50	0.2733	0.34
4	0	0	0.4775	0.15	4	1	0.25	0.4103	0.13
6	0	0	0.4775	0.07	6	1	0.17	0.4459	0.06
8	0	0	0.4775	0.04	8	1	0.13	0.4593	0.04
10	0	0	0.4775	0.02	10	1	0.10	0.4657	0.02

Note: Column $E_i = P/z^2 \times D_i$ (Equation 8.2).

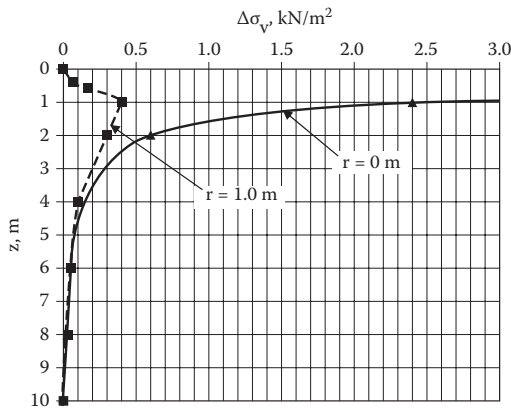


FIGURE 8.5 $\Delta\sigma_v$ distributions under a point load (Exercise 8.2).

directly underneath the point load (that is, $r = 0$ and $z = 0$) as a special case. When distance r takes a non-zero value, $\Delta\sigma_v$ value becomes zero at $z = 0$ as seen in case (b).

8.4 VERTICAL STRESS INCREMENT DUE TO A LINE LOAD

The rest of the solutions are all from the integrations of Boussinesq's point load solution (Equation 8.2) over the area (or line) where the load is applied on the ground surface. As seen in Figure 8.6, line load q is applied on an infinitesimally long line on the ground, and $\Delta\sigma_v$ is obtained in a soil mass at (z, r) , where distance r is measured perpendicular to the line of load. Integration of Equation (8.2) over a loaded line from $-\infty$ to $+\infty$ gives

$$\Delta\sigma_v = \frac{2qz^3}{\pi R^4} = \frac{2q}{\pi z \left[\left(\frac{r}{z} \right)^2 + 1 \right]^2} = \frac{q}{z} \frac{2}{\pi \left[\left(\frac{r}{z} \right)^2 + 1 \right]^2} = \frac{q}{z} I_2 \quad (8.4)$$

$$I_2 = \frac{2}{\pi \left[\left(\frac{r}{z} \right)^2 + 1 \right]^2} \quad (8.5)$$

Table 8.4 tabulates influence factor I_2 as a function of r/z .

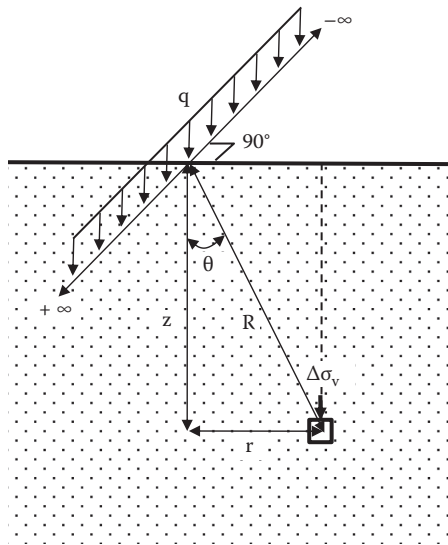


FIGURE 8.6 Vertical stress increment due to a line load.

TABLE 8.4
Influence Factor I_2 by Equation (8.5) (Line Load)

r/z	I_2	r/z	I_2	r/z	I_2
0	0.637	1.1	0.130	2.2	0.019
0.1	0.624	1.2	0.107	2.4	0.014
0.2	0.589	1.3	0.088	2.6	0.011
0.3	0.536	1.4	0.073	2.8	0.008
0.4	0.473	1.5	0.060	3	0.006
0.5	0.407	1.6	0.050	3.2	0.005
0.6	0.344	1.7	0.042	3.4	0.004
0.7	0.287	1.8	0.035	3.6	0.003
0.8	0.237	1.9	0.030	3.8	0.003
0.9	0.194	2	0.025	4	0.002
1	0.159			5	0.001

8.5 VERTICAL STRESS INCREMENT DUE TO A STRIP LOAD

Uniformly distributed strip load q is applied on the ground with footing width B as seen in Figure 8.7. $\Delta\sigma_v$ at point (x, z) can be obtained by an integration of Equation (8.2) over x from $-B/2$ to $+B/2$ and y from $-\infty$ to $+\infty$.

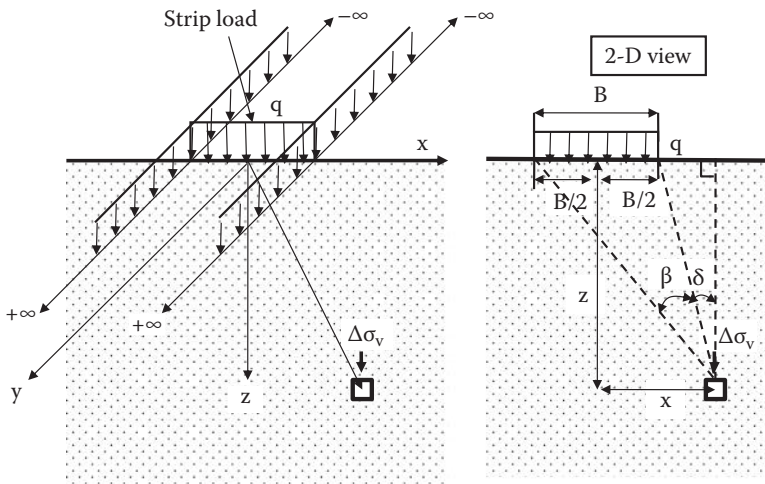


FIGURE 8.7 Vertical stress increment due to a strip load.

The integrated solution is given by

$$\Delta\sigma_v = \frac{q}{\pi} [\beta + \sin\beta \cos(\beta + 2\delta)]$$

$$= \frac{q}{\pi} \left\{ \left[\tan^{-1} \left(\frac{\frac{2z}{B}}{\frac{2x}{B}-1} \right) - \tan^{-1} \left(\frac{\frac{2z}{B}}{\frac{2x}{B}+1} \right) \right] - \frac{\frac{2z}{B} \left[\left(\frac{2x}{B} \right)^2 - \left(\frac{2z}{B} \right)^2 - 1 \right]}{2 \left[\frac{1}{4} \left\{ \left(\frac{2x}{B} \right)^2 + \left(\frac{2z}{B} \right)^2 - 1 \right\}^2 + \left(\frac{2z}{B} \right)^2 \right]} \right\}$$

$$= qI_3 \tag{8.6}$$

Note that in Equation (8.6), when $2x/B < 1$ (i.e., point (x, z) is inside the foundation width B), the value in the first term of the second line becomes negative. To get the correct angle value in that case, π should be added to it. That is, the first term of the second line of Equation (8.6) should be replaced by

$$\text{For } 2x/B < 1, \tan^{-1} \left(\frac{\frac{2z}{B}}{\frac{2x}{B}-1} \right) + \pi \tag{8.7}$$

Table 8.5 shows the values of I_3 as a function of $2x/B$ and $2z/B$ and Figure 8.8 plots those values near the footing.

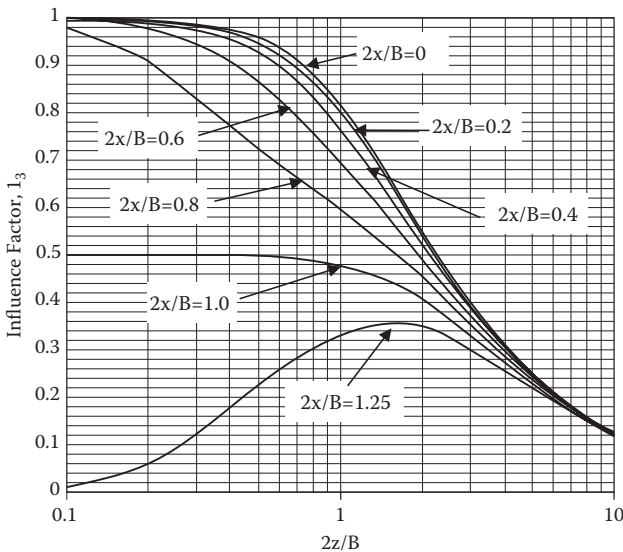


FIGURE 8.8 Influence factor I_3 .

Dr Mohamed Adb Elatif

Exercise 8.3

A strip load $q = 100 \text{ kN/m}^2$ is applied over a footing with width $B = 5 \text{ m}$.

Calculate and plot the vertical stress distribution over the x distance at $z = 5 \text{ m}$ and at $z = 10 \text{ m}$ under the footing.

SOLUTION

At $z = 5 \text{ m}$, $2z/B = 2 \times 5/5 = 2$.

At $z = 10 \text{ m}$, $2z/B = 2 \times 10/5 = 4$

For the preceding $2z/B$ values, I_3 values were read from Table 8.5 and $\Delta\sigma_v$ values were computed in Table 8.6 for various x values. The results are plotted in Figure 8.9 for a half space ($x > 0$ region).

TABLE 8.6
Computation for Exercise 8.3

At $z = 5 \text{ m}$, $2z/B = 2$										
$x, \text{ m}$	0	0.5	1	2	2.5	3.125	3.75	5	7.5	12.5
$2x/B$	0	0.2	0.4	0.8	1	1.25	1.5	2	3	5
I_3	0.550	0.543	0.524	0.455	0.409	0.348	0.288	0.185	0.071	0.013
$\Delta\sigma_v, \text{ kN/m}^2$	55.0	54.3	52.4	45.5	40.9	34.8	28.8	18.5	7.1	1.3

At $z = 10 \text{ m}$, $2z/B = 4$										
$x, \text{ m}$	0	0.5	1	2	2.5	3.125	3.75	5	7.5	12.5
$2x/B$	0	0.2	0.4	0.8	1	1.25	1.5	2	3	5
I_3	0.306	0.304	0.301	0.285	0.275	0.259	0.242	0.205	0.134	0.051
$\Delta\sigma_v, \text{ kN/m}^2$	30.6	30.4	30.1	28.5	27.5	25.9	24.2	20.5	13.4	5.1

Note: I_3 is from Table 8.5; $\Delta\sigma_v = q \times I_3$.

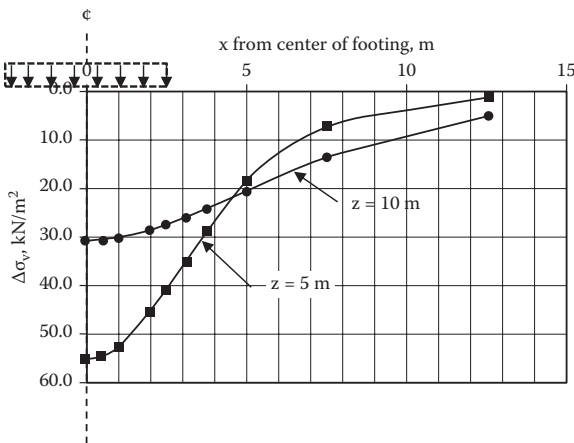


FIGURE 8.9 Solution for Exercise 8.3.

8.6 VERTICAL STRESS INCREMENT UNDER A CIRCULAR FOOTING

A popular footing shape is a circular one, and Boussinesq's solution is integrated for a uniformly loaded circular area as seen in Figure 8.10. Equation (8.8) is the solution for $\Delta\sigma_v$ directly under the center of circular footing.

$$\Delta\sigma_v = q \left[1 - \frac{1}{\left[\left(\frac{r}{z} \right)^2 + 1 \right]^{3/2}} \right] = qI_4 \tag{8.8}$$

$$I_4 = 1 - \frac{1}{\left[\left(\frac{r}{z} \right)^2 + 1 \right]^{3/2}} \tag{8.9}$$

I_4 values are tabulated in Table 8.7 and plotted in Figure 8.11 as a function of z/r .

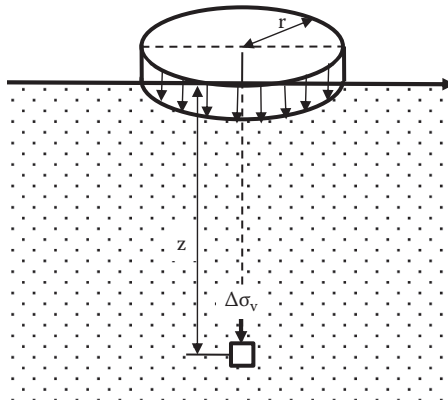


FIGURE 8.10 $\Delta\sigma_v$ under the center of circular footing.

TABLE 8.7
Influence Factor I_4 by Equation (8.9) (Circular Load)

z/r	I_4	z/r	I_4
0	1.000	1.2	0.547
0.1	0.999	1.4	0.461
0.2	0.992	1.6	0.390
0.3	0.976	1.8	0.332
0.4	0.949	2	0.284
0.5	0.911	2.5	0.200
0.6	0.864	3	0.146
0.7	0.811	3.5	0.111
0.8	0.756	4	0.087
0.9	0.701	4.5	0.070
1	0.646	5	0.057

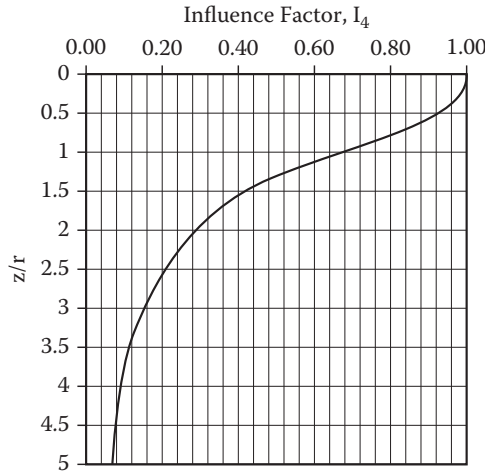


FIGURE 8.11 Influence factor I₄.

8.7 VERTICAL STRESS INCREMENT UNDER AN EMBANKMENT LOAD

Another frequently encountered loading pattern is due to embankments. Figure 8.12 shows a half section of an embankment load. The integrated solution is given by

$$\Delta\sigma_v = \frac{q}{\pi} \left[\frac{B_1 + B_2}{B_1} (\alpha_1 + \alpha_2) - \frac{B_2}{B_1} \alpha_2 \right] = qI_5 \tag{8.10}$$

$$I_5 = \frac{q}{\pi} \left[\frac{B_1 + B_2}{B_1} (\alpha_1 + \alpha_2) - \frac{B_2}{B_1} \alpha_2 \right] \tag{8.11}$$

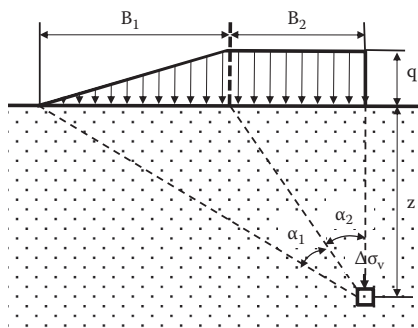


FIGURE 8.12 Vertical stress increment under a half embankment load.

$$\alpha_1 = \tan^{-1}\left(\frac{B_1 + B_2}{z}\right) - \tan^{-1}\left(\frac{B_2}{z}\right) \quad (8.12)$$

$$\alpha_2 = \tan^{-1}\left(\frac{B_2}{z}\right) \quad (8.13)$$

Table 8.8 shows influence factor I_5 as a function of B_1/z and B_2/z and Figure 8.13 plots the results. This is a convenient solution to obtain the stress increment under embankments by using a superposition of solutions as demonstrated in Exercise 8.4. Since the Boussinesq's solution is for an elastic media, the **principle of superposition** of several independent solutions is perfectly legitimate.

Exercise 8.4

An embankment as seen in Figure 8.14 is constructed. Determine $\Delta\sigma_v$ at $z = 12$ m below the ground surface: (a) directly below the centerline of the embankment, and (b) directly under the toe of the embankment. Use the total unit weight of embankment as $\gamma_t = 19.5$ kN/m³.

SOLUTION

$$q = \gamma_t H = 19.5 \times 3 = 58.5 \text{ kN/m}^2$$

- (a) Under the center, the solution is obtained by a superposition of two equal half embankments. For each half embankment,

$$B_1 = 6 \text{ m and } B_2 = 3 \text{ m}$$

$$B_1/z = 6/12 = 0.5, B_2/z = 3/12 = 0.25$$

From Figure 8.13, $I_5 = 0.268$ is read.

From Equation (8.10),

$$\Delta\sigma_v = 2 \times q \times I_5 = 2 \times 58.5 \times 0.268 = \mathbf{31.36 \text{ kN/m}^2} \leftarrow$$

- (b) Under the toe of the embankment, the following superposition is made. That is, (a) = (b) – (c) in Figure 8.15:

For Figure 8.15 (b), $B_1 = 6$ m, $B_2 = 12$ m

$$B_1/z = 6/12 = 0.5, B_2/z = 12/12 = 1.0$$

From Table 8.8, $I_5 = 0.438$ is obtained.

For Figure 8.15 (c), $B_1 = 6$ m, $B_2 = 0$ m

$$B_1/z = 6/12 = 0.5, B_2/z = 0/12 = 0$$

From Figure 8.13, $I_5 = 0.148$ is read.

From Equation (8.10) and superposition of two figures (i.e., (b) – (c)),

$$\Delta\sigma_v = q \times (I_5 (b) - I_5 (c)) = 58.5 \times (0.438 - 0.148) = \mathbf{16.97 \text{ kN/m}^2} \leftarrow$$

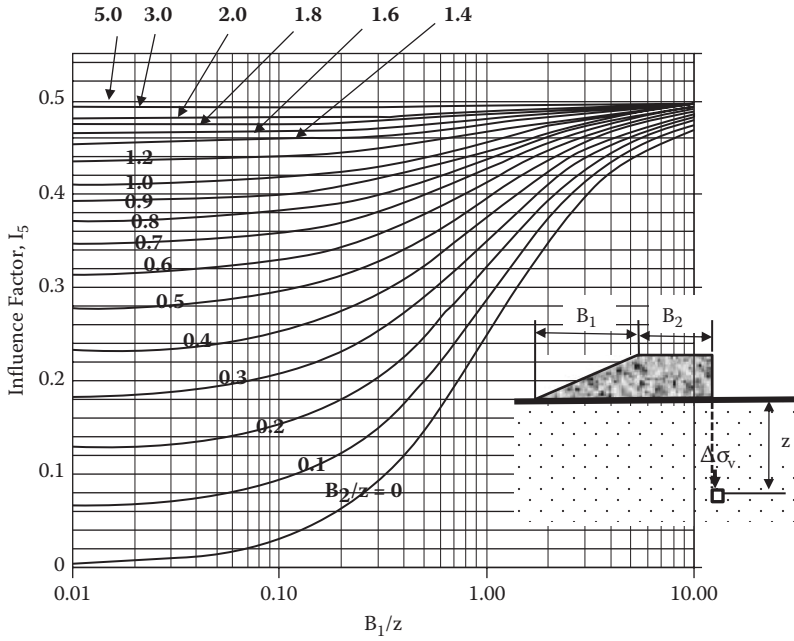


FIGURE 8.13 Influence factor I_5 .

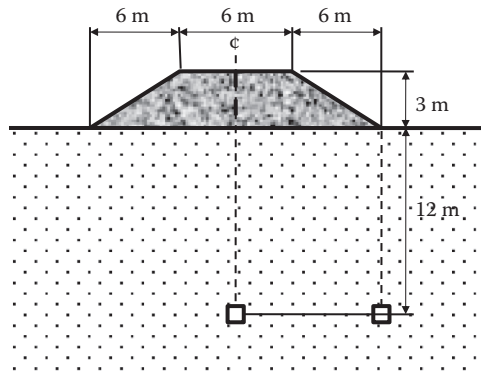


FIGURE 8.14 Exercise 8.4 problem.

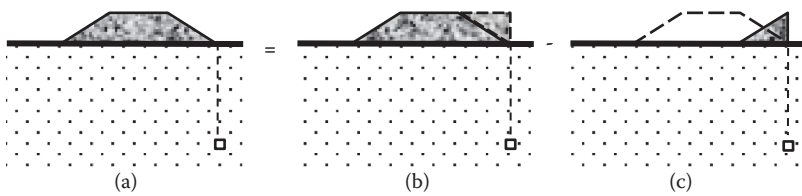


FIGURE 8.15 Superposition to solve Exercise 8.4(b).

8.8 VERTICAL STRESS INCREMENT UNDER CORNER OF RECTANGULAR FOOTING

Newmark (1935) integrated Boussinesq’s equation over a rectangular loading area (Figure 8.16), and the solution under a corner of the footing is given by

$$\Delta\sigma_v = qI_6 \tag{8.14}$$

$$I_6 = \frac{1}{4\pi} \left[\frac{2mn\sqrt{m^2 + n^2 + 1}}{m^2 + n^2 + m^2n^2 + 1} \cdot \frac{m^2 + n^2 + 2}{m^2 + n^2 + 1} + \tan^{-1} \left(\frac{2mn\sqrt{m^2 + n^2 + 1}}{m^2 + n^2 - m^2n^2 + 1} \right) \right] \tag{8.15}$$

where $m = B/z$ and $n = L/z$. Note that when the \tan^{-1} (***) term in Equation (8.15) becomes negative, π should be added to that term to obtain the correct I_6 values, and also B and L (or m and n) are exchangeable parameters so that B or L could be assigned for either side of a footing. Table 8.9 and Figure 8.17 show I_6 values as functions of m and n .

The solution in Equation (8.14) is the one under a corner of rectangular footing. However, the solution can be used to compute $\Delta\sigma_v$ under any point of rectangular footing using the **principle of superposition**. Figure 8.18 shows $\Delta\sigma_v$ computations under various points of footings and real loaded footing areas are shown with darker color and imaginary footing sections drawn with dotted lines. Case (a) is under a corner of a footing, Case (b) is under a midpoint of footing, and Case (c) and Case (d) are under outsides of the footing. By bringing the point of computation to a corner of the footing, including imaginary sections, the following procedures are used:

Case (a): loaded area = I, Equation (8.14) is directly used.

Case (b): Loaded areas = I + II + II + IV

$$\Delta\sigma_v (I + II + III + IV) = \Delta\sigma_v (I) + \Delta\sigma_v (II) + \Delta\sigma_v (III) + \Delta\sigma_v (IV)$$

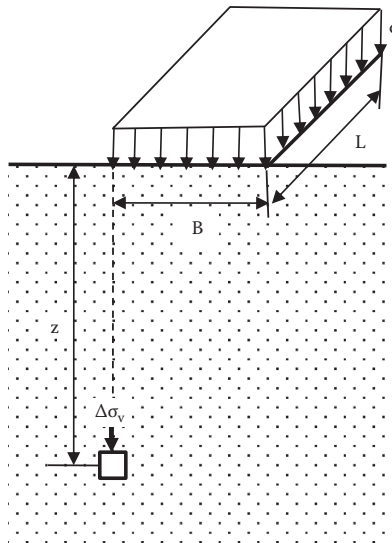


FIGURE 8.16 $\Delta\sigma_v$ under the corner of rectangular footing.

TABLE 8.9
Influence Factor I_6 by Equation (8.15) (Under Corner of Rectangular Footing)

n	m													
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.5	2	5	10
0.1	0.005	0.009	0.013	0.017	0.020	0.022	0.024	0.026	0.027	0.028	0.030	0.031	0.032	0.032
0.2	0.009	0.018	0.026	0.033	0.039	0.043	0.047	0.050	0.053	0.055	0.059	0.061	0.062	0.062
0.3	0.013	0.026	0.037	0.047	0.056	0.063	0.069	0.073	0.077	0.079	0.086	0.089	0.090	0.090
0.4	0.017	0.033	0.047	0.060	0.071	0.080	0.087	0.093	0.098	0.101	0.110	0.113	0.115	0.115
0.5	0.020	0.039	0.056	0.071	0.084	0.095	0.103	0.110	0.116	0.120	0.131	0.135	0.137	0.137
0.6	0.022	0.043	0.063	0.080	0.095	0.107	0.117	0.125	0.131	0.136	0.149	0.153	0.156	0.156
0.7	0.024	0.047	0.069	0.087	0.103	0.117	0.128	0.137	0.144	0.149	0.164	0.169	0.172	0.172
0.8	0.026	0.050	0.073	0.093	0.110	0.125	0.137	0.146	0.154	0.160	0.176	0.181	0.185	0.185
0.9	0.027	0.053	0.077	0.098	0.116	0.131	0.144	0.154	0.162	0.168	0.186	0.192	0.196	0.196
1	0.028	0.055	0.079	0.101	0.120	0.136	0.149	0.160	0.168	0.175	0.194	0.200	0.204	0.205
1.5	0.030	0.059	0.086	0.110	0.131	0.149	0.164	0.176	0.186	0.194	0.216	0.224	0.230	0.230
2	0.031	0.061	0.089	0.113	0.135	0.153	0.169	0.181	0.192	0.200	0.224	0.232	0.240	0.240
5	0.032	0.062	0.090	0.115	0.137	0.156	0.172	0.185	0.196	0.204	0.230	0.240	0.249	0.249
10	0.032	0.062	0.090	0.115	0.137	0.156	0.172	0.185	0.196	0.205	0.230	0.240	0.249	0.250

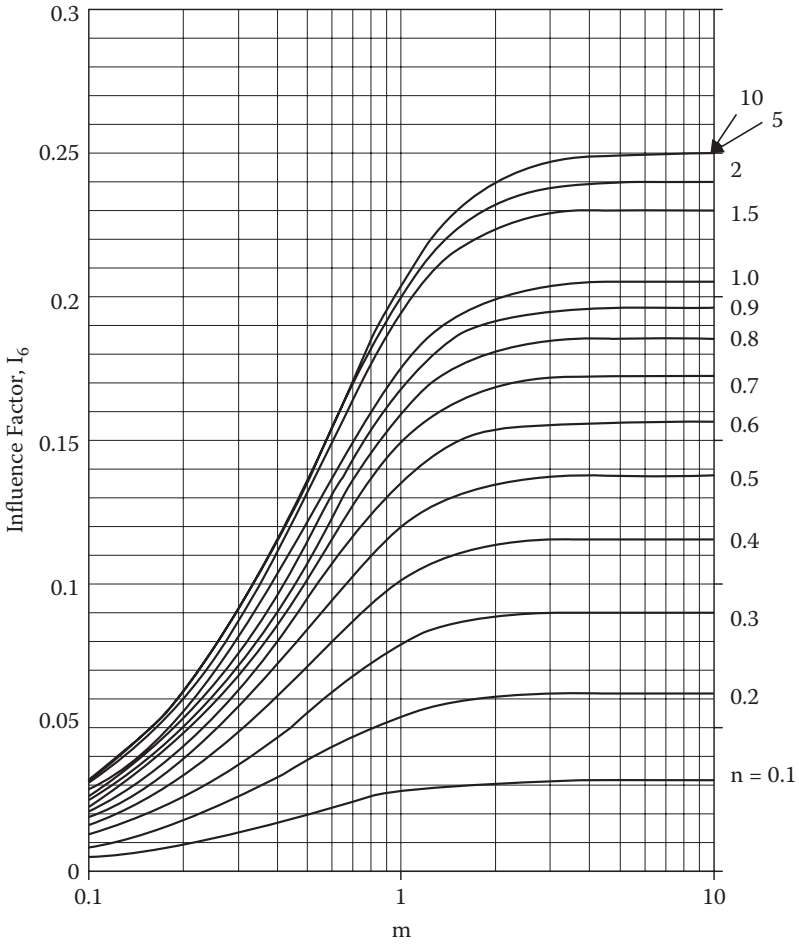


FIGURE 8.17 Influence factor I_6 .

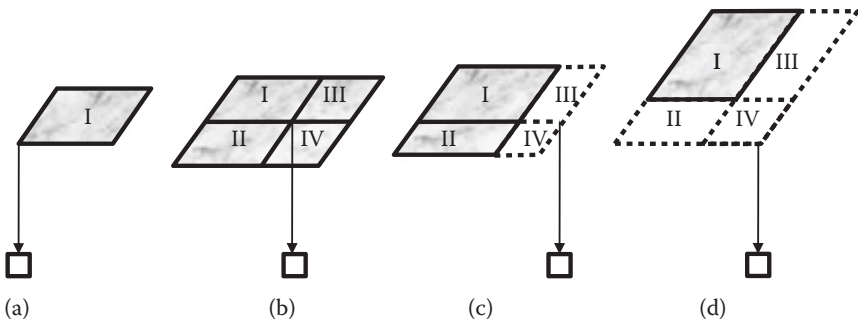


FIGURE 8.18 $\Delta\sigma_v$ computations under various points of footings.

Case (c): Loaded areas = I + II

$$\Delta\sigma_v (I + II) = \Delta\sigma_v (I + III) + \Delta\sigma_v (II + IV) - \Delta\sigma_v (III) - \Delta\sigma_v (IV)$$

Case (d): Loaded areas = I

$$\Delta\sigma_v (I) = \Delta\sigma_v (I + II + III + IV) - \Delta\sigma_v (II + IV) - \Delta\sigma_v (III + IV) + \Delta\sigma_v (IV)$$

In the preceding expression, for example, $\Delta\sigma_v (I + II)$ means the stress increment computation due to the combined footing area I and II. In this manner, all computation points are located at the corners of combined or single footings, and Equation (8.14) is applicable. In case (d), the footing IV is included in the footings (II + IV) and (III + IV) and subtracted twice. Thus, $\Delta\sigma_v (IV)$ is added once. Note that for each real or imaginary footing, B and L values are different, and different I_6 values should be obtained for all those footings.

Exercise 8.5

A loaded footing ABCD with $q = 200 \text{ kN/m}^2$ on the ground is shown in Figure 8.19. Compute $\Delta\sigma_v$ under Points E, F, B, and G at a depth of 5 m.

SOLUTION

- (a) At Point E, there are four equal footings for which Point E corners.
 $B = 1.5 \text{ m}$ and $L = 1 \text{ m}$; thus, $m = B/z = 1.5/5 = 0.3$ and $n = L/z = 1/5 = 0.2$
 From Figure 8.16, $I_6 = 0.026$
 From Equation (8.14), $\Delta\sigma_v = 4 \times q \times I_6 = 4 \times 200 \times 0.026 = \mathbf{20.8 \text{ kN/m}^2} \leftarrow$

- (b) At Point F, there are two equal footings for which Point F corners.
 $B = 1.5 \text{ m}$ and $L = 2 \text{ m}$; thus $m = B/z = 1.5/5 = 0.3$ and $n = L/z = 2/5 = 0.4$
 From Figure 8.16, $I_6 = 0.047$
 From Equation (8.14), $\Delta\sigma_v = 2 \times q \times I_6 = 2 \times 200 \times 0.047 = \mathbf{18.8 \text{ kN/m}^2} \leftarrow$

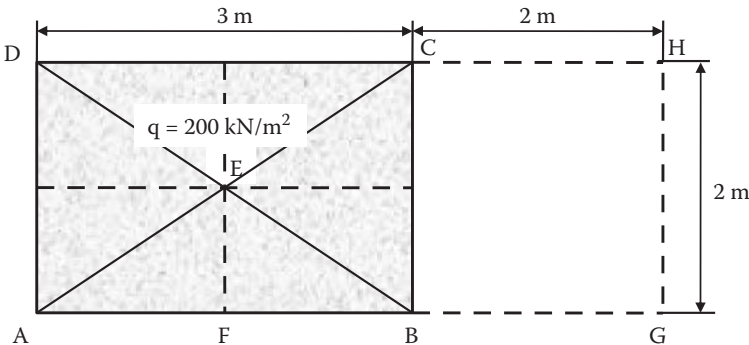


FIGURE 8.19 Exercise 8.5 problem.

Chapter 7 Geotechnical settlement of soil

9.1 INTRODUCTION

Soils, in general, are stable if the stress level is maintained or water content remains constant. However, when stresses applied in soil mass are changed, it deforms and causes settlement or swelling in some instances. A major source of stress change is due to the footing load on the ground. Chapter 8 discussed how to estimate those stress increases in various types of foundation loads. Soil behaves, in some degree, as an elastic material and, at the same time, as a plastic material. Thus, upon the increase in stresses, response is elastic and plastic. The elastic response occurs instantaneously, but the plastic response is a time-dependent phenomenon. The former is more dominant in granular soils, and the latter is more so in cohesive soils. Instantaneous elastic settlement is termed **immediate settlement**, S_i , and the time-dependent settlements are due to **consolidation** phenomena. The consolidation settlement is also categorized into **primary consolidation** settlement, S_c , and **secondary compression** settlement, S_s , as will be discussed later in this chapter. Thus, the **total settlement**, S_t , due to increased stresses is obtained as a summation of these (i.e., $S_t = S_i + S_c + S_s$).

9.2 ELASTIC SETTLEMENTS

For an idealized circular footing on an idealized uniform elastic infinite half-space soil mass as seen in Figure 9.1, an elastic solution for the surface settlement (*Schleicher 1926*) is given by

$$S_i = C_d B \left(\frac{1 - \mu^2}{E_s} \right) \Delta\sigma \quad (9.1)$$

where B is the footing diameter, μ is the **Poisson's ratio** of soil, E_s is the **modulus of elasticity** of soil, and $\Delta\sigma$ is the applied uniform (or average in case of rigid footing) stress on the footing. C_d is the modification factor to count other shapes of $B \times L$ rectangular footing, rigidity of footing, and location of settlement computation, as summarized in Table 9.1.

The ranges of Poisson's ratios and modulus of elasticity of soils are given in Table 9.2 and Table 9.3, respectively.

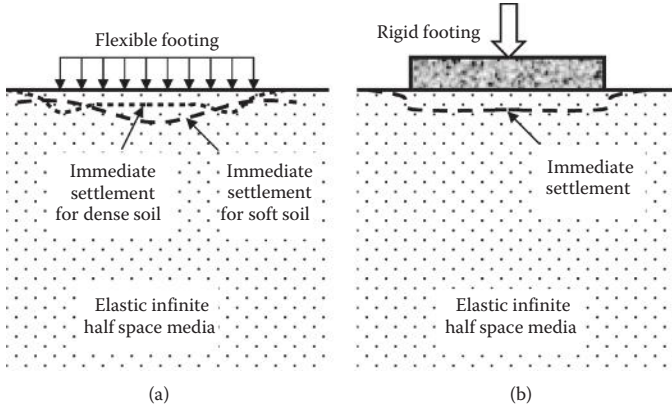


FIGURE 9.1 Flexible and rigid footings on elastic half-space media.

TABLE 9.1
Modification Factor C_d in Equation (9.2)

Shape of Footing	L/B	Flexible Footing		Rigid Footing
		At Center	At Corner	
Circular	—	1.00	0.64	0.79
Rectangular	1 (square)	1.12	0.56	0.99
	1.5	1.36	0.67	1.07
	2	1.52	0.76	1.21
	3	1.78	0.88	1.42
	5	2.10	1.05	1.70
	10	2.53	1.26	2.10
	20	2.99	1.49	2.46
	50	3.57	1.8	3.0
	100	4.00	2	3.43

Source: After Winterkorn, H. F. and Fang, H-Y., 1975, *Foundation Engineering Handbook*, Van Nostrand Reinhold, New York.

Exercise 9.1

A 2 m × 4 m rectangular footing carries 200 kN/m² of surface load. Soil underneath the footing is medium-dense sand. Estimate the immediate settlement under the center of the footing in cases of (a) flexible footing, and (b) rigid footing.

SOLUTION

From Table 9.2 and Table 9.3, choose $\mu = 0.3$ and $E_s = 40$ MPa for medium-dense sandy soil. From Table 9.1, for L/B = 2.0, $C_d = 1.52$ for case (a), and $C_d = 1.21$ for case (b). From Equation (9.1):

Dr Mohamed Adb Elattief

(a) Flexible footing

$$S_i = 1.52 \times 2 \times \left(\frac{1 - 0.3^2}{40000} \right) \times 200 = 0.0138\text{m} = \mathbf{13.8\text{mm} \leftarrow}$$

(b) Rigid footing

$$S_i = 1.21 \times 2 \times \left(\frac{1 - 0.3^2}{40000} \right) \times 200 = 0.0110\text{m} = \mathbf{11.0\text{mm} \leftarrow}$$

The solutions make sense that a rigid footing gave a smaller settlement than for a flexible footing under the same footing load.

TABLE 9.2
Ranges of Poisson's Ratios of Soils

Soil Type	Poisson's Ratio, μ
Most clay soils	0.4–0.5
Saturated clay soils	0.45–0.50
Cohesionless—medium and dense	0.3–0.4
Cohesionless—loose to medium	0.2–0.35

Source: After **Bowles, J. E., 1996, Foundation Analysis and Design**, 5th ed., McGraw–Hill, New York.

TABLE 9.3
Ranges of Modulus of Elasticity of Soils

Soil	Modulus of Elasticity (MPa)	
Clay	Very soft	2–15
	Soft	5–25
	Medium	15–50
	Hard	50–100
	Sandy	25–250
Glacial till	Loose	10–150
	Dense	150–720
	Very dense	500–1440
Loess	15–60	
Sand	Silty	5–20
	Loose	10–25
	Dense	50–81
Sand and gravel	Loose	50–150
	Dense	100–200
Shale	150–5000	
Silt	2–20	

Source: After **Bowles, J. E., 1996, Foundation Analysis and Design**, 5th ed., McGraw–Hill, New York.

Equation (9.1) is the solution for footings on the ground surface. In many cases, footings are embedded in a certain depth from the ground surface, and thus Equation (9.1) would give a rather conservative solution for such cases. Also, an infinitely deep half-space elastic medium is an idealized situation. In reality, there would be a hard layer at a certain depth, which would reduce the immediate settlement. Readers are referred to extended solutions (*Janbu, Bjerrum, and Kjaernsli 1956; Mayne and Poulos 1999*) for problems in such cases.

As can be seen in Equation (9.1), the amount of settlement is largely influenced by properties of soils μ and E_s . In particular, the estimation of E_s is not an easy task for a given soil and thus computed settlement should be treated as a rough estimate unless E_s is properly assessed by proper laboratory tests or field methods. Fortunately, immediate settlement occurs during or right after the construction, and thus contractors can do proper corrective measures at the construction site if needed.

9.3 PRIMARY CONSOLIDATION SETTLEMENT

Upon increase in stresses in an element, a time-delayed deformation and then settlement occur for saturated cohesive soils due to plastic behavior. When additional boundary stresses are applied to an element, it tends to be compressed. However, due to low permeability of the clay, water cannot escape from the element at once, but eventually water goes out and the volume of the element decreases. This is a time-delay process of volume decrease (settlement) and is classified as a **primary consolidation** process.

9.4 ONE-DIMENSIONAL PRIMARY CONSOLIDATION MODEL

Terzaghi (1925) developed a model that explains well the primary consolidation process as seen in Figure 9.2. The model consists of a water-filled cylinder with a piston, which is supported by a spring. In the piston, there is a small hole to allow drainage. There is also a standpipe to monitor the water pressure inside the cylinder.

In the model, stress increment $\Delta\sigma$ is applied on top of the piston at time 0_+ . At time 0_+ , there is no settlement since there is no time for water to escape from the small hole. If there is no settlement ($S = 0$), stress in the spring σ_{spring} is zero. And thus applied stress $\Delta\sigma$ is totally carried by the water pressure ($u = \Delta\sigma$) as seen in Figure 9.2(a). When time is allowed for some water to drain from the hole in the piston, the piston moves down and settlement starts; at the same time, applied stress $\Delta\sigma$ is transferred more to the spring σ_{spring} from the water pressure u as seen in Figure 9.2(b). When a sufficient time is allowed for water to drain completely in Figure 9.2(c) (at the infinite time in the theory), all water is drained ($u = 0$) and the final consolidation settlement is attained ($S = S_f$). At this stage, all external pressure $\Delta\sigma$ is carried by the stress in the spring ($\sigma_{\text{spring}} = \Delta\sigma$).

Note that this is exactly the same model as the one used in the effective stress discussion in Chapter 7. The soil's skeleton is modeled by the spring, and the effective

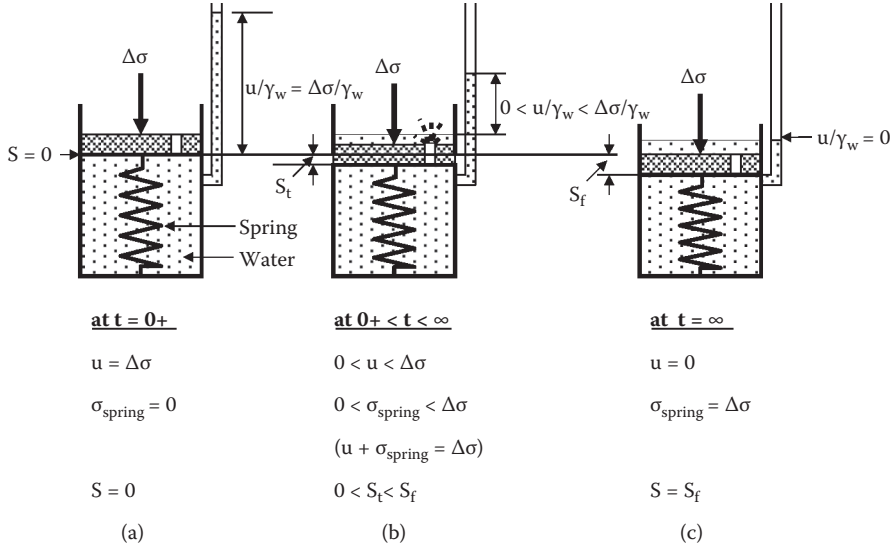


FIGURE 9.2 Terzaghi's one-dimensional primary consolidation model.

stress σ' is represented by σ_{spring} . The model clearly demonstrates that the time-delayed volume change and thus *the settlement occur due to the escape of water from the element and the stress transfer from the total water pressure at $t = 0+$ to the total effective stress at $t = \infty$ during the consolidation process.*

9.5 TERZAGHI'S CONSOLIDATION THEORY

Terzaghi developed a theory for the previously mentioned consolidation model. It assumes the following:

1. The specimen is fully saturated.
2. Water and solid components are incompressible.
3. Darcy's law is strictly applied.
4. Flow of water is one dimensional.

Note that those assumptions are nearly all valid for one-dimensional consolidation for fully saturated soils. Figure 9.3 shows a three-phase diagram of a fully saturated soil. In the model, the original total volume is 1.0, and the original volume of void (water) is initial porosity, n_0 . During the consolidation process, when the effective stress increased from initial σ'_0 to the current σ' , water is squeezed out at the amount of Δn , and the current volume of void becomes n as seen. Thus,

$$\Delta n = n_0 - n = m_v \Delta\sigma' = m_v (\sigma' - \sigma'_0) \tag{9.2}$$

where $\Delta\sigma'$ is the effective stress change, and m_v is defined as the **coefficient of volume change**, which is a parameter to connect the effective stress change to

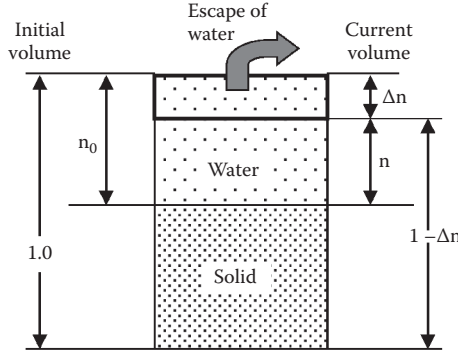


FIGURE 9.3 Three-phase model for consolidation process.

the volume change. By taking the first derivative of Equation (9.3) with respect to time t ,

$$\frac{\partial \Delta n}{\partial t} = \frac{\partial n_o}{\partial t} - \frac{\partial n}{\partial t} = 0 - \frac{\partial n}{\partial t} = m_v \left(\frac{\partial \sigma'}{\partial t} - \frac{\partial \sigma'_o}{\partial t} \right) = m_v \left(\frac{\partial \sigma'}{\partial t} - 0 \right) \quad (9.3)$$

Then,

$$\frac{\partial n}{\partial t} = -m_v \frac{\partial \sigma'}{\partial t} \quad (9.4)$$

Figure 9.4 shows a square tube element with $1 \times 1 \times dz$ dimensions. Water flows toward the upper z direction, and the inflow water velocity v and the outflow velocity $v + (\partial v/\partial z)dz$ are shown. q_{in} and q_{out} are the inflow water rate and the outflow flow rate, respectively. If q_{in} and q_{out} are the same, there is no volume change. When q_{out} is larger than q_{in} , the volume of the tube decreases, resulting in settlement. By knowing that $q_{out} - q_{in}$ is the volume change per unit time for $1 \times 1 \times dz$ total volume, and that $\partial n/\partial t$ in Equation (9.4) is also the volume change per unit time for 1.0 total volume, the following equation is obtained:

$$\begin{aligned} q_{out} - q_{in} &= (v_{out} - v_{in})A = (v_{out} - v_{in}) \cdot 1 \cdot 1 = \left(v + \frac{\partial v}{\partial z} dz \right) - v \\ &= \frac{\partial v}{\partial z} dz = -\frac{\partial n}{\partial t} dz = m_v \frac{\partial \sigma'}{\partial t} dz \end{aligned} \quad (9.5)$$

where A is the cross-sectional area for water flow (i.e., 1×1). In Equation (9.5), note that the positive value of $(q_{out} - q_{in})$ is the volume decrease and the positive value of $(-\partial n/\partial t)dz$ is also a volume decrease. From Equation (9.5), Equation (9.6) is obtained:

$$\frac{\partial v}{\partial z} = m_v \frac{\partial \sigma'}{\partial t} \quad (9.6)$$

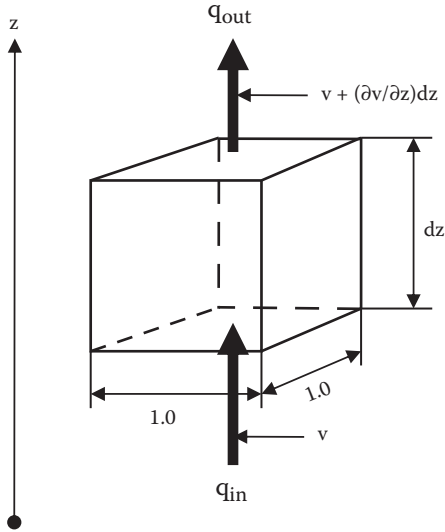


FIGURE 9.4 Vertical water flow through a square tube (1 × 1 × dz).

Now, in Chapter 7, the effective stress is defined as $\sigma' = \sigma - u$, and taking the first derivative with respect to time t ,

$$\frac{\partial \sigma'}{\partial t} = \frac{\partial \sigma}{\partial t} - \frac{\partial u}{\partial t} = 0 - \frac{\partial u}{\partial t} = - \frac{\partial u}{\partial t} \tag{9.7}$$

where $\partial \sigma / \partial t = 0$ since the applied total stress σ is constant during the consolidation process. Thus,

$$\frac{\partial \sigma'}{\partial t} = - \frac{\partial u}{\partial t} \tag{9.8}$$

Here, Darcy’s law for water flow (Chapter 6) is introduced:

$$v = k \cdot i = k \frac{-\partial h_p}{\partial z} = k \frac{-\partial \left(\frac{u}{\gamma_w} \right)}{\partial z} = - \frac{k}{\gamma_w} \frac{\partial u}{\partial z} \tag{9.9}$$

where k is the coefficient of permeability, and i is the hydraulic gradient. ∂h_p is the pressure head difference and is negative for positive water flow velocity v in Figure 9.4. From Equation (9.9),

$$\frac{\partial v}{\partial z} = - \frac{k}{\gamma_w} \frac{\partial^2 u}{\partial z^2} \tag{9.10}$$

By equating Equations (9.6) and (9.10) and by substituting Equation (9.8),

$$\frac{\partial u}{\partial t} = \frac{k}{m_v \gamma_w} \frac{\partial^2 u}{\partial z^2} = C_v \frac{\partial^2 u}{\partial z^2} \tag{9.11}$$

where

$$C_v = \frac{k}{m_v \gamma_w} \tag{9.12}$$

Equation (9.11) is called the **consolidation equation**. C_v is the **coefficient of consolidation** with a unit of length²/time (m²/s or ft²/s, etc.) and is a key material parameter in consolidation theory.

Equation (9.11) expresses the change of pore water pressure (u) relative to time (t) and space (z) domains, and it takes a popular form of partial differential equations. The equation is in the same form as the thermal diffusion equation. To solve the second order of partial differential equations, four boundary (or initial) conditions are required. Figure 9.5(a) plots the pore water pressure u with depth z as a fraction of time t . The top and bottom layers are assigned as drainage layers like sand or gravel, and a clay layer ($2H$ thickness) is sandwiched between them. Excess pore water pressure can only be drained through the drainage layers, and thus at the mid-depth H , the highest pore water pressure remains for $0 < t < \infty$ as seen. The initial and boundary conditions for this case are

1. u (at any z , at $t = 0$) = $\Delta\sigma$
2. u (at any z , at $t = \infty$) = 0
3. u (at $z = 0$, at any t) = 0
4. u (at $z = 2H$, at any t) = 0

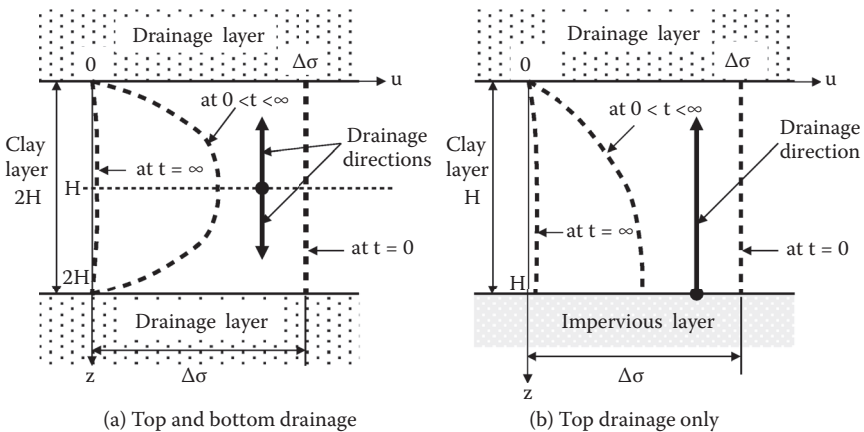


FIGURE 9.5 Initial and boundary conditions for the consolidation equation.

These conditions can also be applied to Figure 9.5(b), where the bottom layer is impervious, so that water drainage occurs only at the top boundary. In this case, the clay thickness is treated as H and then it becomes mathematically equivalent to the Figure 9.5(a) condition. Note that the drawing of the upper half (down to $z = H$) of Figure 9.5(a) is the same as the entire drawing (down to $z = H$) of Figure 9.5(b).

Pore water pressure u is assigned to have the following form:

$$u(z,t) = Z(z) \cdot T(t) \quad (9.13)$$

where $Z(z)$ and $T(t)$ are independent functions of z and t , respectively. By using the initial and boundary conditions and substituting Equation (9.13) into the consolidation equation, Equation (9.11), the following solution is obtained:

$$u(z,t) = \frac{4}{\pi} \Delta\sigma \sum_{N=0}^{\infty} \left[\frac{1}{2N+1} \sin \frac{(2N+1)\pi z}{2H} \cdot e^{-\frac{(2N+1)^2 \pi^2 C_v t}{4H^2}} \right] \quad (9.14)$$

By substituting $N = 0$ to several higher values, the solution converges and the numerical solution is obtained for given z and t . To make an operation much simpler, **time factor** T_v is introduced as

$$T_v = \frac{C_v t}{H^2} \quad [0] \quad (9.15)$$

This is a nondimensional variable to express the time relative to material parameter C_v and drainage distance H . In this equation, H should be taken as the longest distance to the drainage layer, as seen in Figure 9.5.

When T_v is substituted into Equation (9.14), it becomes

$$\begin{aligned} u(z,t) &= \frac{4}{\pi} \Delta\sigma \sum_{N=0}^{\infty} \left[\frac{1}{2N+1} \sin \frac{(2N+1)\pi z}{2H} \cdot e^{-\frac{(2N+1)^2 \pi^2}{4} T_v} \right] \\ &= f \left(\Delta\sigma, \frac{z}{2H}, T_v \right) \end{aligned} \quad (9.16)$$

In this equation form, the pore water pressure u is expressed as a function of three independent parameters: $\Delta\sigma$, $z/2H$, and T_v .

By referring to the three-phase diagram of Figure 9.3 and using Equation (9.2), the final consolidation settlement S_f (at $t = \infty$) for a clay layer of thickness H can be obtained as

$$S_f = \Delta n_f \cdot H = m_v \Delta\sigma'_f H = m_v \Delta\sigma H \quad (9.17)$$

In the preceding expression, subscript “f” stands for “final.” Meanwhile, the settlement S_t at any arbitrary time, t , is obtained from an integration of settlement

$\Delta n \times dz$ for a small clay thickness dz over the total clay layer thickness H as seen in Figure 9.6. Thus,

$$S_t = \int_0^H \Delta n \cdot dz = \int_0^H m_v \Delta \sigma' dz = m_v \int_0^H (\Delta \sigma - u) dz = m_v \Delta \sigma H - m_v \int_0^H u dz \quad (9.18)$$

where $\Delta \sigma$ is the increased stress at depth z . Since u is given in Equation (9.16), Equation (9.18) becomes

$$S_t = m_v \Delta \sigma H \left[1 - \frac{8}{\pi^2} \sum_{N=0}^{\infty} \left(\frac{1}{2N+1} \cdot e^{-\frac{(2N+1)^2 \pi^2 T_v}{4}} \right) \right] \quad (9.19)$$

Now, the **degree of consolidation** U is defined as the percentage of settlement at an arbitrary time t to its final settlement at $t = \infty$ and it is computed from Equations (9.17) and (9.19) as

$$U = \frac{S_t}{S_f} = \left[1 - \frac{8}{\pi^2} \sum_{N=0}^{\infty} \left(\frac{1}{2N+1} \cdot e^{-\frac{(2N+1)^2 \pi^2 T_v}{4}} \right) \right] = f(T_v) \quad (9.20)$$

As seen in Equation (9.20), the degree of consolidation U is only a function of time factor T_v . There are unique relationships between U and T_v , and they are shown in Table 9.4 and plotted in Figure 9.7.

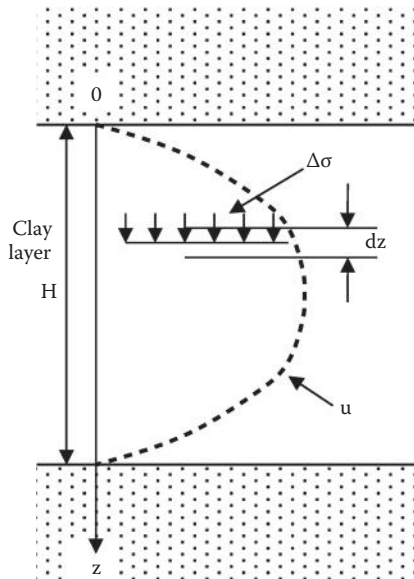


FIGURE 9.6 Settlement computation model.

TABLE 9.4
Relationships between U and T_v

U (%)	T_v	U (%)	T_v
0	0	3.751	0.001
5	0.00196	5.665	0.0025
10	0.00785	7.980	0.005
15	0.0177	9.772	0.0075
20	0.0314	11.28	0.01
25	0.0491	17.84	0.025
30	0.0707	25.23	0.05
35	0.0962	30.90	0.075
40	0.126	35.68	0.1
45	0.159	56.22	0.25
50	0.197	76.40	0.5
55	0.239	87.26	0.75
60	0.286	93.13	1
65	0.340	99.83	2.5
70	0.403	100	5
75	0.477	100	7.5
80	0.567	100	9.5
85	0.684		
90	0.848		
95	1.129		
100	∞		

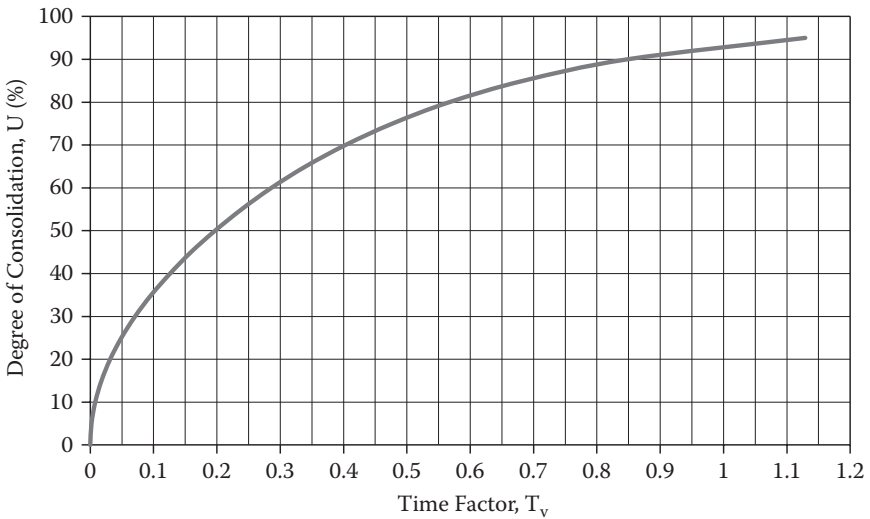


FIGURE 9.7 U versus T_v relationship.

In the first two columns of Table 9.4, $T_{50} = 0.197$ and $T_{90} = 0.848$ are often used to evaluate the midterm (50%) consolidation and near final (90%) consolidation, respectively. Also note that it takes the infinite time to accomplish 100% consolidation from the theory, although practically it reaches to 100.000% when T_v is 5.0 and above.

Exercise 9.2

In a laboratory consolidation test, a 12.7 mm ($\frac{1}{2}$ in.) thick clay specimen was tested with top and bottom drained condition, and 90% consolidation was accomplished in 15.8 min ($t_{90} = 15.8$ min). In the field, the same clay material with the thickness of 6.5 m is sandwiched by top sand and bottom gravel layers for drainage. How long does the field clay take to accomplish 50% and 90% consolidation, respectively?

SOLUTION

In the laboratory test, top and bottom are drainage layers, so the clay thickness 12.7 mm = $2H$ and $T_{90} = 0.848$ from Table 9.4. Inserting these values into Equation (9.15),

$$C_v = \frac{H^2}{t_{90}} T_{90} = \frac{\left(\frac{12.7}{2}\right)^2}{15.8} 0.848 = 2.164 \text{ mm}^2/\text{min}$$

From the field drainage condition, $2H = 6.5$ m. Also $T_{50} = 0.197$ from Table 9.4. Utilizing Equation (9.15), 50% consolidation time, t_{50} , is

$$t_{50} = \frac{H^2}{C_v} T_{50} = \frac{\left(\frac{6.5 \times 1000}{2}\right)^2}{2.164} 0.197 = 9.615 \times 10^5 \text{ min} = \mathbf{667.7 \text{ days} \leftarrow}$$

Similarly, for 90% consolidation time t_{90} ,

$$t_{90} = \frac{H^2}{C_v} T_{90} = \frac{\left(\frac{6.5 \times 1000}{2}\right)^2}{2.164} 0.848 = 41.39 \times 10^5 \text{ min}$$

$$= 2874 \text{ days} = \mathbf{7.87 \text{ years} \leftarrow}$$

or, from Equation (9.15) and by using a common C_v value,

$$C_v = \frac{H^2}{t_{50}} T_{50} = \frac{H^2}{t_{90}} T_{90}, \quad \text{then,} \quad t_{90} = \frac{T_{90}}{T_{50}} t_{50} = \frac{0.848}{0.197} \times 667.7 = 4.305 \times 667.7$$

$$= 2874 \text{ days} = \mathbf{7.87 \text{ years} \leftarrow}$$

Exercise 9.2 demonstrates the usefulness of a nondimensional time factor T_v . Equation (9.15) was used first to determine the C_v value from the laboratory

experiment, and then the same equation was used to determine the real time of consolidation in the field. It is most important to realize that ***H in Equation (9.15) is half the thickness of the clay layer for double drainage, or full thickness for single layer drainage.*** If H is taken as the full thickness (i.e., H = 6.5 m) in Exercise 9.2 by mistake, the result would be four times different from the correct one.

Exercise 9.3

A clay layer had a thickness of 4.5 m. After 6 months, it settled to 30% of the total settlement and 50 mm of the settlement was observed. For a similar clay layer and loading condition, if the thickness of clay is 20 m, how much settlement occurs at the end of 3 years? Assume that the top of the clay layer is a drainage layer and the bottom is an impervious layer for both 4.5 m and 20 m thick clay layers.

SOLUTION

For the 4.5 m thick clay, since 30% settlement is 50 mm, the final settlement will be

$$S_{f,4.5m} = 50/0.30 = 166.7 \text{ mm}$$

H = 4.5 m since the top is only a drainage layer in this case and thus,

$$C_v = \frac{H^2}{t_{30}} T_{30} = \frac{(4.5)^2}{6} 0.0707 = 0.239 \text{ m}^2/\text{month}$$

For the 20 m thick clay, the final settlement $S_{f,20m}$ is proportional to the one for 4.5 m clay; thus,

$$S_{f,20m} = 166.7 \times (20/4.5) = 740 \text{ mm} \leftarrow$$

At the end of 3 years,

$$T_v = \frac{C_v t}{H^2} = \frac{0.239 \times (3 \times 12)}{20^2} = 0.0215$$

From the right two columns of Table 9.4 corresponding to $T_v = 0.0215$, $U = 16.3\%$ was obtained by a linear interpolation of data points. Thus, 20 m thick clay settles at the end of 3 years in the amount of

$$S_{3yrs, 20m} = S_{f,20m} \times U_{3yrs} = 740 \times 0.163 = \mathbf{120.6 \text{ mm}} \leftarrow$$

In Exercise 9.3, the degree of saturation U was found from the T_v value, and this is another example of the versatility of the T_v equation (Equation 9.15).

9.6 LABORATORY CONSOLIDATION TEST

Small-scale laboratory consolidation tests are performed for clay specimens to determine several key consolidation parameters, including the coefficient of consolidation C_v value. An undisturbed thin-wall tube specimen is obtained from the field, where consolidation settlement is anticipated due to future footing load. The specimen is carefully trimmed to fit inside a rigid consolidation ring, as seen in Figure 9.8. A typical dimension of the ring is 63.5 mm (2.5 in.) diameter and 25.4 mm (1.0 in.) high. The consolidation ring filled with the specimen is placed inside the consolidation device, and the upper porous stone and a loading cap are placed on top of the specimen. The device is normally filled with water to avoid drying out of the specimen during the test. The whole device is set up on a stable platform. A consolidation load is applied and the vertical deformation is monitored with a dial gauge. In this system, soil deforms only in a vertical direction due to the escape of water during the consolidation process.

The first consolidation stress σ (load divided by specimen area) is applied at time zero and the vertical deformation dial (δ_v) is continuously read at $t = 0, 0.1, 0.25, 0.5, 1, 2, 4, 8, 15,$ and 30 minutes, and 1, 2, 4, 8, and 24 hours. Thus, one consolidation stress application takes a whole day. At the end of 24 hours (not necessarily exactly 24 hours, but elapsed time should be recorded), consolidation stress σ is normally doubled, and δ_v at the similar time intervals are recorded. Then σ is doubled again for the third consolidation stress. The preceding process is repeated with typical σ values of 25, 50, 100, 200, 400, 800, and 1600 kPa until the design maximum consolidation stress value is attained. This increasing stress process is called the **loading process** of consolidation, which takes about 1 week.

At the end of the test with the maximum consolidation stress, an **unloading process** is performed. The σ value is deduced to 1600, 400, 100, 25 kPa, or such. During this process, rebound on the specimen occurs and only the final dial gauge readings are recorded at several hours after each unloading process. Thus, the whole unloading process takes about a day. After the unloading process, wet and dry weights and water content of the specimen are measured by weighing and drying it in an oven overnight.

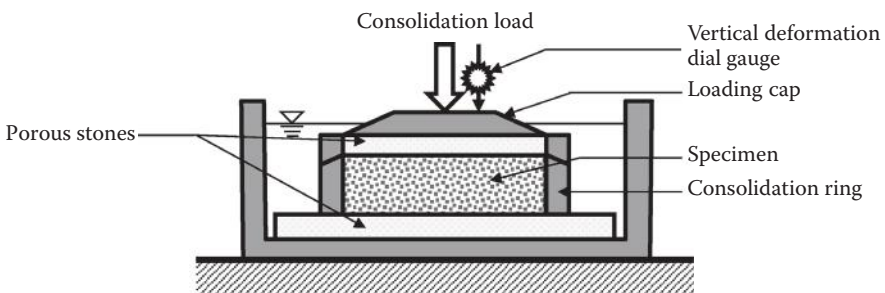


FIGURE 9.8 Consolidation test setup.

9.7 DETERMINATION OF C_v

From each consolidation stress, a set of data with δ_v and t is obtained. Table 9.5 shows a sample set of data obtained.

There are two practical methods available to determine the coefficient of consolidation C_v from laboratory consolidation tests: the **log t method** and the **\sqrt{t} method**.

9.7.1 LOG t METHOD

The log t versus δ_v data from Table 9.5 are plotted in Figure 9.9. The middle section of data points shows a linear relation and a straight line is drawn as a **primary consolidation curve**. The later section of the curve also shows a linear relation, and the second straight line is drawn as the **secondary compression curve**. The intersection of the two straight lines is considered as the **end of the primary consolidation** and leveled as δ_{100} in the figure. The initial section of the data points is a curve that is assumed to be parabolic. By this assumption, t_1 and $4t_1$ points are chosen on the curve as seen (in the example, $t_1 = 0.1$ minute and $4t_1 = 0.4$ minute). Corresponding δ_v values are identified as B and C levels in the figure, respectively. Since $t = 0.0$ cannot be plotted in the log scale, the initial vertical deformation δ_0 (Point A level) is then determined by taking $\overline{AB} = \overline{BC}$. Once δ_0 and δ_{100} are determined, the mid-point δ_{50} is obtained. Corresponding time is designated as t_{50} or the time to reach 50%

TABLE 9.5
Sample Consolidation Test Data, δ_v and t ($\sigma = 1566$ kPa)

Elapsed Time, t (minute)	Reading in Vertical Dial Gauge, δ_v (mm)	\sqrt{t} ($\sqrt{\text{min}}$)
0	17.74	0.00
0.1	17.56	0.32
0.25	17.47	0.50
0.5	17.33	0.71
1	17.17	1.00
2	16.96	1.41
4	16.76	2.00
10	16.45	3.16
15	16.38	3.87
30	16.25	5.48
120	16.14	10.95
250	16.11	15.81
520	16.10	22.80
1400	16.08	37.42

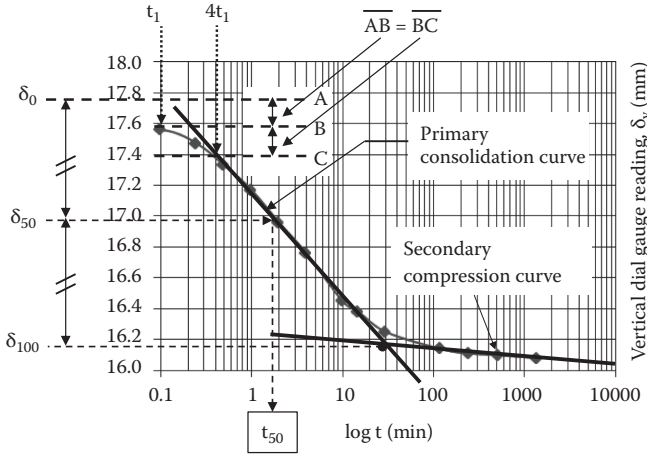


FIGURE 9.9 Log t method.

primary consolidation. From Equation (9.15) for T_v , the coefficient of consolidation C_v is obtained as

$$C_v = \frac{H^2}{t_{50}} T_{50} = \frac{H^2}{t_{50}} 0.196 \tag{9.21}$$

Note that H in Equation (9.21) should be the longest drainage distance so that it should be half of the specimen thickness in common laboratory consolidation tests.

9.7.2 \sqrt{t} METHOD

The same data in Table 9.5 are plotted with \sqrt{t} versus δ_v in Figure 9.10.

In the figure, at the initial portion of the data, a linear relation is observed and a straight line is drawn. The intersection with the $\sqrt{t} = 0$ axis is assigned as δ_0 . Starting from the δ_0 point, the second straight line with the inverse slope of 1.15 times the first line is drawn as seen with a broken line. The interception of the second line with the data curve is assigned as 90% of the primary consolidation point. The corresponding time is read as $\sqrt{t_{90}}$ and thus t_{90} value is obtained. From Equation (9.15), the C_v value is computed as

$$C_v = \frac{H^2}{t_{90}} T_{90} = \frac{H^2}{t_{90}} 0.848 \tag{9.22}$$

In both methods, the coefficient of consolidation C_v values are determined with various consolidation stresses, since δ_v and t relations are obtained for each consolidation stress. Those values might be different when the consolidation stress is varied, or those are nearly the same. Engineers have to make a decision on the selection of C_v values for design based on the analyzed data. The selection between these two methods depends very much on a preference of the engineers. As a common rule,

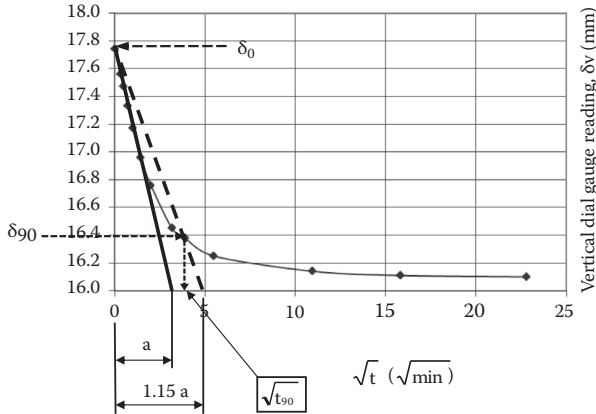


FIGURE 9.10 \sqrt{t} Method.

the log t method (Equation 9.21) could be better used for evaluating the earlier stage of the consolidation process since it uses 50% consolidation time, while the \sqrt{t} method (Equation 9.22) could be better suited for the final stage consolidation estimation because it uses 90% consolidation.

9.8 e-LOG σ CURVE

First of all, note that in traditional consolidation theory and practice, “e-log p curve” has been used as common terminology, where p donates the stress symbol. However, throughout this book, σ is used as the symbol for stress. Therefore, “e-log σ curve” replaces the old term “e-log p curve” throughout.

Laboratory data are analyzed for the final settlements achieved under given consolidation stresses σ at the end of 24 hours of consolidations. Example data are analyzed in Table 9.6.

In the table, δ values in Column B are the final vertical dial gauge readings under corresponding consolidation stresses σ , and the rest can be readily computed using the spreadsheet format shown.

Exercise 9.4

Based on a three-phase diagram, find the solid height H_s of the example data in Table 9.6 for this fully saturated clay. Specific gravity G_s was found to be 2.69.

SOLUTION

For the three-phase diagram in Figure 2.4 (Chapter 2), $W_s = 109.68$ gf, $G_s = 2.69$, and thus Equation (2.13) in Chapter 2 gives

$$V_s = \frac{W_s}{G_s \gamma_w} = \frac{109.68 \text{ gf} \times \text{g}}{2.69 \times 9.81 \text{ kN/m}^3} = \frac{0.10968 \text{ kgf} \times 9.81 \text{ m/s}^2}{2.69 \times 9.81 \times 10^3 \text{ N/m}^3} = 4.077 \times 10^{-5} \text{ m}^3$$

Since $V_s = H_s \cdot$ (area of specimen) and the diameter of the specimen $D = 76.04$ mm,

$$H_s = \frac{V_s}{\pi \left(\frac{D}{2}\right)^2} = \frac{4.077 \times 10^{-5}}{\pi \left(\frac{76.04 \times 10^{-3}}{2}\right)^2} = 0.00898 \text{ m} = \mathbf{8.98 \text{ mm} \leftarrow}$$

TABLE 9.6
Example of e-log σ Curve Analysis

(a) Specimen Information					
Description of soil:	Silty organic clay	Specimen diameter D:	76.04 mm		
Location:	Craney Island, Virginia	Initial specimen height H_0 :	19.06 mm		
Water contents:					
At beginning of test (whole specimen)			42.3%		
At beginning of test (auxiliary specimen)			42.4%		
At end of test (whole specimen)			31.3%		
Weight of dry specimen, W_s :	109.68 gf	Solid height, H_s :	8.98 mm		
(b) e-log σ Computation					
A	B	C	D	E	F
Consolidation Stress, σ (kPa)	Final Vertical Dial Reading, δ (mm)	Change in Specimen Height, $\Delta\delta$ (mm)	Final Specimen Height, H (mm)	Height of Void, H_v (mm)	Final Void Ratio, e
0.00	22.86	0	19.06	10.08	1.122
14.21	22.71	0.15	18.91	9.93	1.106
28.53	22.34	0.37	18.54	9.56	1.064
53.84	21.76	0.58	17.96	8.98	0.999
107.69	20.82	0.93	17.02	8.04	0.895
215.31	19.41	1.41	15.61	6.63	0.738
430.69	17.74	1.67	13.94	4.96	0.553
861.39	16.08	1.66	12.28	3.30	0.368
430.69	16.17	-0.09	12.37	3.39	0.377
107.69	16.42	-0.25	12.62	3.64	0.405
53.81	16.65	-0.23	12.85	3.87	0.431
28.52	16.82	-0.17	13.02	4.04	0.450

Note: Height of solid $H_s = W_s / (\gamma_{\text{wat}} G_s A_{\text{specimen}}) = W_s / (\gamma_{\text{wat}} G_s \pi D^2 / 4)$.

Column A: Applied consolidation stress.

Column B: Final vertical dial reading at the end of each stress σ_i .

Column C: $\Delta\delta_i = \delta_{i-1} - \delta_i$ (positive number for loading and negative number for unloading).

Column D: $H_i = H_{i-1} - \Delta\delta_i$.

Column E: $H_{v,i} = H_i - H_s$.

Column F: $e_i = H_{v,i} / H_s$.

Now, from Table 9.6, relationships between consolidation stress σ (in log scale) and final void ratio e are plotted in Figure 9.11. This curve is called the **e-log σ curve** and is a key relationship to determine final consolidation settlement. The loading curve (decreasing e with increase in σ) and unloading curve (increasing e with decrease in σ) are seen in the figure. In the loading section, a linear relation is observed at higher stress level and a straight line is drawn as a **virgin compression curve**. The virgin curve is the e and σ relations for naturally deposited soils on the bottoms of lakes (or rivers), which are fully consolidated under their own gravity. The slope of the virgin curve is read as **compression index** C_c and is given by

$$C_c = \frac{-(e - e_i)}{\log \sigma - \log \sigma_i} = \frac{-(e - e_i)}{\log \frac{\sigma}{\sigma_i}}, \quad \text{thus,} \quad e = e_i - e = C_c \log \frac{\sigma}{\sigma_i} \quad (9.23)$$

where (e_i, σ_i) and (e, σ) are arbitrary points on the virgin curve as seen. Equation (9.23) is used to estimate consolidation settlement for soils that follow only the virgin curve relation. Table 9.7 shows typical values of C_c for various soil types.

Skempton (1944) proposed the following equation to calculate C_c :

$$C'_c = 0.007 (LL-10) \text{ for remolded clays} \quad (9.24)$$

Terzaghi and Peck (1967) proposed the follow equation for low- to medium-sensitive undisturbed clays:

$$C_c \approx 1.3 C'_c = 0.009 (LL-10) \text{ for undisturbed clays} \quad (9.25)$$

These equations are used only as a guideline to evaluate approximate consolidation settlement initially. A more accurate value should be determined from the laboratory consolidation tests.

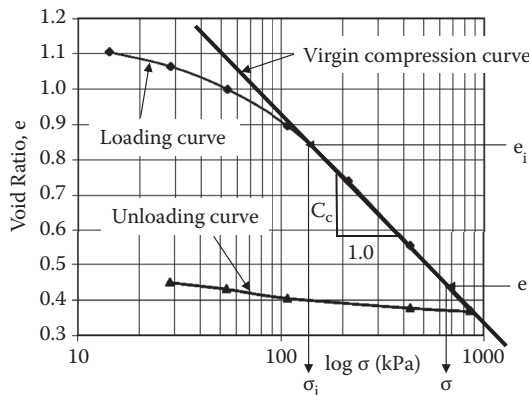


FIGURE 9.11 e-log σ curve.

TABLE 9.7
Typical Values of Compression Index C_c

Soil	Liquid Limit	Plastic Limit	C_c	
			Undisturbed	Remolded
Boston blue clay	41	20	0.35	0.21
Chicago clay	58	21	0.42	0.22
Louisiana clay	74	26	0.33	0.29
New Orleans clay	79	26	0.29	0.26
Fort Union clay	89	20	0.26	
Mississippi loess	23–43	17–29	0.09–0.23	
Delaware organic silty clay	84	46	0.95	
Indiana silty clay	36	20	0.21	0.12
Marine sediment, B.C., Canada	130	74	2.3	

Source: *Winterkorn, H. F. and Fang, H-Y., 1975, Foundation Engineering Handbook*, Van Nostrand Reinhold, New York.

9.9 NORMALLY CONSOLIDATED AND OVERCONSOLIDATED SOILS

Any specimen that will be tested comes from a certain depth at the site; thus, it has been subjected to a prior effective overburden stress and that stress has been relaxed during the sampling process. Therefore, it makes the laboratory e -log σ curve different from the virgin curve. At the beginning of a loading curve, the compression rate (reduction in e) is much smaller than the one of the virgin curve, and the curve merges to the virgin curve at a higher σ value. ***The stress at the turning point on the e -log σ curve is the stress at which soil has previously experienced the historical maximum consolidation in the field.*** This stress is called **preconsolidation stress**.

Casagrande (1936) developed an empirical method to determine preconsolidation stress σ_c for a given e -log σ curve. Referring to Figure 9.12, the maximum curvature (or minimum radius) Point M is first identified on the curve. Starting from M, the tangential line MT to e -log σ curve and the horizontal line MH are drawn. Lines MT and MH are bisected by MB. The stress at the intersection of MB and the virgin curve is defined as preconsolidation stress σ_c , as seen.

When preconsolidation stress is found to be the same as the current effective overburden stress σ'_0 at the site from which the sample is obtained, the soil is called **normally consolidated**. Referring to Figure 9.13, a soil has been consolidated at the site under its own weight till sampling takes place (Point A). During the sampling process (A to B), the in-situ stress (at A) is reduced to nearly zero (at B) and reloaded in the laboratory consolidation process (B, C to D). The loading path

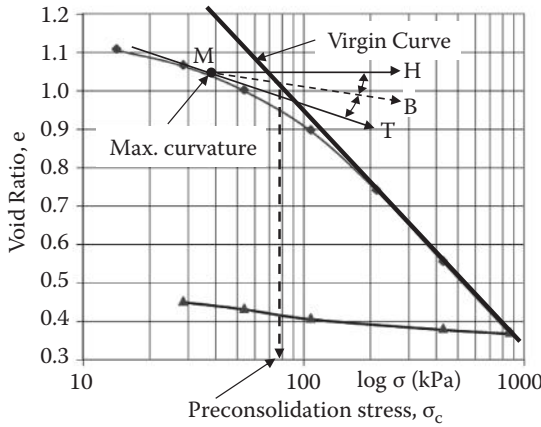


FIGURE 9.12 Casagrande's preconsolidation stress determination.

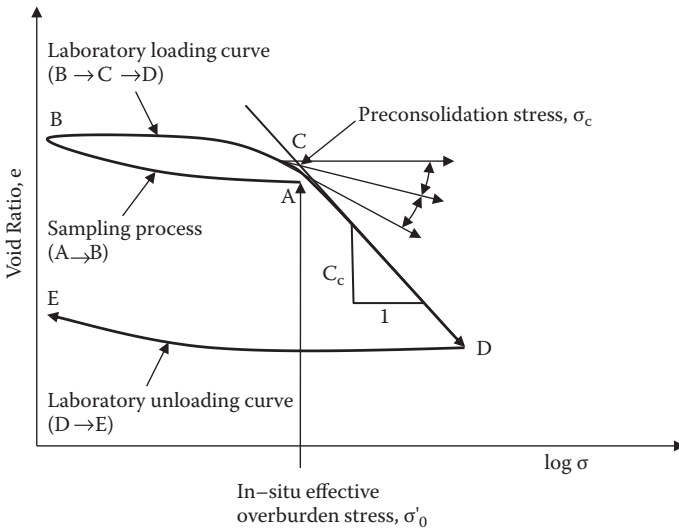


FIGURE 9.13 e - $\log \sigma$ curve for normally consolidated soils.

B to C is a reloading process and thus the slope is rather small. After passing the preconsolidation stress Point C, soil enters a new stress territory and the slope becomes steeper (to the value of C_c). After the maximum stress Point D in the laboratory, the unloading process (D to E) takes place and its slope is similar to the one of the A to B curve.

On many occasions, however, the in-situ effective overburden stress σ'_0 is found to be smaller than laboratory-obtained preconsolidation stress σ_c , as seen in Figure 9.14. The site might have been subjected to stress higher than σ'_0 during

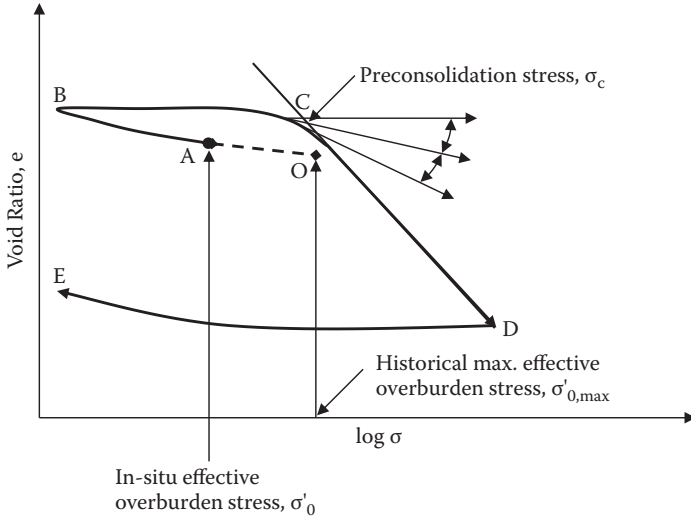


FIGURE 9.14 e-log σ curve for overconsolidated soils.

its geological history. This soil is called **overconsolidated soil**. The e-log σ curve starts from Point O with the consolidation stress with $\sigma_{0,max}$ (historical maximum effective overburden stress), and a portion of the stress has been reduced to Point A (current σ'_0). Note that its historical maximum stress cannot be seen visually at the present time. Similarly, the sampling process follows A to B and the laboratory loading process goes from B, C, and D. Preconsolidation stress found on the laboratory e-log σ curve at Point C is similar to the value of the historical maximum effective overburden stress $\sigma_{0,max}$. The removal of consolidation stress from $\sigma_{0,max}$ to σ_0 is due to meltdown of glacial ice, excavation, erosion of top soils, permanent rise of ground water tables, etc. In particular, past glacial ice loads that covered the North American continent resulted in heavily overconsolidated soils in the region.

Overconsolidation ratio (OCR) is defined as

$$OCR = \frac{\text{historical maximum effective overburden stress}}{\text{current effective overburden stress}} = \frac{\sigma_{0,max}}{\sigma_0} \quad (9.26)$$

The OCR value for normally consolidated soils is 1.0, and it is higher than 1.0 for overconsolidated soils.

Exercise 9.5

In a northern city in the United States, the area had been covered with a 100 m thick ice load in an early historical time. Some soil in the city is obtained from 10 m deep below the ground surface. The water table was near the ground surface.

Estimate the value of OCR for this soil specimen. Assume that the area had not been subjected to any major erosion or excavation.

SOLUTION

Assume that soil's unit weight is 19 kN/m^3 and that ice's unit weight is the same as that of water (9.81 kN/m^3); thus,

$$\sigma_{0,\max} = 9.81 \times 100 + (19 - 9.81) \times 10 = 981 + 91.9 = 1073 \text{ kPa}$$

$$\sigma_0 = (19 - 9.81) \times 10 = 91.9 \text{ kPa}$$

$$\text{Thus, OCR} = \sigma_{0,\max} / \sigma_0 = 1073 / 91.9 = \mathbf{11.7} \leftarrow$$

Like the example in Exercise 9.5, many soils in the northern regions of the United States and Canada are heavily overconsolidated due to the historical glacial ice load. This created unique soil properties. For example, **glacial till** is a highly compacted mixture of gravel, sand, silt, and clay (glacier-carried and -deposited materials) that has an excellent bearing capacity for foundations. However, since the lateral stress has not been relaxed as much as the vertical stress upon release of the ice load, there are rather high lateral stresses trapped in the soil elements. This creates rather high lateral stress against earth structures. Also, when the area is excavated or slopes are cut, relaxation in the lateral stress causes gradual increase of the volume (swelling), and thus the water migrates into the soil elements to make weaker soil zones, which may possibly lead to gradual failure (**creep failure**) of the slope.

Normally consolidated and overconsolidated soils show quite different behaviors in many aspects, such as shear strength, settlement, swell, lateral earth pressure, etc., and thus identification of σ_c relative to σ_0 is an important practice during the consolidation data analyses.

9.10 FINAL CONSOLIDATION SETTLEMENT FOR THIN CLAY LAYER

Assume that a relatively thin clay layer with total thickness of H is subjected to an incremental stress $\Delta\sigma$ due to a new footing, and that its initial vertical effective stress is σ_0 at its mid-depth as seen in Figure 9.15. Final primary consolidation settlement can be computed as follows.

9.10.1 NORMALLY CONSOLIDATED SOILS

As shown in Figure 9.16, σ_0 and $\sigma_0 + \Delta\sigma$ are on the virgin curve, and its slope is C_c . In this case, Equation (9.24) is used to calculate Δe as

$$e = e_i - e = C_c \log \frac{\sigma_0 + \sigma}{\sigma_0} \quad (9.27)$$

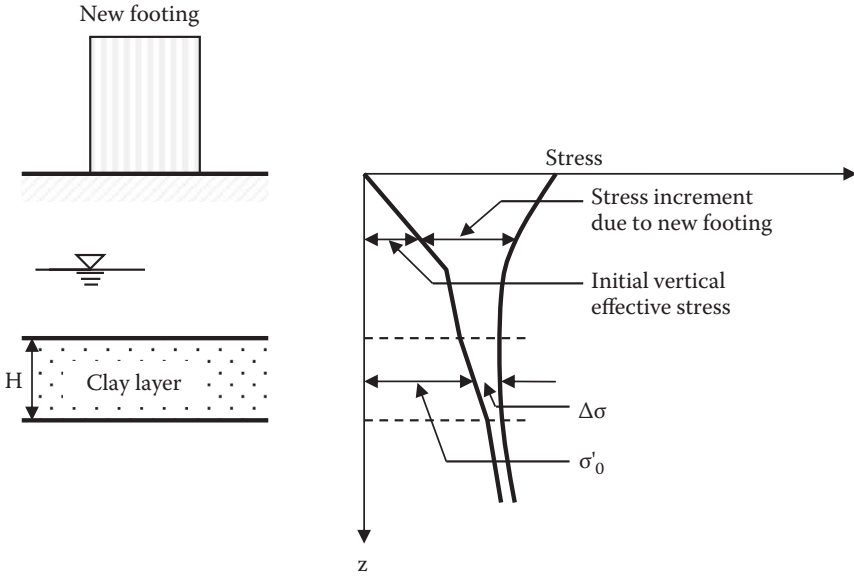


FIGURE 9.15 Consolidation settlement computation for a thin single clay layer.

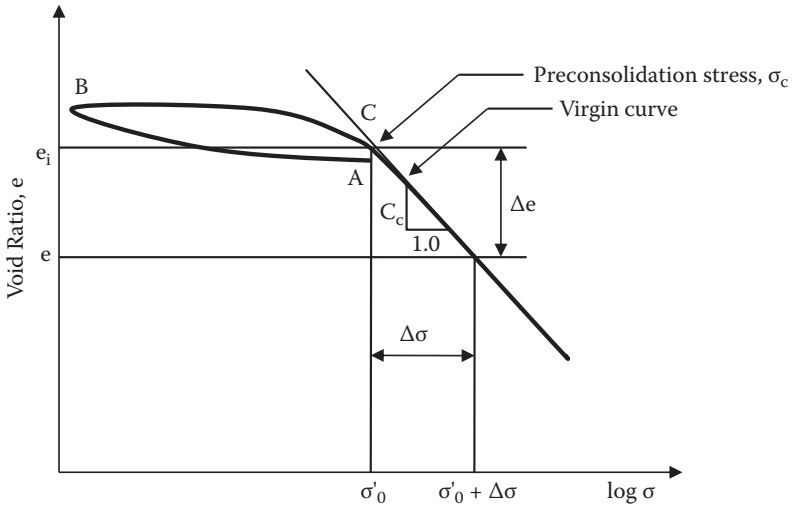


FIGURE 9.16 Settlement computation for normally consolidated soils.

The void ratio change Δe occurs to the total initial height $(1 + e_0)$. Thus, proportionally, the final settlement S_f to the total initial clay thickness H is

$$\frac{e}{1 + e_0} = \frac{S_f}{H}, \text{ thus, } S_f = \frac{H}{1 + e_0} \Delta e = \frac{H}{1 + e_0} C_c \log \frac{\sigma_0 + \sigma}{\sigma_0} \quad (9.28)$$

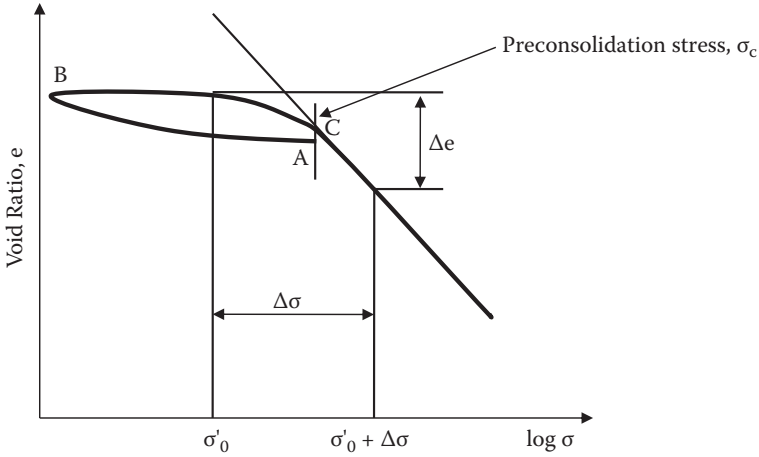


FIGURE 9.17 Settlement computation for overconsolidated soils.

Or the Δe value can be directly read from the e - $\log \sigma$ curve, and it is applied to the first term of Equation (9.28) to obtain final total consolidation settlement S_f .

9.10.2 OVERCONSOLIDATED SOILS

For these types of soils, σ_0 and $\sigma_0 + \sigma$ are not necessarily on the virgin curve, as seen in Figure 9.17, and thus the constant C_c value with Equation (9.28) cannot be used for the settlement computation. In this case, the Δe value is directly read from the e - $\log \sigma$ curve and substituted into the first term of Equation (9.28):

$$S_f = \frac{H}{1 + e_0} \Delta e \quad (9.29)$$

It should be noted that *for the amount of settlement computation* in Equations (9.28) and (9.29), “ H ” is always the total thickness of clay layer regardless of top and bottom drainage conditions. This was a key factor in choosing H or $2H$ in the T_v equation (Equation 9.15).

Exercise 9.6

A 3 m thick clay layer is sandwiched between dry sand on the top and saturated gravel on the bottom as seen in Figure 9.18. Soil properties are given in the figure. On top of the sand layer, 1000 kN of a point load is applied. Estimate final consolidation settlement of the clay layer directly under the loading point. Handle the clay layer as a single layer and assume that it is normally consolidated.

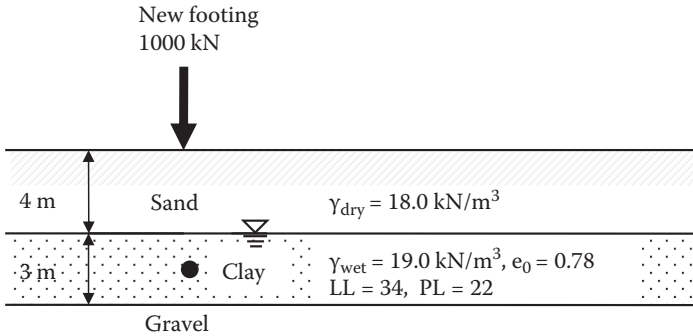


FIGURE 9.18 Exercise 9.6 problem.

SOLUTION

Initial effective vertical stress at the midpoint (depth at 5.5 m) of the clay layer is

$$\sigma_0 = 18.0 \times 4 + (19.0 - 9.81) \times 1.5 = 85.79 \text{ kPa}$$

Incremental stress $\Delta\sigma$ at $z = 5.5 \text{ m}$ is obtained from Boussinesq's point load equation (Equation 8.2 in Chapter 8) under the center ($r = 0$).

$$\sigma = \frac{3}{2\pi} \cdot \frac{P}{z^2} = \frac{3}{2\pi} \cdot \frac{1000}{5.5^2} = 15.78 \text{ kPa}$$

From Equation (9.25), $C_c = 0.009(LL-10) = 0.009 \times (34 - 10) = 0.216$ for undisturbed soil.

By substituting these values into Equation (9.28) for this normally consolidated clay,

$$S_f = \frac{H}{1+e_0} C_c \log \frac{\sigma_0 + \sigma}{\sigma_0} = \frac{3}{1+0.78} 0.216 \cdot \log \frac{85.79 + 15.78}{85.79} = \mathbf{0.0267 \text{ m} \leftarrow}$$

9.11 CONSOLIDATION SETTLEMENT FOR MULTILAYERS OR A THICK CLAY LAYER

When clay layers are thick or consist of several different clay layers, one-step computation by Equations (9.28) and (9.29) is not suitable, since σ_0 and $\Delta\sigma$ are not considered to be constant values throughout the depth of the clay layers, as seen in Figure 9.19. In this case, the whole clay layer is divided into several sublayers as seen in the figure. Final settlement $S_{f,i}$ for each sublayer is computed from the methods described in Section 9.10, using H_i , $\sigma_{0,i}$, and $\Delta\sigma_i$ values, which can be obtained at the midpoints of each sublayer i . The final total settlement S_f is the summation of $S_{f,i}$.

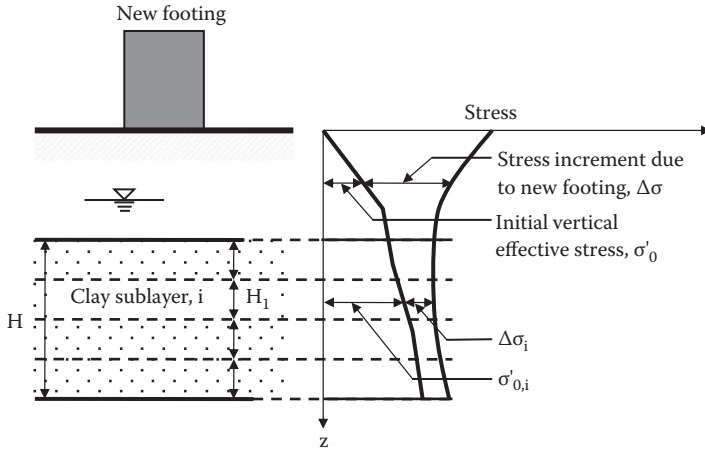


FIGURE 9.19 Consolidation settlement computation for multilayers or a thick layer.

Exercise 9.7

A 20 m thick uniform clay layer as shown in Figure 9.20 is anticipated to settle after a new footing is placed on the site. Distributions of computed initial vertical effective stress σ'_0 and incremental stress $\Delta\sigma$ due to the new footing under the center of the footing are plotted also in the figure. The e-log σ curve is obtained from a laboratory consolidation test for a clay sample at the site in Figure 9.21. Compute total final consolidation settlement of the clay layer under the center of the footing.

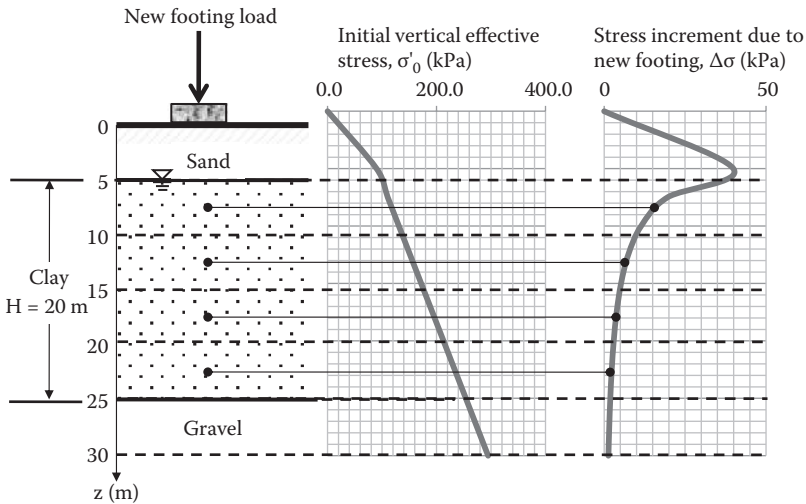


FIGURE 9.20 Exercise 9.7 problem.

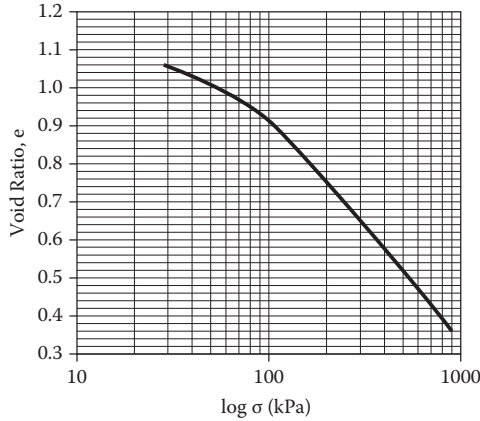


FIGURE 9.21 e-log σ curve for Exercise 9.7.

SOLUTION

A 200 m thick clay layer is divided into four equal sublayers and σ_0 and incremental stress $\Delta\sigma$ for each sublayer are read at the midpoints in Figure 9.20. Corresponding e_0 to σ_0 and e_f to $\sigma_0 + \sigma$ of each sublayer are read from Figure 9.22 (enlarged version of Figure 9.21). Note that in Figure 9.22 only $\sigma_0 + \sigma$ and e_f lines for the first sublayer are shown. The results are summarized in Table 9.8, where Equation (9.29) was used to compute $S_{e,i}$.

Thus, estimated total final settlement of the 20 m thick clay layer is **0.273 m**. ←

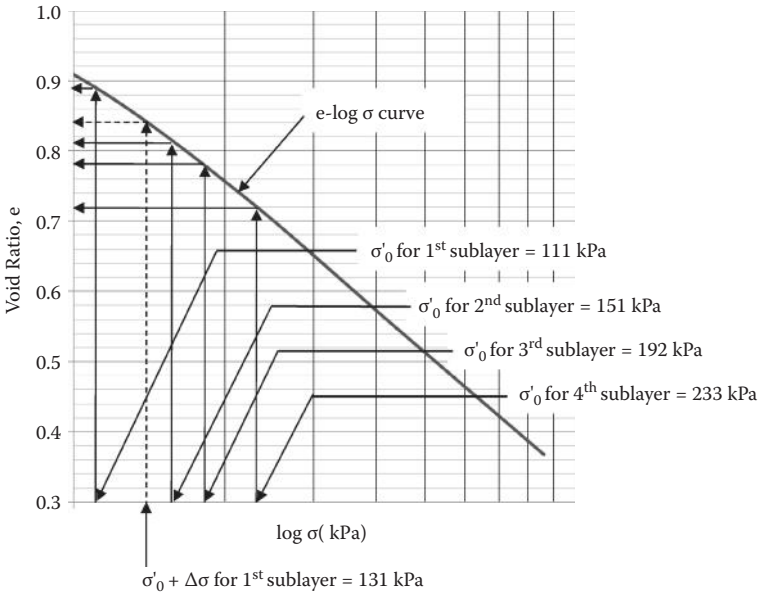


FIGURE 9.22 Enlarged curve of Figure 9.21.

TABLE 9.8
Settlement Computation for Thick or Multi-Clay Layers

Sublayer i	H (m)	$\sigma_{o,i}$ (kPa)	σ_i (kPa)	$\sigma_{o,i} + \sigma_i$ (kPa)	$e_{o,i}$	$e_{i,i}$	Δe_i	$S_{f,i}$ (m)
1	5	111	20	131	0.89	0.84	0.05	0.132
2	5	151	7	158	0.81	0.79	0.02	0.055
3	5	192	4	196	0.78	0.76	0.02	0.056
4	5	233	2.5	235.5	0.720	0.714	0.01	0.030
Σ	20	—	—	—	—	—	—	0.273

Table 9.8 is based on utilization of the e-log σ curve to obtain Δe_i . If clays are normally consolidated, $S_{f,i}$ can be calculated from Equation (9.28). In such cases, C_c values for all sublayers should be assigned if they vary with several different clay layers. Selection of sublayer thickness depends on the engineer's judgment by allowing rather small variations of σ_0 and $\Delta\sigma$ within sublayers.

9.12 SUMMARY OF PRIMARY CONSOLIDATION COMPUTATIONS

As discussed so far, in primary consolidation theory and practice, there are two different computations: (1) amount of final consolidation settlement, and (2) time to reach a certain percentage of the consolidation. It is practical and less confusing to separate consolidation problems into two categories: (1) *how much*, and (2) *how soon* (rate problem). By doing so, the selection of key consolidation equations and the handling of the clay layer thickness (H or 2H) become easier. The following summarize these two different procedures.

9.12.1 THE "HOW MUCH" PROBLEM

Depending on whether soil is normally consolidated or overconsolidated, Equation (9.28) or Equation (9.29) is used, respectively:

$$S_f = \frac{H}{1 + e_0} e = \frac{H}{1 + e_0} C_c \log \frac{\sigma_0 + \sigma}{\sigma_0} \quad \text{for normally consolidated soils} \quad (9.28 \text{ [repeated from Subsection 9.10.1]})$$

or

$$S_f = \frac{H}{1 + e_0} e \quad \text{for over consolidated soils} \quad (9.29 \text{ [repeated from Subsection 9.10.2]})$$

In Equation (9.28) for normally consolidated soils, Δe can be read directly from the e-log σ curve, which consists of only the virgin curve, or it is obtained from computed C_c , σ_0 and $\Delta\sigma$ values. Meanwhile, in Equation (9.29) for

overconsolidated soils, only the e-log σ curve is available to compute the final settlement. ***In both cases, H is the full thickness of the clay layer regardless of top and bottom drainage conditions.***

9.12.2 THE “HOW SOON” PROBLEM (RATE PROBLEM)

This problem always utilizes the relationship between the time factor T_v (Equation 9.15) and the degree of consolidation U (Table 9.4):

$$T_v = \frac{C_v t}{H^2} \quad (9.15 \text{ [repeated from Section 9.5]})$$

In this case, H is the longest drainage distance, and thus ***if only one boundary layer is pervious and the other is impervious, H in Equation (9.15) is the full clay layer thickness. On the other hand, if both the top and bottom layers are pervious, H in Equation (9.16) is half the clay layer thickness.***

9.13 SECONDARY COMPRESSION

In Terzaghi's consolidation theory discussed so far, when generated pore water pressure is fully dissipated, it is the end of consolidation and that part of consolidation is called the **primary consolidation**. In the theory, it takes infinite time, but practically, it will be completed in a certain finite time, observed as δ_{100} in Figure 9.9. After the primary consolidation is over, soil continues to compress with a slower rate as seen as the secondary compression curve in Figure 9.23 (replot of Figure 9.9 in terms of void ratio e and $\log t$). **Secondary compression** is not due to dissipation of pore water pressure, but rather due to slow rearrangement of fine particles and to other reasons.

The amount of secondary compression is sometimes very significant since it continues for a long time. The slope of the secondary compression curve in e versus $\log t$ is taken as C_α and defined as a **secondary compression index** in Figure 9.23. Thus, from the figure,

$$C_\alpha = \frac{e}{\log t - \log t_p} = \frac{e}{\log \frac{t}{t_p}}, \text{ thus } e = C_\alpha \log \frac{t}{t_p} \quad (9.30)$$

where t is any arbitrary time and t_p is the time at the end of primary consolidation; e_p is also defined as the void ratio at the end of primary consolidation in Figure 9.23. By using a similarity law of settlement Δe to the total height $1 + e_p$ and the secondary compression settlement S_s to the total clay layer thickness H ,

$$\frac{e}{1 + e_p} = \frac{S_s}{H}, \text{ thus, } S_s = \frac{e}{1 + e_p} H = \frac{H}{1 + e_p} C_\alpha \log \frac{t}{t_p} = C_\alpha H \log \frac{t}{t_p} \quad (9.31)$$

where C_α is $C_\alpha/(1 + e_p)$ and is called **modified secondary compression index**. Once C_α or C_α is determined from laboratory e-log t curve, Equation (9.31) is rewritten to

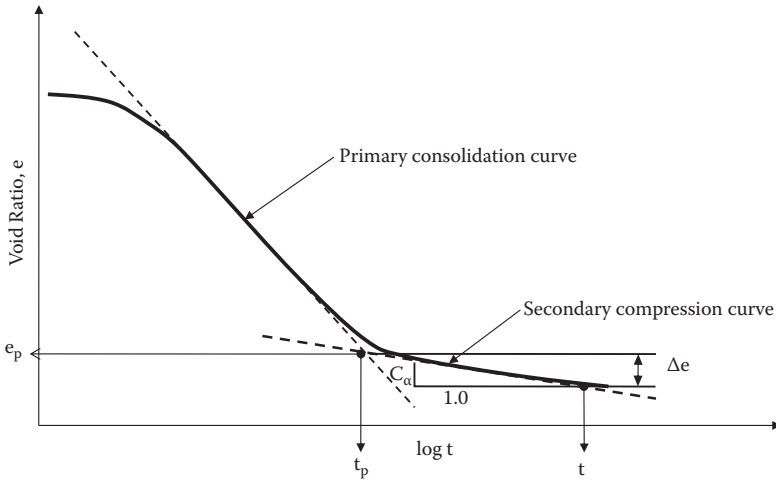


FIGURE 9.23 Secondary compression curve.

Equation (9.32) to take any arbitrary time interval t_1 to t_2 ($t_1 < t_2$). Note that t_1 should be larger than t_p :

$$S_s = \frac{H}{1 + e_p} C_\alpha \log \frac{t_2}{t_1} = C_\alpha H \cdot \log \frac{t_2}{t_1} \tag{9.32}$$

Equation (9.32) is used to estimate S_s for any time interval t_1 to t_2 during the secondary compression process.

The C_α or C_{α} value can be obtained from laboratory consolidation tests. Also, it was found empirically that the ratio of C_α to the compression index C_c is rather constant for a given group of materials. For inorganic clays and silts, the ratio (C_α/C_c) is about 0.04 ± 0.01 , and the ratio for organic clays and silts is about 0.05 ± 0.01 for most natural soils (*Terzaghi, Peck, and Mesri 1996*).

Exercise 9.8

For the same problem as in Exercise 9.6 (Figure 9.18), estimate the secondary compression settlement S_s from the year 20 to the year 40. The e - $\log t$ curve at a similar stress condition to the in-situ value is given in Figure 9.24.

SOLUTION

From Figure 9.24, the secondary compression index C_α is read as the slope of the secondary compression curve:

$$C_\alpha = \frac{-(e_2 - e_1)}{\log t_2 - \log t_1} = \frac{-(0.365 - 0.395)}{\log \frac{10000}{0.1}} = 0.006$$

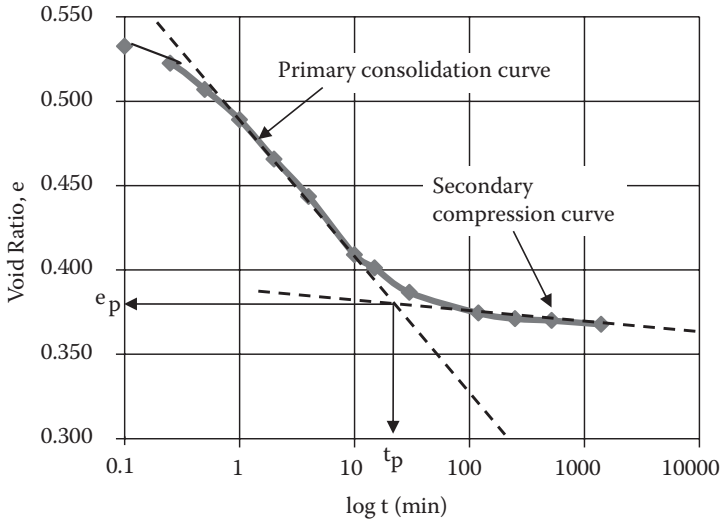


FIGURE 9.24 Exercise 9.8 (e-log t curve).

and $e_p = 0.378$ from the figure. By substituting these values with $t_1 = 20$ and $t_2 = 40$ years into Equation (9.32),

$$S_s = \frac{H}{1 + e_p} C_a \log \frac{t_2}{t_1} = \frac{3}{1 + 0.378} 0.006 \log \frac{40}{20} = \mathbf{0.00393 \text{ m} \leftarrow}$$

In Exercise 9.6, primary consolidation settlement was 0.0267 m; thus, the previously obtained secondary compression settlement for a period of 20 years adds an additional 14.7% settlement, and it cannot be simply neglected.

9.14 ALLOWABLE SETTLEMENT

Theoretically speaking, if a building settles evenly, it would not cause any damages to the structure. However, in practice, most foundations settle unevenly (**differential settlement**) due to uneven load distributions and non-uniform subsurface soil conditions. If differential settlement occurs, walls may crack, doors and windows may jam, and tall structures may tilt and tumble. In other situations, for example, if a paved parking lot settles by a certain amount (**total settlement**), this would create problems for access to the roads and buildings. Accordingly, the total and the differential settlements are to be controlled during and after construction. **Allowable settlements** are specified by design engineers based on the safety and the functionality of buildings and earth structures. Table 9.9 gives guidance about various allowable settlements based on both theory and observations of structures that have suffered damage.

TABLE 9.9
Guidance for Allowable Settlement

Type of Movement	Limiting Factor	Maximum Settlement
Total settlement	Drainage and access	150–600 mm
	Probability of differential settlement	
Tilting	Masonry walls	25–50 mm
	Framed buildings	50–100 mm
	Tower, stacks	0.004B ^a
Differential	Rolling of trucks, stacking of good	0.01S ^a
	Crane rails	0.003S ^a
	Brick walls in buildings	0.0005S–0.002S ^a
	Reinforced-concrete building frame	0.003S ^a
Maximum permissible settlement	Steel building frame, continuous	0.002S ^a
	Steel frame, simple	0.005S ^a
	Front slab, 100 mm thick	0.02S ^a

Source: After *Sowers, G. F., 1979, Introductory Soil Mechanics and Foundations: Geotechnical Engineering*, 4th ed., MacMillan, New York.

Note: B = footing base width; S = column spacing.

^a Differential settlement in distance B or S.

9.15 GROUND-IMPROVING TECHNIQUES AGAINST CONSOLIDATION SETTLEMENT

When the estimated settlement exceeds the allowable settlement, some remedial measures are necessary. Depending on the in-situ situations, several options are available:

1. Replace the whole compressible soils with less compressible soils if the thickness of the soils is rather thin and if it is economically feasible.
2. Change the soil property into a less compressible one by **chemical grouting, cement grouting, or lime mixing**.
3. Reinforce soft ground by utilizing **geosynthetics** materials.
4. Accelerate consolidation time prior to major structural construction by means of **vertical drain (paper drain, wick drain, or sand drain)** techniques.
5. **Preload** the area and induce consolidation prior to construction.
6. Apply **vacuum** in the soft clay to induce negative pore water, thus increasing the effective stress. The increased effective stress works as a preloading as in item (5).

Detailed procedures of these techniques can be found in other literature such as *Hausmann (1990)* for ground modification, *Koerner (2005)* for geosynthetics reinforcement, etc.

The techniques mentioned in items (4), (5), and (6) are addressed in the following subsections based on the consolidation concept learned in this chapter.

9.15.1 VERTICAL DRAIN (PAPER DRAIN, WICK DRAIN, AND SAND DRAIN) TECHNIQUES

The time factor equation (Equation 9.15) is rewritten as

$$t = \frac{H^2}{C_v} T_v \tag{9.33}$$

Equation (9.33) implies that consolidation time t is proportional to H^2 , where H is the longest drainage distance. The techniques call for shortening H in the field with inserted vertical drainage materials.

The drainage materials may be paper, wick, or sand columns. The technique was first developed in the 1960s by using long strips of colligated cardboards (paper), which were inserted into soft ground. As seen in Figure 9.25, strips of vertical drains are arranged so that the drainage distance H_d is much shorter than the original drainage distance $H/2$ without vertical drain installation. For example, if the H_d to $H/2$ ratio becomes $1/5$, the consolidation time reduces to $1/25$, according to Equation (9.33). Presently, the wick drain replaces the paper drain in most applications. The wick is made of synthetic materials. It consists of a long core material in the center with drain ribs in the longitudinal direction and geofabric filter material around it, as seen in Figure 9.26.

A sand drain works with the same principle. Bore holes are drilled through soft soil layers and sand is filled in the holes to make sand columns. At the **Kansai International Airport** project (first phase) in Japan (see Figure 1.4 in Chapter 1), 1,000,000 sand columns were installed in order to stabilize 20 m thick soft soils underwater to construct 511 ha of a man-made island. The sand columns had 400 mm diameter and were placed with 2.5 m pitches.

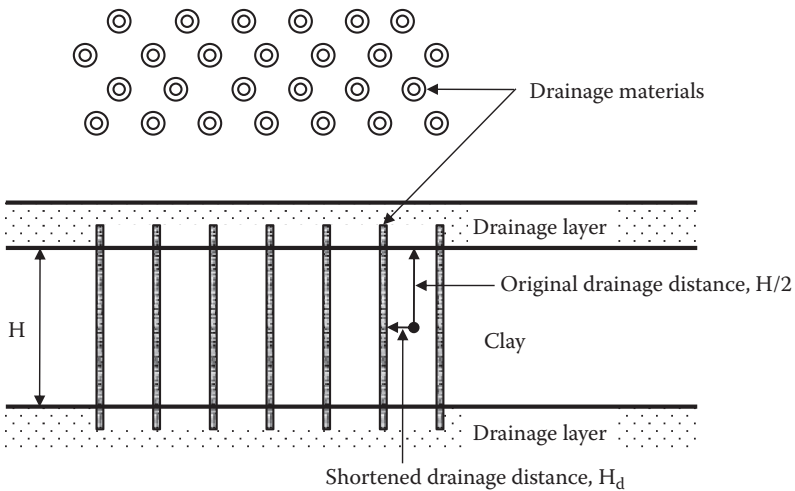


FIGURE 9.25 Principle of vertical drain (paper, wick, and sand drain) techniques.

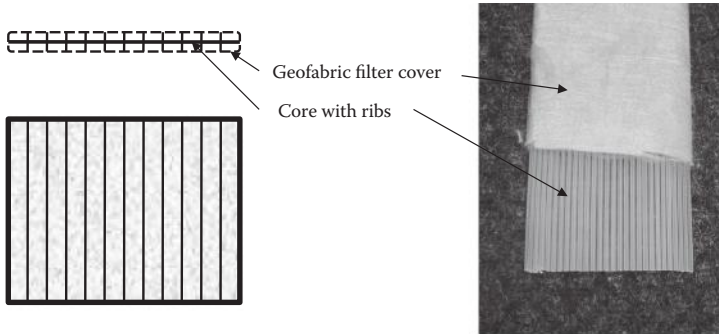


FIGURE 9.26 Wick drain.

9.15.2 PRELOADING TECHNIQUE

In the **preloading technique**, dirt (mostly soil) of a few meters high is placed to cover the future building site. It is left for several months to a year. The dirt is then removed and buildings are constructed on the site.

This technique reduces future consolidation settlement. The entire processes of the loading and unloading of dirt and the building construction are plotted in an e -log σ diagram as seen in Figure 9.27. Point A is the starting point at the site before construction. During the preloading process of the dirt with $\Delta\sigma_{\text{preload}}$, it moves from Point A to Point B. During removal of the dirt, it moves back to Point C. During the building construction process with $\Delta\sigma_{\text{building}}$, it moves back again from Point C to Point D. When the resulting changes in void ratio are compared in the cases of with-preloading ($\Delta e_{\text{with preload}}$) and without-preloading ($\Delta e_{\text{w/o preload}}$), it is clear from the figure that $\Delta e_{\text{with preload}}$ is much smaller than $\Delta e_{\text{w/o preload}}$. Thus, consolidation settlement during building construction will be considerably reduced.

This cost-effective technique is often used for rather small-scale projects (office buildings, shopping mall construction, etc.), for which there is some flexibility in the construction schedule, since the preloading process requires some extra time to achieve. Also, the preloading technique is used together with vertical drain methods to reduce the consolidation time as well as the amount of consolidation settlement.

9.15.3 VACUUM CONSOLIDATION TECHNIQUE

Recently, a new technique called **vacuum consolidation** has been developed. This induces a vacuum in soft clay soil through pipes in a sealed environment by impermeable membranes. Typically, 80 kPa (-78% of the atmospheric pressure) or greater vacuum (negative pore water pressure) can be applied by this method. Because of induced vacuum pressure inside the soil mass, the atmospheric pressure works the same way as in the preloading technique through the impermeable membrane on top of the ground. An 80 kPa vacuum pressure is equivalent to about 4.0 m height of soil load in the preloading technique. Readers are referred to other references on the topics (e.g., *Carter, Chai, and Hayashi 2005; Chu and Yan 2005*).

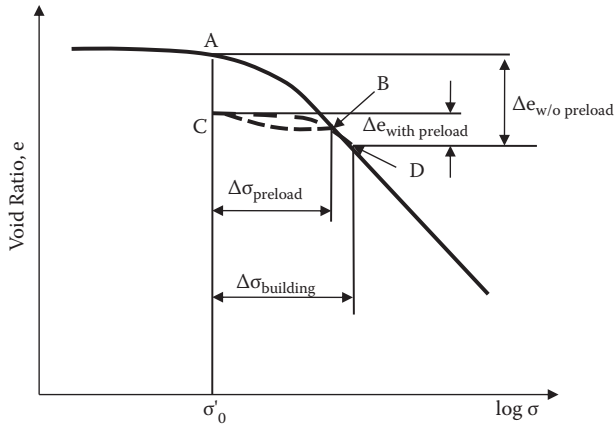


FIGURE 9.27 Principle of preloading technique.

9.16 SUMMARY

Settlements, in particular, consolidation settlements, are a major soil mechanics problem, as shown in Chapter 1 (examples are Pisa's tower and Kansai International Airport). The theory and practice were fully presented in this chapter. Normally consolidated versus overconsolidated soils are quite different in their behaviors and thus differences should be clearly recognized. The ways to handle the thickness of clay layer H or $2H$ were clearly distinguished in this chapter (Section 9.12) by always taking H for "how much" problems and H or $2H$ for "how soon" problems.

REFERENCES

- Bowles, J. E. (1996), *Foundation Analysis and Design*, 5th ed., McGraw-Hill, New York.
- Carter, J. P., Chai, J. C., and Hayashi, S. (2005), Ground deformation induced by vacuum consolidation, *Journal of Geotechnical and Geoenvironmental Engineering*, ASCE, vol. 131, no. 12, 187–192.
- Casagrande, A. (1936), Determination of the preconsolidation load and its practical significance, *Proceedings of 1st International Conference on Soil Mechanics and Foundation Engineering*, Cambridge, MA, vol. 3, 60–64.
- Chu, J., and Yan, S. W. (2005), Estimation of degree of consolidation for vacuum preloading projects, *International Journal of Geomechanics*, vol. 5, no. 2, 187–192.
- Hausmann, M. R. (1990), *Engineering Principles of Ground Modification*, McGraw-Hill, New York.
- Janbu, N., Bjerrum, L., and Kjaernsli, B. (1956), *Norwegian Geotechnical Institute Publication*, no. 16.
- Koerner, R. M. (2005), *Designing with Geosynthetics*, 5th ed., Pearson/Prentice Hall, Upper Saddle River, NJ.
- Mayne, P. W., and Poulos, H. G. (1999), Approximate displacement influence factors for elastic shallow foundations, *Journal of Geotechnical and Environmental Engineering*, ASCE, vol. 125, no. 6, 453–460.
- Schleicher, F. (1926), Zur Theorie Des Baugrundes, *Der Bauingenieur*, no. 48, 49.

Chapter 8 Shear Strength of Soils

11.1 INTRODUCTION

The strength of soil is a key design parameter in designing building foundations, embankments, retaining structures, and other earth structures. In shallow foundation design, the capacity of the foundation to support footing load is given by the soil's **bearing capacity** (Chapter 14), which is a function of its strength parameters. **Lateral earth pressure** theories (Chapter 12) at ultimate stages (that is, active or passive failure stages) use the strength parameters of the soil. **Slope stability analysis** (Chapter 16) also requires the strength of the soil as a resisting force against sliding along potential sliding surfaces. In this chapter, soil strength is defined and laboratory and field determination techniques on the shear strength parameters are presented. Proper interpretation of these parameters and the application to field problems are presented and critically reviewed.

11.2 FAILURE CRITERIA

Soil strength may be attributed to two distinctly different mechanisms of materials: one is its **frictional resistance** and the other is **cohesive resistance** along the shearing zone. As seen in Figure 11.1, shearing of a soil assemblage in (a), which is subjected to normal stress and shear stress, is modeled with a block on a solid plate with a rough surface as seen in (b). In the model, shear stress τ is resisted by a frictional mechanism and cohesive resistance between the interface of the block and the solid plate. Frictional resistance τ_{friction} follows Coulomb's friction law ($\tau_{\text{friction}} = \sigma \tan\phi$), where σ is the normal stress and ϕ is called the **angle of internal friction** of soil. The angle ϕ can be interpreted as the friction angle between facing soil elements along the shear surface. Cohesion resistance c is called **cohesion** of soil. In the block model, it could be simulated by heavy grease coated between the block and the plate, and thus it is independent of the applied normal stress σ . In soils, normal stress-independent cohesion comes from particle-to-particle close-range interactive forces as studied in Chapter 3, and it is a material property of fine particles (clays or cohesive soils).

Accordingly, the total shear stress at failure τ_f is expressed as

$$\tau_f = c + \sigma \tan\phi \quad (11.1)$$

Equation (11.1) is a linear relationship between σ and τ_f and plotted as a straight line in Figure 11.2(a). The line defined by Equation (11.1) is called the **failure envelope**, which implies that if any stress combination of σ and τ on any arbitrary plane (as shown in Figure 11.2(b) plots below the failure envelope line, there is no failure.

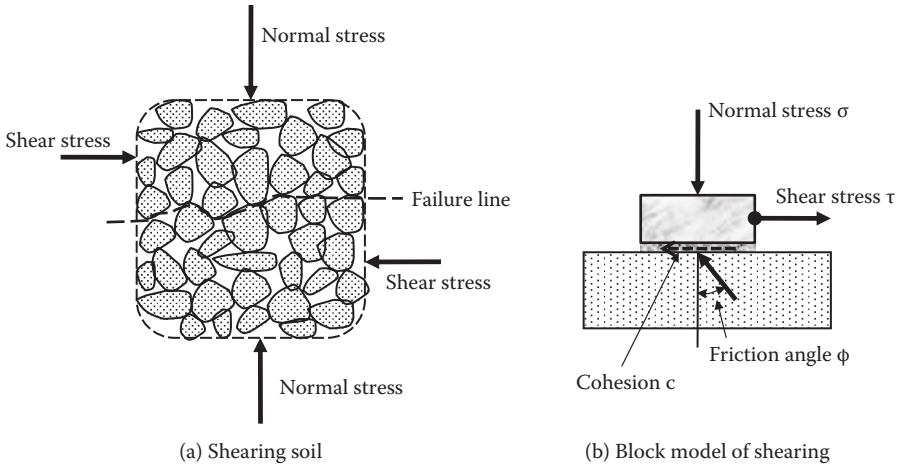


FIGURE 11.1 Shearing in soil mass.

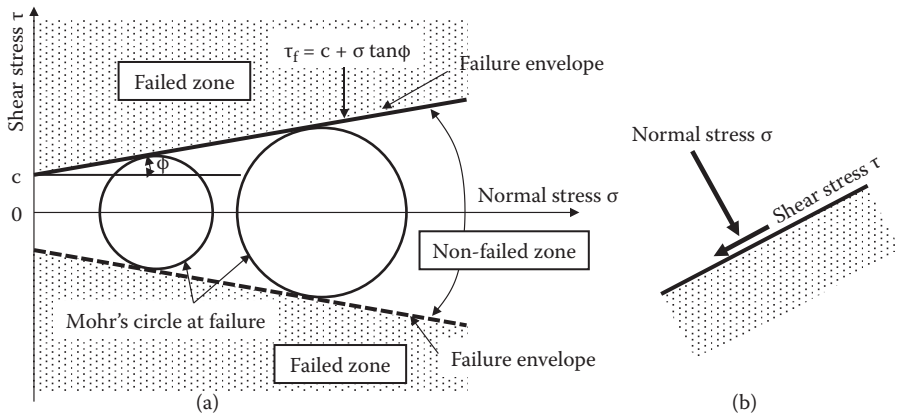


FIGURE 11.2 Failure criteria.

On the other hand, if the stress combination of σ and τ goes above the envelope, the failure occurs on that plane. In practice, combinations of σ and τ cannot go beyond the envelope, and thus the envelope defines the upper limit of stress combination on any plane of an element. Figure 11.2(a) also plots a mirror image of the failure envelope in the negative domain of the shear stress with a dotted line, since the negative shear stress merely changes its direction, and thus these two failure envelopes define the safe limits of the stress combination of σ and τ . In the figure, two Mohr's circles at failure are drawn that make tangent at the failure envelopes as seen. In other words, Mohr's circles cannot cross the failure envelopes.

Equation (11.1) is called the **Mohr–Coulomb failure criteria**, which is attributed to Coulomb's contribution on frictional law and linear representation of its relation

and Mohr's contribution on defining failure with a unique combination of normal stress σ and shear stress τ .

Figure 11.2 also suggests the importance of shear stress rather than normal stress in failure of soils. As an example, imagine a soil element in deep earth such as in a deep salt mine (e.g., 1000 m deep). The vertical normal stress of the dry soil element at 1000 m deep is very high ($\sigma_v = \gamma_{\text{soil}} \cdot z \approx 20.0 \text{ kN/m}^3 \times 1000 \text{ m} = 20,000 \text{ kPa}$). How can that soil element survive under such high normal stress? At that element, lateral normal stress σ_h is about a half of σ_v (see K_0 discussion in Chapter 12), and thus $\sigma_h \approx 10,000 \text{ kPa}$. A Mohr's circle is drawn in Figure 11.3 for this soil element. Even though the circle has large σ values, it is still below the failure envelope, and thus it is safe in any plane directions. Increasing normal stress is not a critical factor for failure as seen in the preceding example. However, if shear stress τ is increased, it will easily touch the failure envelope, and thus shear stress is critically important in the failure of soils. This is the reason why soil strength is often referred to as **shear strength**.

Terzaghi (1925) modified the Mohr–Coulomb equation to include his **effective stress concept** as

$$\tau_f = c' + \sigma' \tan\phi' = c' + (\sigma - u) \tan\phi' \tag{11.2}$$

where all strength parameters c' and ϕ' are expressed in terms of the effective normal stress σ' ($= \sigma - u$). His concept is that soil strength is controlled by the effective stress (stresses in the soil's skeleton) rather than the total stress. It is found to govern the failure mechanism of soils, which is examined in detail later in this chapter.

There are many different soil testing devices to determine c and ϕ or c' and ϕ' in the laboratory as well as in the field. Commonly used shear testing devices and their interpretation of results are discussed in the following sections.

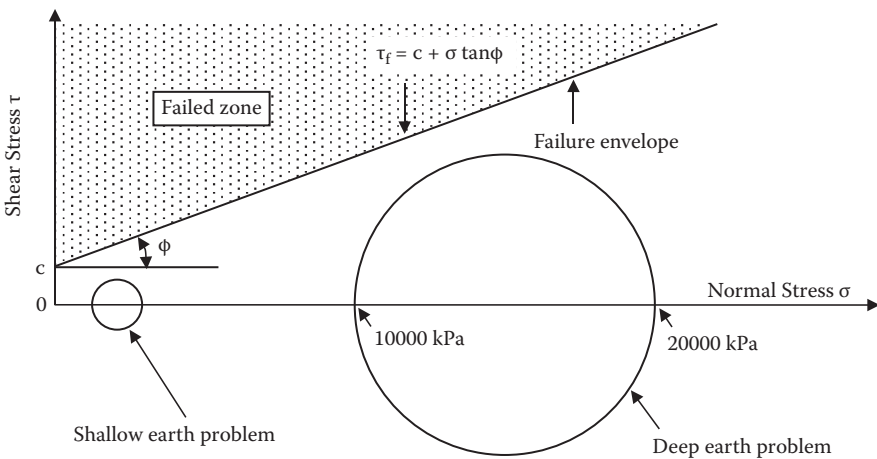


FIGURE 11.3 Deep earth and high normal stress problem.

11.3 DIRECT SHEAR TEST

This is the earliest and simplest device to determine soil strength parameters. As seen in Figure 11.4, it consists of upper and lower shear boxes, and a soil specimen is placed inside the box. Vertical normal force F_v and hence the normal stress $\sigma (= F_v/\text{specimen area})$ is applied and kept constant. In most devices, the upper box is fixed, and the lower box is movable on low-friction rollers at the base. Also, special care is taken to minimize friction at contacting surfaces between the upper and the lower shear boxes such as with low-friction Teflon push bolts. The lower box is pulled or pushed to apply shear force T , and hence the shear stress $\tau (= T/\text{specimen area})$ is induced along the middle plane of the specimen.

In this device, shear failure surface is forced to develop in a near-horizontal direction. Measurements during the test are constant σ , and changes in τ , vertical deformation δ_v , and horizontal shear deformation δ_h . The change in δ_v measurement is directly proportional to the volume change of the specimen $\Delta V (= \Delta\delta_v \cdot \text{specimen area})$ since the cross-sectional area of the specimen remains the same. Thus, under a given normal stress σ , τ versus $\Delta\delta_h$ and ΔV versus $\Delta\delta_h$ are plotted as seen in Figure 11.5.

Figure 11.5(a) defines the **peak shear strength** and the **residual shear strength**. The former is generally used as the shear strength of the soil τ_f . The latter is the strength after a large deformation, and it may be used to evaluate the stability of earth structures when large deformation is allowed beyond its peak strength.

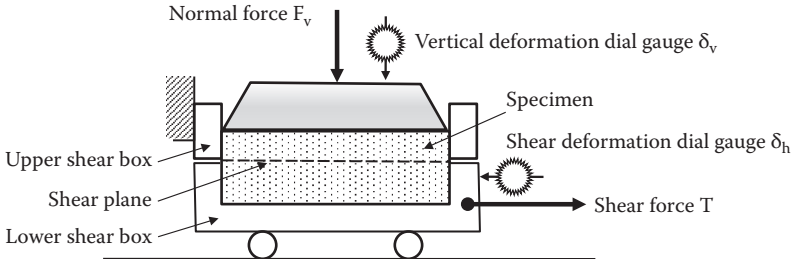


FIGURE 11.4 Direct shear test setup.

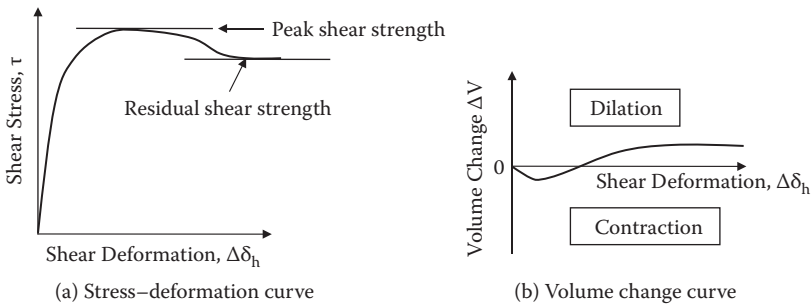


FIGURE 11.5 Direct shear test result.

Soil may contract or dilate during shearing, as seen in Figure 11.5(b), mostly depending on its initial density. It is interesting to notice that soil is a very unique material, which increases its volume upon application of shear stress (**dilatancy**), particularly for dense sands and heavily overconsolidated clays. It is because densely packed grains or particles have to move or roll over neighboring grains to change their relative positions during shearing, as seen in Figure 11.6.

Accordingly, shear stress–deformation relations and their volume change characteristics during shear are largely influenced by initial density of specimens. Figure 11.7 shows these for dense, medium dense, and loose soils. As seen in the figure, the shear stress–deformation curves emerge to the residual shear strength at a large shear deformation. The void ratios also emerge to a certain value at a large shear deformation. When soil assemblage is sheared at large deformation, certain zones within the specimen (**shear zone**) are subjected to large shear deformation. Along these shear zones, where shear failure is taking place, particles are oriented to a preferred direction, which changes from their original loose or dense configurations, and a steady-state flow (failure) mechanism is created. This is the reason why all strengths emerge to the residual strength and all void ratios become a certain value at large shear deformation, regardless of their original denseness. In Figure 11.7(b), initial dense soils undergo initial contraction and then dilation. On the other hand,

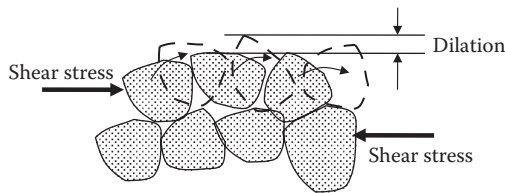


FIGURE 11.6 Dilatancy model.

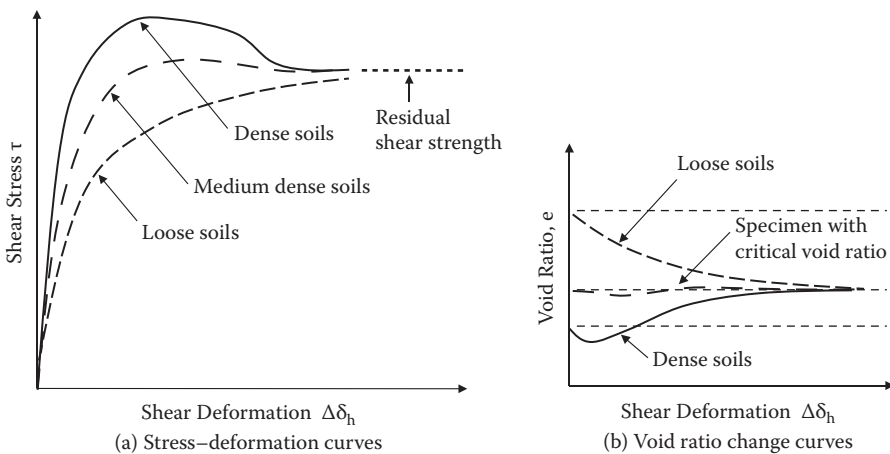


FIGURE 11.7 Shear stress–deformation and void ratio for loose to dense soils.

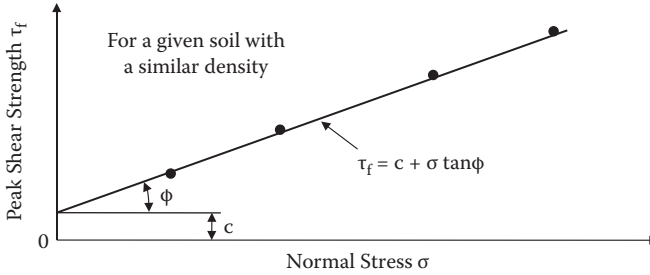


FIGURE 11.8 Determination of j and c from direct shear tests.

loose soils contract all the way till failure. For a specimen in between dense and loose, there is a specimen of which the void ratio remains nearly the same during the shear. That void ratio is called **critical void ratio** and this specimen does not contract or dilate during shear.

For a given soil with a similar density, several direct shear tests are conducted under different normal stresses. Peak shear strength values τ_f are measured for each test. Then σ and τ_f relations are plotted as in Figure 11.8. A linear relation is obtained through the data points and the intersection on the τ_f axis gives the cohesion component c ; the slope of the line makes the internal friction angle ϕ . For different soils and different densities, lines are different, so different c and ϕ values are obtained.

11.4 UNCONFINED COMPRESSION TEST

This rather simple test is used for cohesive specimens only, which can stand alone without any lateral confinement of the specimen during the test. As seen in Figure 11.9, a specimen is trimmed to have a cylindrical shape and placed on a loading platform. The specimen height-to-diameter ratio should be at least 2.0 or more to avoid the end boundary effect during the shear. Axial compressive force F_v is gradually increased until failure with a measurement of axial deformation δ_v .

In general, the test is completed within 10 to 20 minutes, so during this process the water content of the specimen remains nearly constant. Pore water pressure may build up inside the specimen, but it will not have enough time to dissipate during a short period of shearing time. This process is called an **undrained shear test** and is discussed later in this chapter.

Axial normal stress σ_v ($= F_v/\text{specimen area}$) and axial strain ϵ_v ($= \delta_v/\text{initial specimen height}$) are plotted in Figure 11.10. Two curves for typical soils are seen: (a) heavily overconsolidated or dense soils with a clear peak value, and (b) normally consolidated or loose soils without a clear peak value. The peak σ_v values or σ_v values at a certain defined failure strain ϵ_v (e.g., at $\epsilon_v = 10\%$ or 15% strain, etc.) are taken as **unconfined compression strength** q_u . In this experiment, q_u is the major principal stress at failure. Lateral normal stress is the minor principal stress and is zero with no lateral confinement (unconfined). Accordingly, Mohr's circle is drawn

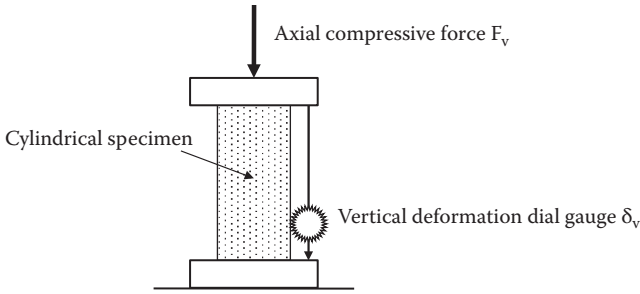


FIGURE 11.9 Unconfined compression test setup.

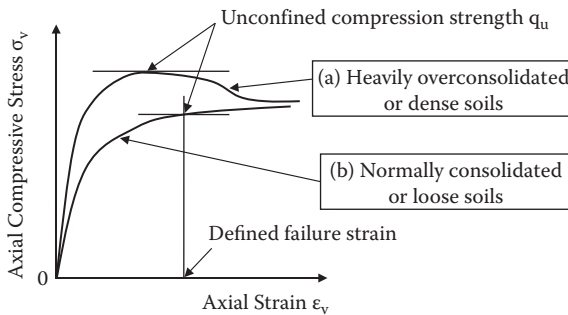


FIGURE 11.10 Unconfined compression test result.

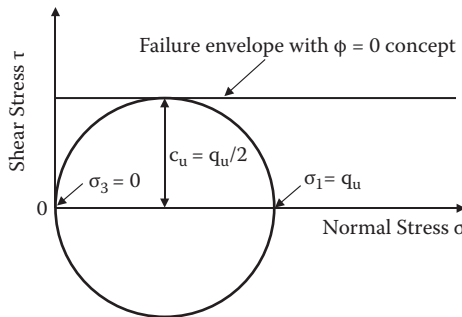


FIGURE 11.11 Determination of C_u from unconfined compression test.

as in Figure 11.11. A horizontal failure envelope ($\phi = 0$) is drawn to contact the failure Mohr's circle. Thus, the maximum shear stress at failure C_u is equal to

$$C_u = q_u/2 \tag{11.3}$$

In Figure 11.11, a horizontal failure envelope is drawn and is called the $\phi = 0$ **concept** in determining shear strength of cohesive soils. It will be discussed later in the section on unconsolidated undrained tests in this chapter.

11.5 TRIAXIAL COMPRESSION TEST

11.5.1 GENERAL CONCEPT AND TEST SETUP

A triaxial compression test device is routinely used to determine the shear strength of soils for more general stresses and drainage conditions. It applies three principal stresses, σ_1 , σ_2 , and σ_3 , to a cylindrical specimen; the intermediate principal stress σ_2 is equal to the minor principal stress σ_3 , as seen in Figure 11.12. The axial stress is increased until failure, while the lateral confining pressure is kept constant during the shear. Thus, the axial stress is the major principal stress σ_1 and the lateral confining pressure is the minor principal stress σ_3 . Note that since σ_2 is always equal to σ_3 , this is not three-axial (triaxial) test equipment in the true sense of the term.

A specimen is enclosed in a thin rubber membrane (typically 8–15 μm thick) and placed on a loading platform. Figure 11.13 shows the schematic setup of a typical triaxial compression test device. In this system, the lateral confining pressure is applied through a thin rubber membrane to the specimen via chamber pressure. During the test, the confining pressure, in general, is kept constant and the axial compressive force F_v is increased to failure. The vertical deformation δ_v is measured to compute the axial strain.

For a free body diagram of the upper section of soil specimen as seen in Figure 11.14, the vertical force equilibrium is established by neglecting weights of soil, loading cap, and loading piston as follows:

$$F_v + \sigma_3 \cdot A_s = \sigma_1 \cdot A_s, \quad \text{and thus,} \quad F_v/A_s = \sigma_1 - \sigma_3 \quad (11.4)$$

where F_v is the applied vertical force on the top of the piston and A_s is the specimen's cross-sectional area. $\sigma_1 - \sigma_3$ is called the **deviatoric stress**, and it is increased from zero to failure stress during the shear test.

In a typical triaxial test, the confining stress σ_3 is kept constant and the vertical force F_v is increased until failure. The deviatoric stress $\sigma_1 - \sigma_3 (= F_v/A_s)$ and the vertical strain $\epsilon_1 (= \delta_v/L_0)$ is monitored, where δ_v is the measured vertical deformation increment and L_0 is the initial specimen height.

Exercise 11.1 demonstrates how to utilize triaxial test data to obtain shear strength parameters c and ϕ in a general sense.

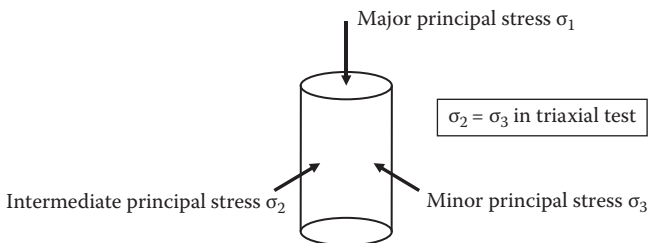


FIGURE 11.12 Triaxial stresses on a cylindrical specimen.

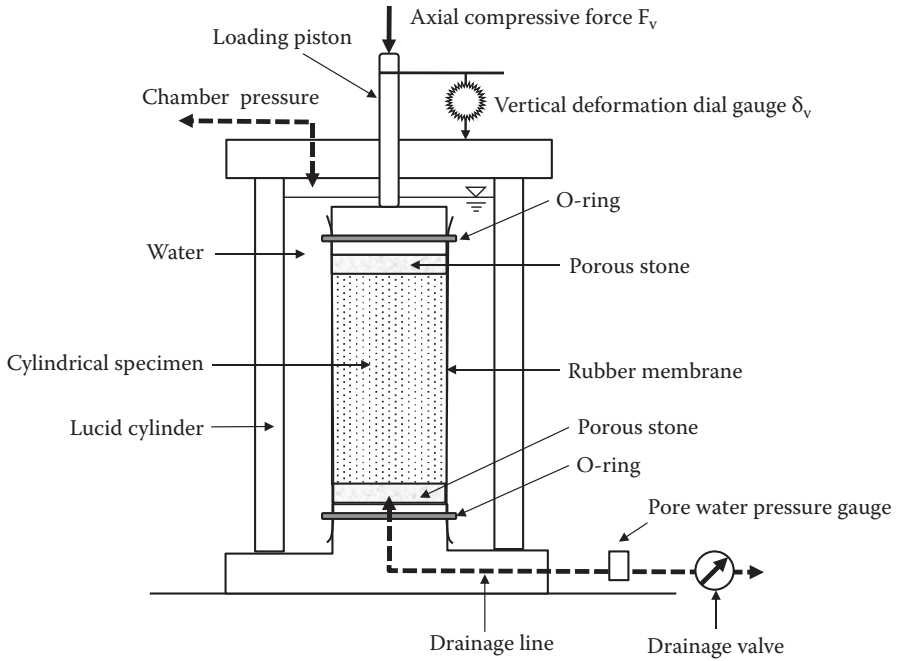


FIGURE 11.13 A typical triaxial test setup.

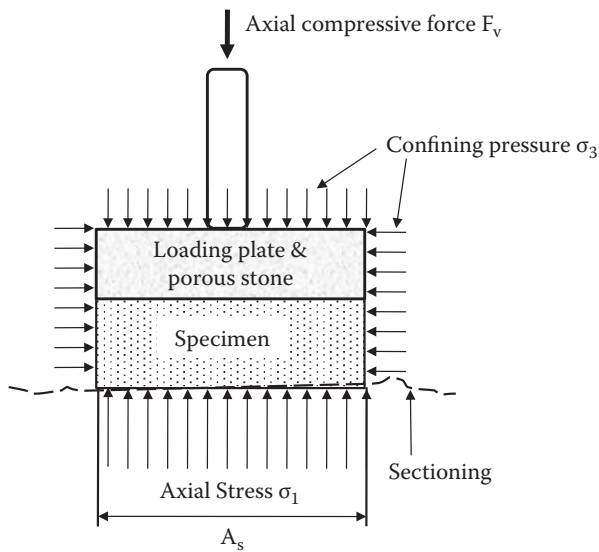


FIGURE 11.14 Free body diagram of triaxial specimen.

Exercise 11.1

Triaxial test data with three different confining pressures for a similar soil are shown in Figure 11.15. The deviatoric stress $(\sigma_1 - \sigma_3)$ is plotted with the vertical strain ϵ_1 and the failure strengths $(\sigma_1 - \sigma_3)_f$ are identified for those tests. After drawing Mohr's circles at failure for three specimens, determine cohesion component c and the angle of internal friction ϕ of this soil.

SOLUTION

From the data, for specimen 1:

$$\sigma_3 = 80 \text{ kPa}$$

$$(\sigma_1 - \sigma_3)_f = 174 \text{ kPa and thus,}$$

$$\sigma_{1f} = (\sigma_1 - \sigma_3)_f + \sigma_3 = 174 + 80 = 254 \text{ kPa}$$

From the data, for specimen 2:

$$\sigma_3 = 120 \text{ kPa}$$

$$(\sigma_1 - \sigma_3)_f = 202 \text{ kPa and thus,}$$

$$\sigma_{1f} = (\sigma_1 - \sigma_3)_f + \sigma_3 = 202 + 120 = 322 \text{ kPa}$$

From the data, for specimen 3:

$$\sigma_3 = 160 \text{ kPa}$$

$$(\sigma_1 - \sigma_3)_f = 248 \text{ kPa and thus,}$$

$$\sigma_{1f} = (\sigma_1 - \sigma_3)_f + \sigma_3 = 248 + 160 = 408 \text{ kPa}$$

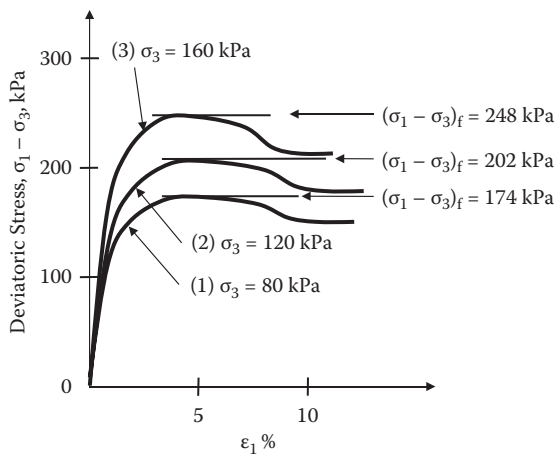


FIGURE 11.15 Exercise 11.1 problem (results from triaxial tests).

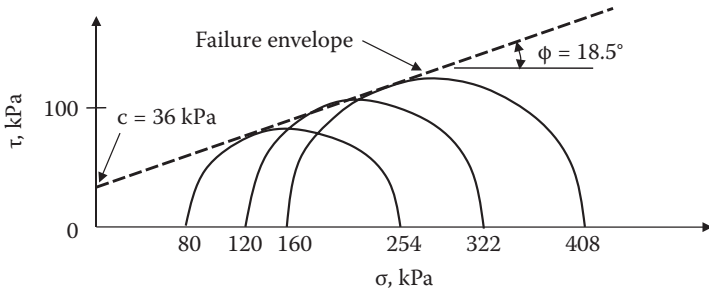


FIGURE 11.16 Exercise 11.1 (determination of ϕ and c).

Based on the preceding σ_{1f} and σ_3 values, Mohr's circles at failure are constructed in Figure 11.16. A failure envelope is also drawn by just touching these Mohr's circles at failure, and the cohesion c and the angle of internal friction ϕ are read as 36 kPa and 18.5° , respectively, as seen.

Real practice of a triaxial test requires more detailed techniques on how the specimen is prepared and how it is sheared in terms of pore water pressure dissipation during the shear. In this respect, the drainage line and the drainage valve in Figure 11.13 play significant roles. As seen in the figure, the drainage line is connected from the inside of the specimen through the porous stone. During the preshearing process, the drainage valve is kept open to allow the dissipation of induced pore water pressure for a **consolidated test** or kept closed for an **unconsolidated test**. During the shear, the drainage valve could be either closed for an **undrained test** or opened for a **drained test**. When the valve is closed, the pore water pressure gauge monitors the pore water pressure buildup inside the specimen.

11.5.2 INITIAL CONSOLIDATION PROCESS AND DRAINAGE CONDITION DURING SHEAR

In the following discussions, it is assumed that soils are fully saturated. A triaxial specimen is, in general, brought to the laboratory in a thin wall tube. It is then extruded, and trimmed into a cylindrical specimen with approximately 2:1 sample height-to-diameter ratio. The specimen is then placed in the device, and the confining pressure through the thin membrane is increased to a certain value. At this stage, if the drainage valve is open, the specimen will be compressed by the confining pressure, and the water in pore space of the specimen will be squeezed out through the drainage line and a consolidation process will take place. When the specimen is left for several hours to overnight, primary consolidation will be completed under an applied confining pressure. The process is called a **consolidated test** for the sample preparation process. When the previously described consolidation process is not allowed, by keeping the drainage valve closed or by not allowing sufficient time for consolidation, the test is called an **unconsolidated test**.

During the shear, the drainage line could be kept open for a **drained test** or closed for an **undrained test**. Note that a drained test cannot be simply accomplished by opening the drainage valve. Rather, this is accomplished by allowing generated pore water pressure to be dissipated fully during the process of shear, and thus a drained test takes, in general, a long time (slow shearing)—such as a few days to a week or more—for cohesive soils. On the other hand, an undrained test does not require a long shearing time as in a drained test, and it could be completed in an hour or so (quick shearing). In general, undrained tests are accompanied with pore water pressure measurement.

Thus, types of triaxial tests are any combination of **preshear conditions** (either consolidated or unconsolidated) and **drainage conditions during shear** (either drained or undrained). Four combinations are possible, but, practically, the three listed here are used:

Consolidated drained test (CD test, S test)

Consolidated undrained test (CU test, Q_c test) with/without pore water pressure measurement

Unconsolidated undrained test (UU test, Q_u test)

In this list, “S test” stands for “slow test” since the drainage process during the shear takes a long time, and “Q test” stands for “quick test” since an undrained test could be finished in a short time. Note that S and Q designations will be applicable for cohesive soils only. For granular soils (sands and gravels), both drained and undrained tests do not require a large amount of time due to the high permeability of the materials.

11.5.3 CONSOLIDATED DRAINED (CD) TRIAXIAL TEST

First, the specimen is fully consolidated and then it is sheared slowly to allow the generated pore water pressure to be fully dissipated. This requires at least 1 day for the consolidation process and several days for conducting drained shear. Let us assume that a clay specimen is prepared with enough water to have the water content above its liquid limit, and then the consolidated drained test is performed. For nearly zero consolidation pressure, the strength of the specimen is nearly equal to zero since the specimen’s initial water content was above the liquid limit. With a small consolidation pressure, the specimen gains some strength due to the consolidation process. During the shear it also gains some more strength due to the drainage of water. With higher consolidation pressure, it gains more strength due to its higher reduction in water content through the consolidation and drained shear processes. Accordingly, sizes of failure Mohr’s circles nearly proportional to their consolidation pressures are drawn to define the failure envelope of the soil as seen in Figure 11.17.

The preceding specimens started from a very high water content (above their liquid limit) and gradually gained strength due to consolidation in a process similar to the natural forming of soil deposits under bottoms of lakes or rivers. These are called **normally consolidated soils**, as discussed in the consolidation

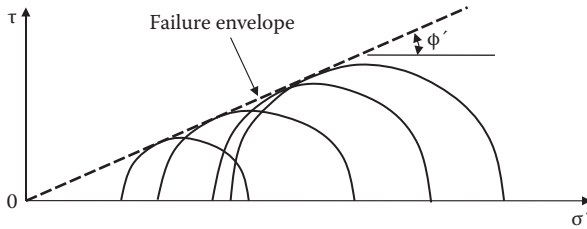


FIGURE 11.17 Failure envelope from CD test for normally consolidated soils.

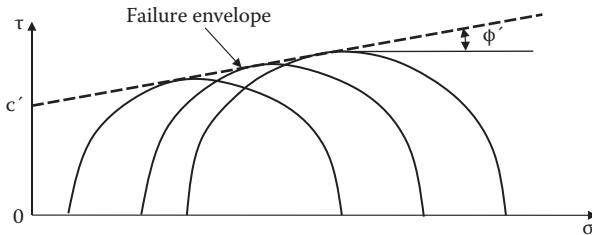


FIGURE 11.18 Failure envelope from CD test for overconsolidated soils.

section of Chapter 9. Accordingly, consolidated drained strength of normally consolidated soils is expressed as

$$\tau_f = \sigma' \tan\phi' \tag{11.5}$$

This is the same equation as in Equation (11.2) (Terzaghi's effective stress equation) with $c' = 0$. During the drained test, pore water pressure is zero, and thus the applied total stress is also the effective stress. **Note that zero cohesion component c' in this case does not necessarily mean that soil is resisted purely by friction. In fact, shear resistance of clays is mostly contributed from cohesive resistance, but its expression merely implies that, when consolidation pressure is zero, there will be no strength.** This discussion suggests that failure criteria in Equations (11.1) and (11.2) should be considered as just expressions to determine failure shear strength τ_f , and that **the strength parameters (c , ϕ , and c' , ϕ') are not the cohesion and friction of the materials in the true sense, but rather the cohesive and frictional components in those expressions.**

A soil specimen brought from the field has been subjected to at least in-situ effective overburden stress or even higher effective stress during its historical time, as discussed for normal or overconsolidated soils in Chapter 9. Thus, when the soil is sheared in a consolidated drained test, it has some amount of shear strength due to its preconsolidation stress, even for small laboratory consolidation stress. When the consolidation stress in the test is less than the preconsolidation stress, the specimen is overconsolidated. Figure 11.18 plots Mohr's circles at failure for overconsolidated specimens. The failure envelope has the form of Equation (11.2) with cohesion component c' and frictional component ϕ' .

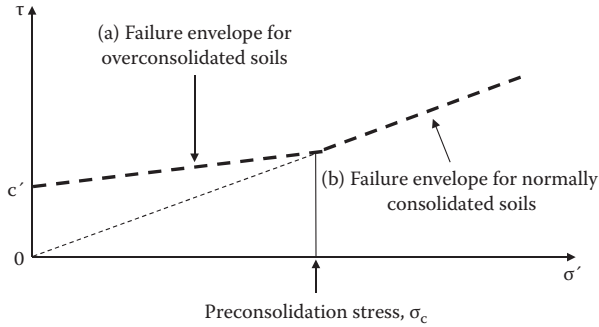


FIGURE 11.19 Failure envelope from CD test for full range of consolidation stresses.

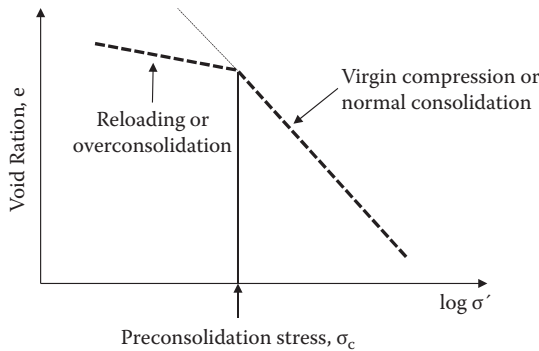


FIGURE 11.20 e - $\log \sigma'$ curve from consolidation test.

When the consolidation stress exceeds its preconsolidation stress, then the shear strength will follow the failure envelope observed in Figure 11.17 or Equation (11.5). Accordingly, the entire failure envelope from the consolidated drained test consists of two straight lines as shown in Figure 11.19: curve (a) or Equation (11.2) for the consolidation stress up to its preconsolidation stress, and curve (b) or Equation (11.5) for the consolidation stress above the preconsolidation stress. A bilinear failure envelope is analogous to the bilinear e - $\log \sigma$ curve of the consolidation test as seen in Figure 11.20. Below and above the preconsolidation stress, the rates of soils' volume change and shear strength are quite different.

11.5.4 CONSOLIDATED UNDRAINED (CU) TRIAXIAL TEST WITH PORE WATER PRESSURE MEASUREMENT

This is the most widely used triaxial shear test in practice. The specimen is first fully consolidated in the triaxial cell. Then the drainage valve is closed and sheared by increasing the deviatoric stress $\sigma_1 - \sigma_3$ to failure while σ_3 is kept constant. At the same time, pore water pressure u is measured during shearing process. Measurements during the test are $\sigma_1 - \sigma_3$, and u with the vertical strain ϵ_1 .

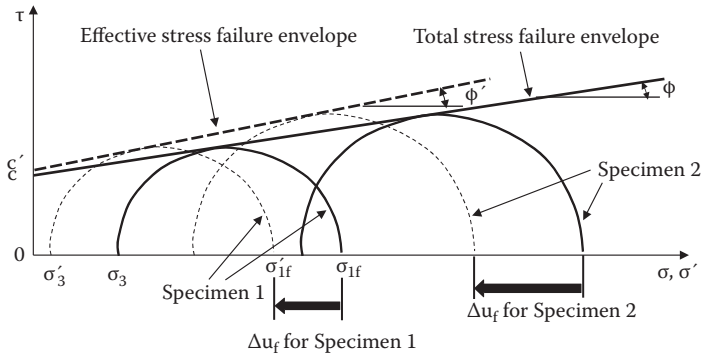


FIGURE 11.21 Total stress and effective stress analyses from CU test.

By obtaining σ_3 , σ_{1f} , and u_f at the failure for a given specimen, the total principal stresses (σ_3 , σ_{1f}), as well as the effective principal stresses ($\sigma'_3 = \sigma_3 - u_f$, $\sigma'_{1f} = \sigma_{1f} - u_f$), are calculated and Mohr's circles at failure are drawn in the total stress and in the effective stress. These circles are shown in Figure 11.21, where solid lines are in the total stress and dotted lines are in the effective stress. The diameters of Mohr's circles are the same for both total stress and effective stress, but the latter circle is shifted toward the left with an amount of u_f for a positive pore water pressure at failure.

Similarly, another set of failure Mohr's circles are drawn for differently consolidated specimen 2. The failure envelopes, which are tangent to those circles, are then drawn to determine the total stress strength parameters c and ϕ as well as the effective stress parameters c' and ϕ' as seen. To obtain these strength parameters, at least two CU tests with different consolidation stresses are needed, as in Figure 11.21. In practice, however, three or more CU tests with different consolidation stresses for similar specimens are performed to determine reliable c , ϕ , c' , and ϕ' values.

Exercise 11.2

Consolidated undrained triaxial tests for two similar specimens with different consolidation stresses were performed and the data in Figure 11.22 obtained, which includes pore water pressure measurements. Plot Mohr's circles at failure for two specimens in both the total stress and in the effective stress and determine the shear strength parameters c and ϕ from the total stress failure envelope and c' and ϕ' from the effective stress failure envelope.

SOLUTION

From the data plot, for specimen 1:

$$\sigma_3 = 150 \text{ kPa}$$

$$(\sigma_1 - \sigma_3)_f = 197 \text{ kPa}$$

$$\Delta u_f = +78 \text{ kPa}$$

and thus,

$$\sigma_{1f} = (\sigma_1 - \sigma_3)_f + \sigma_3 = 197 + 150 = 347 \text{ kPa}$$

Effective stresses are

$$\sigma'_3 = \sigma_3 - \Delta u_f = 150 - 78 = 72 \text{ kPa}$$

$$\sigma'_{1f} = \sigma_{1f} - \Delta u_f = 347 - 78 = 269 \text{ kPa}$$

From the data plot, for specimen 2:

$$\sigma_3 = 300 \text{ kPa}$$

$$(\sigma_1 - \sigma_3)_f = 295 \text{ kPa}$$

$$\Delta u_f = 121 \text{ kPa}$$

and thus,

$$\sigma_{1f} = (\sigma_1 - \sigma_3)_f + \sigma_3 = 295 + 300 = 595 \text{ kPa}$$

Effective stresses are

$$\sigma'_3 = \sigma_3 - \Delta u_f = 300 - 121 = 179 \text{ kPa}$$

$$\sigma'_{1f} = \sigma_{1f} - \Delta u_f = 595 - 121 = 474 \text{ kPa}$$

From these values, Mohr's circles at the failure are drawn in Figure 11.23. The total stress failure envelope is drawn to touch the total stress Mohr's circles (solid lines) and the effective stress failure envelope is drawn to touch the failure circles in effective stresses (dotted lines). Accordingly, $c = 42 \text{ kPa}$, $\phi = 14^\circ$, for the total stress, and $c' = 53 \text{ kPa}$, $\phi' = 18^\circ$, for the effective stress, are read from the plot.

From the CU test with pore water pressure measurement, similar bilinear failure envelopes are obtained independently for the total stress (solid line) and the effective stress (dotted line) as seen in Figure 11.24. It should be noted in Figure 11.24 that the breaking points of the two bilinear failure envelopes that separate the normally consolidated and the overconsolidated specimens could not be at the same normal stress since the Mohr failure circle for the effective stress moves toward the left from the Mohr failure circle for the total stress with the same diameter when positive pore water pressure is generated. Also, it can be seen that for a small consolidation stress region, where the overconsolidation ratio (OCR) is high, there could be

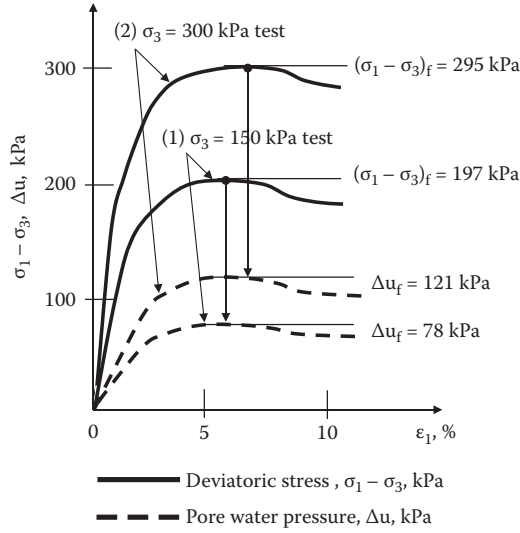


FIGURE 11.22 Exercise 11.2 problem (results from CU tests).

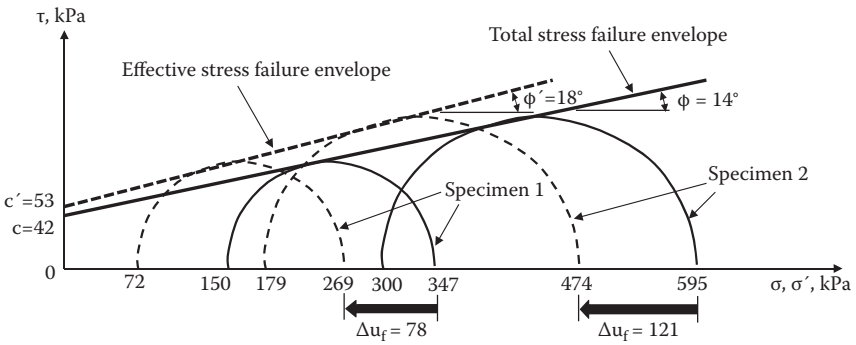


FIGURE 11.23 Exercise 11.2 (determination of c, ϕ and c', ϕ').

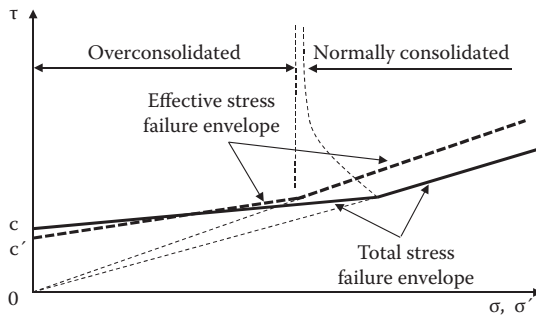


FIGURE 11.24 Failure envelopes from CU test for full range of consolidation stresses.

negative pore water pressure buildup at failure, and thus the Mohr failure circle for the effective stress could be drawn to the right side of the Mohr failure circle for the total stress. This could result in crossover of the failure envelopes at a low consolidation stress, as seen.

11.5.5 EFFECTIVE STRESS PARAMETERS FROM CU AND CD TESTS

The CU test result provides c' and ϕ' values, in addition to c and ϕ values, when analyzed in the effective stress using measured pore water pressure. These c' and ϕ' parameters have the same meanings of c' and ϕ' obtained from the CD shear test, since CD test data are always analyzed in the effective stress (= total stress) with zero pore water pressure. In fact, **Rendulic (1936)** showed experimentally that *the effective stress failure envelope is unique regardless of testing methods (drained or undrained)*. This fact enables engineers to save testing time by running a rather quick CU test and obtaining drained parameters c' and ϕ' by doing the effective stress analysis, instead of running a slow CD test. In other words, *the effective stress analysis of a CU test result can be substituted for a CD test*.

Another important conclusion on the shear strength is that *the unique effective failure envelope is the one to govern the failure mechanism of soils*. For example, when the undrained total stress strength is known, the amount of pore water pressure generation can be estimated with the knowledge of the uniqueness of the effective failure envelope, as demonstrated in Exercise 11.3.

Exercise 11.3

For a normally consolidated specimen, it was found that $\phi = 16^\circ$ and $\phi' = 28^\circ$. If a similar specimen is sheared with $\sigma_3 = 120$ kPa in a CU test with pore water pressure measurement, estimate the deviatoric stress $(\sigma_1 - \sigma_3)_f$ and pore water pressure Δu_f at failure.

SOLUTION

- Since the specimen is normally consolidated, draw total and effective stress failure envelopes from the origin ($c = c' = 0$) with $\phi = 16^\circ$ and $\phi' = 28^\circ$, respectively, as seen in Figure 11.25.
- Draw the total stress failure Mohr's circle with $\sigma_3 = 120$ kPa, which touches the total stress failure envelope, and read $\sigma_{1f} = 205$ kPa on the graph. Thus, the deviatoric stress at failure $(\sigma_1 - \sigma_3)_f = 205 - 120 = \mathbf{85 \text{ kPa}}$ (diameter of the Mohr's circle). ←
- With the same diameter as with the preceding circle, draw the effective stress failure Mohr's circle, which contacts at the effective stress failure envelope and reads $\sigma'_3 = 48$ kPa and $\sigma'_{1f} = 133$ kPa on the graph.
- Pore water pressure generation at the failure is the amount of horizontal shift on two Mohr's circles and thus $\Delta u_f = 205 - 133 = \mathbf{72 \text{ kPa}}$ as seen in the graph. ←

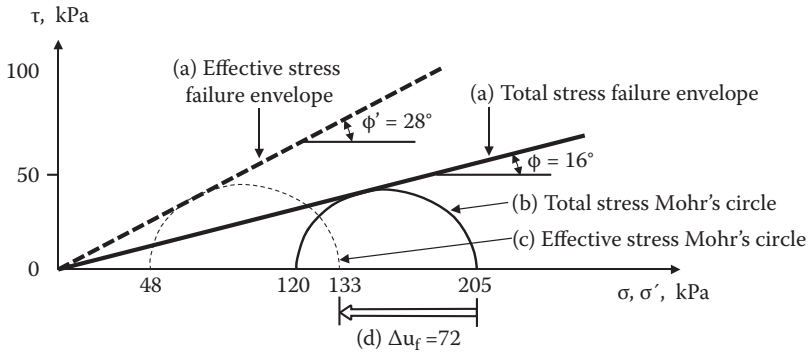


FIGURE 11.25 Exercise 11.3 solution.

In Exercise 11.3, since the effective stress failure envelope is unique, the amount of positive pore water pressure generation (shift of failure Mohr's circle to left) is governed by the effective stress failure envelope.

11.5.6 UNCONSOLIDATED UNDRAINED (UU) TEST

The simplest triaxial compression test would be a **UU test**. During sample preparation time, the specimen is not allowed any consolidation process. Shortly after the specimen is placed in the chamber, the confining stress is applied through the specimen membrane and sheared under undrained conditions; that is, the drainage valve is kept closed, and it is sheared in a short time. During the sample preparation and shearing processes, there will be no escape of the pore water from the specimen; thus, no change in the water content of the specimen will take place. Therefore, the anticipated soil strengths are the same for any confining stress. Figure 11.26 shows Mohr's circles at failure for similar fully saturated specimens under different confining stresses from the UU test. The diameters of Mohr's circles at failure are the same and the drawn failure envelope is horizontal, which implies $\phi = 0$ (**$\phi = 0$ concept**).

When the $\phi = 0$ concept is adapted, there will be no need to run several UU tests to determine c value. In Figure 11.26, Mohr's circle at failure from an unconfined compression test ($\sigma_3 = 0$) is seen as a special case of a UU test. If an unconfined compression test is run properly, then the cohesion component c is determined by drawing a horizontal line to make a tangent with Mohr's circle at failure, as seen in Figure 11.11. In this case, UU tests with several confining stresses will no longer be needed. Note that the $\phi = 0$ concept is only applicable to soils that have been fully saturated. If the specimen's void contains air, the air would be easily compressed by increased confining stress and thus reducing the specimen's whole volume, which makes the sample stronger. Thus, the failure envelope forms a convex shape. With higher confining stresses, all air voids would be compressed, and it reaches a flat failure envelope to obey the $\phi = 0$ concept.

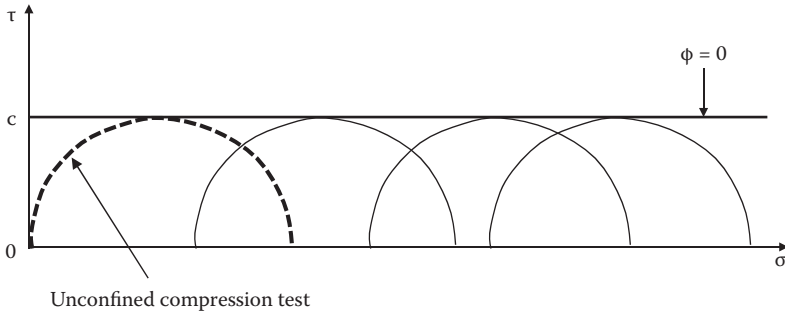


FIGURE 11.26 UU test results and $\phi = 0$ concept.

11.6 OTHER SHEAR TEST DEVICES

There are many other laboratory as well as in-situ shear devices to determine soil’s strength parameters. They could include true triaxial devices, plane strain triaxial devices, torsional shear devices, simple shear devices, ring shear devices, etc. They are mostly used as research tools and not routinely used by practicing engineers. Readers can refer to related literature on them. In this section, popularly used small shear devices (vane shear device, tor-vane shear device, and pocket penetrometer) are introduced.

11.6.1 VANE SHEAR DEVICE

For the **vane shear device**, a rigid, cross-shaped vane such as seen in Figure 11.27 is often used in the field as well as in the laboratory. A vane is inserted into the soil. In the field, it is generally installed to the tip of the boring rod. The shaft is twisted, and applied torque T is measured at failure. Shear resistance comes from the perimeter area and the top and the bottom surfaces of the vane. Upon the application of the torque T , full resistance C_u will be developed along the perimeter, where C_u is the same as the undrained shear strength.

On the top and the bottom shear surfaces of the vane, shear resistance could be the maximum value of C_u at $r = D/2$ and is zero at the center of the torque since there is no rotation there. If the distribution of the shear resistance on the top and the bottom shear surfaces is assumed as a linearly increasing function with the radius r , $\tau = C_u \cdot r/(D/2)$, as seen in Figure 11.27. The measured maximum torque T_f is related to the maximum shear resistance C_u by integrating local torque $\tau \times r$ over the top and the bottom shear surfaces and the perimeter area. Accordingly, T_f and C_u are related by

$$T_f = \pi C_u \left[\frac{D^3}{8} + \frac{D^2}{2} H \right] \tag{11.6}$$

From Equation (11.6), the undrained shear strength C_u can be obtained. A field vane shear test is generally considered to be very useful and a reliable tool, since the stress

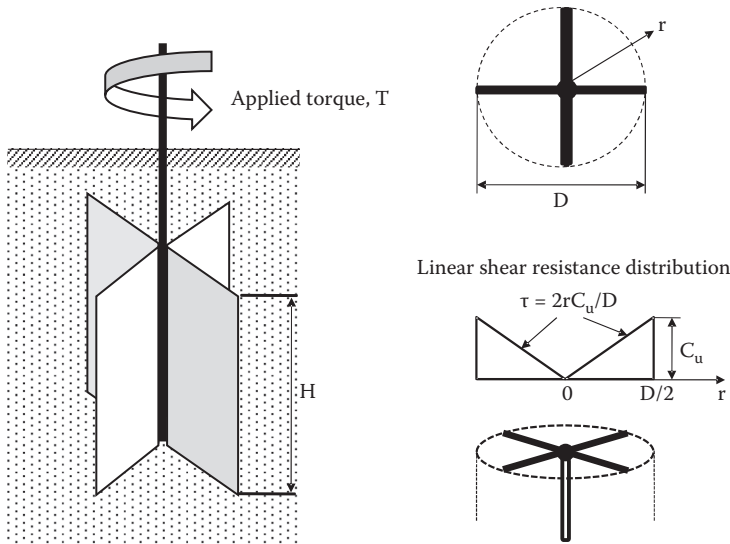


FIGURE 11.27 Vane shear test device.

conditions are in-situ and samples are less disturbed in comparison with laboratory specimens. Also, it is rather economical to perform. Note that the vane shear test is conducted within a short period of time, and thus it can be categorized as an unconsolidated undrained test.

11.6.2 TOR-VANE SHEAR TEST

Figure 11.28 shows a schematic drawing of a hand-twisting **tor-vane shear device**. Similarly to the vane shear test, rigid fins are inserted into the surface of soil and twisted until failure. The measured torque is converted into the shear resistance near the surface of the soil specimen. This is mostly used in the laboratory to provide supplementary shear strength information or for quality control of sampled specimens. The nature of shear strength obtained from this test is similar to that from the UU test.

11.6.3 POCKET PENETROMETER

The **pocket penetrometer** is a punching probe as seen in Figure 11.29. The device is pushed by hand into the soil until failure. The maximum deformation of the spring is registered to identify the punching force. The failure punching force is an indication of the soil's **bearing capacity** (Chapter 14) and is calibrated to UU shear strength. The device is mostly used in the laboratory to provide supplementary shear strength information or to check the quality of specimens.

Note that the vane shear, the tor-vane shear, and the pocket penetrometer test are all equivalent to the UU test, since no additional consolidation takes place

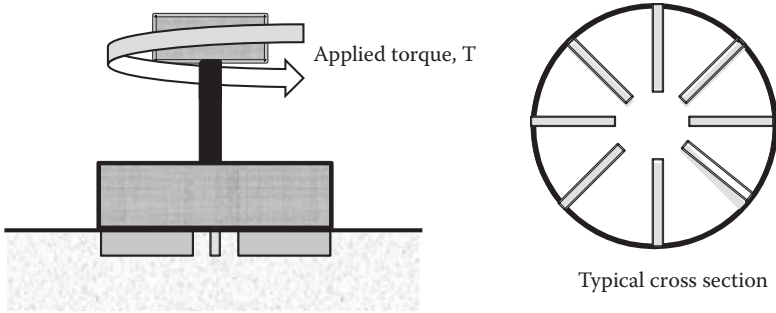


FIGURE 11.28 Tor-vane shear test device.

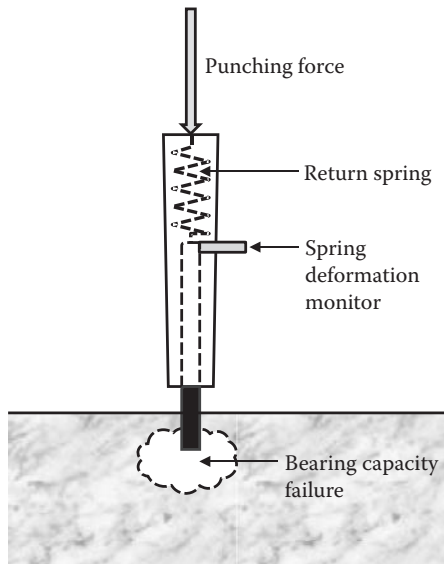


FIGURE 11.29 Pocket penetrometer.

and shearing time is very quick. Thus, results obtained from these tests are compared with the shear strength C_u ($= q_u/2$) obtained from an unconfined compression test or a UU test.

11.7 SUMMARY OF STRENGTH PARAMETERS FOR SATURATED CLAYS

As discussed in the previous sections, shear strength parameters c and ϕ , and c' and ϕ' , depend on types of shear test (UU, CD, or CU test) and consolidation history (normally consolidated or overconsolidated). They are summarized next.

11.7.1 UU TEST

The $\phi = 0$ concept is applicable (for fully saturated soils) and thus ϕ is always zero. UU strength C_u is conveniently expressed as C_u/σ'_{vo} , where σ'_{vo} is the effective overburden stress. A couple of empirical correlations are available:

Skempton and Henkel (1953) have developed the following for normally consolidated clays:

$$C_u/\sigma'_{vo} = 0.11 + 0.0037 \text{ PI} \tag{11.7}$$

Bjerrum and Simons (1960) have developed the following for normally consolidated clays:

$$C_u/\sigma'_{vo} = 0.045(\text{PI})^{0.5} \text{ for PI} > 50\% \tag{11.8}$$

$$C_u/\sigma'_{vo} = 0.018(\text{LI})^{0.5} \text{ for LI} > 50\% \tag{11.9}$$

In these equations, the plasticity index (PI) and the liquidity index (LI) are expressed in percentages. Note that the C_u/σ'_{vo} values obtained from these equations should be considered as approximate ones.

11.7.2 CD TEST AND CU TEST (EFFECTIVE STRESS)

For normally consolidated clays, $c' = 0$ and ϕ' is in a range of 20° – 42° (*Bowles 1996*). For overconsolidated clays, there are various combinations of non-zero c' and ϕ' values.

11.7.3 CU TEST (TOTAL STRESS)

For normally consolidated clays, $c = 0$ and ϕ is non-zero values. For overconsolidated clays, there are various combinations of non-zero c and ϕ values. Table 11.1 summarizes shear strength parameters from different types of shear tests.

TABLE 11.1
Shear Strength Parameters from Different Shear Tests

Shear Strength Parameters	Types of Shear Tests
c and ϕ	CU test (total stress analysis)
c' and ϕ'	CD test, CU test (effective stress analysis)
C_u ($= q_u/2$) ($\phi = 0$ concept)	Unconfined compression, UU, vane shear, tor-vane shear, pocket penetrometer

11.8 APPLICATIONS OF STRENGTH PARAMETERS FROM CD, CU, AND UU TESTS TO IN-SITU CASES

Now a big question arises: *how to utilize those different strength parameters in various field problems*. Simple but not always easy answers to this question are

1. Observe field phenomenon carefully in terms of preshearing conditions (consolidated or unconsolidated) and shearing mode of anticipated failure (quick or slow failure).
2. Based on these observations, use appropriate soil strength parameters that match the anticipated field phenomena.

The following typical cases are discussed as practical problems that are usually encountered in the field.

11.8.1 CONSTRUCTION OF EMBANKMENT ON SOFT CLAY SOIL AT ONCE (UU CASE)

Figure 11.30 shows a case in which an embankment is constructed on soft foundation clay in a rather short period of time—for example, in few days, a week, or so on. In this case, there will not be enough time for the foundation clay to be consolidated. If the soil fails, the failure will occur suddenly, and thus it will be the undrained condition. Shear strength C_u should be the one from unconfined compression strength or its equivalent for stability analysis along the potential failure surface.

11.8.2 FOUNDATION DESIGN FOR RAPIDLY CONSTRUCTED SUPERSTRUCTURES

If superstructures are constructed in a rather short period of time, foundations should be designed based on UU soil parameters, since there will be very little consolidation time and failure mode will be rather sudden if it fails as shown in Figure 11.31.

11.8.3 STAGED CONSTRUCTION OF EMBANKMENT ON SOFT CLAY (CU CASE)

When an embankment cannot be constructed at once due to rather low shear strength of the foundation clay, **staged construction** will be the choice. As seen in Figure 11.32, the first-stage embankment is placed on the ground. Since the load

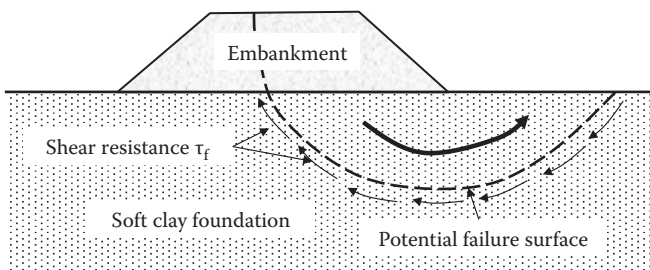


FIGURE 11.30 Quick construction of embankment on soft clay.

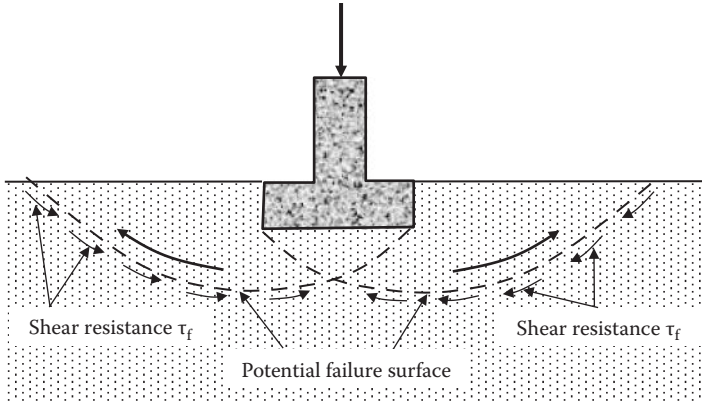


FIGURE 11.31 Construction of a footing in a short period of time.

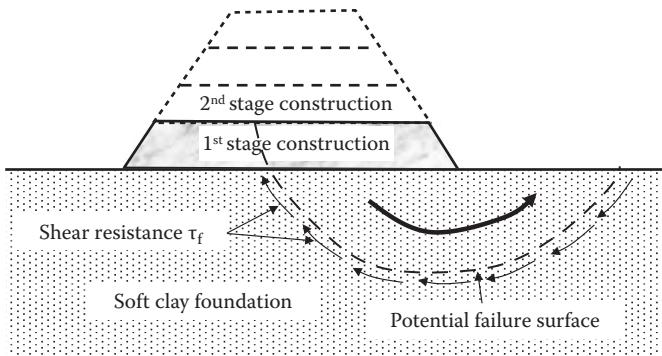


FIGURE 11.32 Staged construction of embankment on soft clay.

increment is small, the original foundation clay could have enough strength to carry its weight. At this stage, stability analysis should be carried out by using shear strength from a UU test. Then the first-stage load is left there for a while (e.g., for a month or more). During this period, the consolidation process begins and the foundation clay will gain some strength. When the gained strength is enough to carry the second-stage load, a new load is applied and it is left for another period for further consolidation. The process is repeated until the final height of the embankment is attained. In this case, total stress soil parameters by CU test are the strength parameters for each stage of construction. Strength gain occurs during the consolidation process, but anticipated failure will be sudden if it fails (undrained).

11.8.4 STABILITY OF CUT SLOPE (CD CASE)

Figure 11.33 shows a vertical cut made in a clayey soil. Due to the cohesion component, it is possible to do a vertical cut safely to a certain depth (Chapter 12). Assume that the **cut slope** is stable at the time of the excavation. Near the cut section, soil

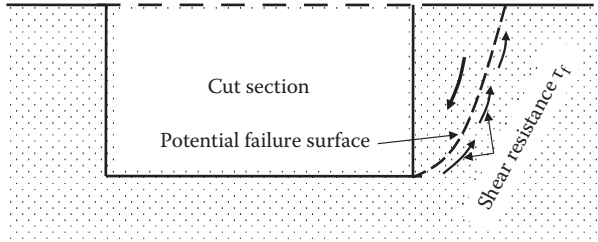


FIGURE 11.33 Cut-slope and potential slope failure.

is subjected to stress decrease, and this is a reversed phenomenon of consolidation. Stress relaxation will promote gradual swelling of the soil and it starts to attract pore water to the zone. When the water content increases, the soil's strength decreases. An initially stable slope will gradually increase the danger of sliding with time due to the preceding phenomenon. The failure mode may be progressive and slow due to gradual increase in the water content of soils along the potential failure surface. Thus, CD strength parameters will be the proper ones to be used in such cases. However, it should be noted that if the anticipated failure mode is sudden in the preceding case, CU parameters should be used.

As observed in the previous cases, the selection of CD, CU, and UU parameters depends on the preshearing condition (consolidated or unconsolidated) and the shearing process (slow failure or sudden failure). In particular, the evaluation of failure mode is very significant. Most failure modes would be sudden if the soil does not have enough shear strength, and slow failure would occur in very limited cases such as possible progressive failure in the cut-and-creep type of failure mode. It should be noted that, though UU or equivalent tests are rather simple ones, there are many applications of the UU case as seen in the previous examples.

11.9 STRENGTH PARAMETERS FOR GRANULAR SOILS

Granular soils (sand and gravel, and maybe some silt) have very little interparticle interactive forces due to rather larger particle sizes (Chapter 3). Therefore, most of their shear resistance comes from a frictional component. Thus, c can be assumed to be zero for granular soils. In addition, these soils have a rather high permeability, so the pore water pressure will normally dissipate very quickly in most applications. Accordingly, for both wet and dry granular soils, Equation (11.10) is used without the “ c ” component:

$$\tau_f = \sigma \tan \phi \quad (11.10)$$

The failure envelope starts at the origin of the σ - τ diagram with ϕ angle inclination, and the angle of internal friction ϕ is the sole parameter to determine the shear strength. The ϕ values are influenced by soil's various properties, such as density or void ratio, gradation (uniform or well graded), angularity (rounded or angular), grain

surface roughness, etc. Among these, density (or void ratio) would most significantly influence the ϕ values of granular materials. The typical values of ϕ are given in Table 11.2.

A straight line failure envelope or a constant ϕ angle for a given soil with a given density is generally assumed for granular soils as discussed. However, in reality, it was empirically observed that the failure envelope for granular soils is slightly curved, as seen in Figure 11.34. This implies that a slightly higher ϕ angle is at a lower confining stress and a lower ϕ angle is at a higher stress. The importance of this fact is addressed by a non-conservative application of small-scale model test data in the evaluation of in-situ earth structure behavior. In a small-scale model test, the stress level, which mostly comes from the gravitational force of soils, is small, and thus the failure phenomenon is controlled by a rather high ϕ value, while in an in-situ earth structure, the stress level is high, which provides a lower ϕ value.

The geotechnical **centrifuge model test** is meant to overcome the previously mentioned shortcomings of the small-scale model test. For example, if a 0.5 m high model earth dam is subjected to 20 g of the centrifugal gravity on a rotating

TABLE 11.2
Typical Ranges of Angle of Internal Friction ϕ for Sandy Soils

Type of Soil	Density	Peak ϕ Value	Residual ϕ Value
Sand, rounded	Loose	28° to 30°	
	Medium	30° to 35°	26° to 30°
	Dense	35° to 38°	
Sand, angular	Loose	30° to 35°	
	Medium	35° to 40°	30° to 35°
	Dense	40° to 45°	
Sandy gravel		34° to 48°	33° to 36°

Source: After *Murthy, V. N. S., 2003, Geotechnical Engineering*, Marcel Dekker, New York.

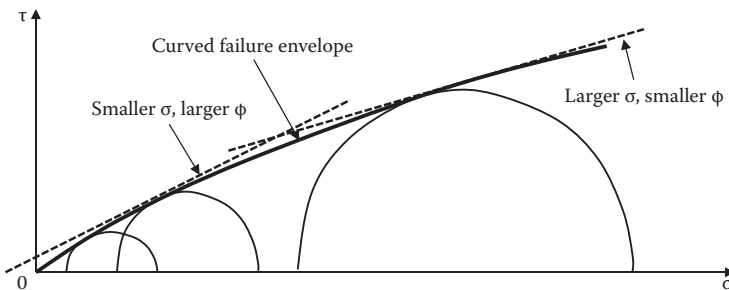


FIGURE 11.34 Curved failure envelope for granular soils.

platform, its stress level in the model increases to the level of a 10 m (0.5 m × 20) high earth dam, and thus similar ϕ values would be utilized in both the 0.5 m high model under 20 g centrifugal force and the 10 m high prototype under 1 g conditions. Readers are referred to other references (e.g., *Taylor 1995*) for details of geotechnical centrifuge testing.

11.10 DIRECTION OF FAILURE PLANES ON SHEARED SPECIMEN

Mohr's circle and the concept of the **pole** can be effectively utilized in order to evaluate the direction of anticipated (or observed) failure planes on a sheared specimen. A specimen is sheared under triaxial conditions (σ_1 on a horizontal plane and σ_3 on a vertical plane), as seen in Figure 11.35(a), and the effective stresses σ'_1 and σ'_3 are calculated based on measured pore water pressure Δu . First, assume that the soil strength is defined by c' and ϕ' in general; a Mohr's circle at failure is drawn in Figure 11.35(b). To find the pole, by referring to Section 10.7, draw a line from a known stress point σ'_1 on Mohr's circle parallel to the plane (horizontal) on which these stresses act. Find the intersection on the circle as the pole. The pole is at the same point as σ'_3 in the figure. Alternatively, draw a line from a known stress point σ'_3 on Mohr's circle parallel to the plane (vertical) on which these stresses act. Find the intersection on the circle as the pole. The pole is also at the σ'_3 point.

Next, to know the direction of a stress point on Mohr's circle, connect a line from the pole to that stress point on the circle. This is the direction of the plane on which these stresses (σ', τ) act. In this case, points F+ and F- are the stress points for the failure planes, and thus PF+ and PF- are the directions of potential failure planes

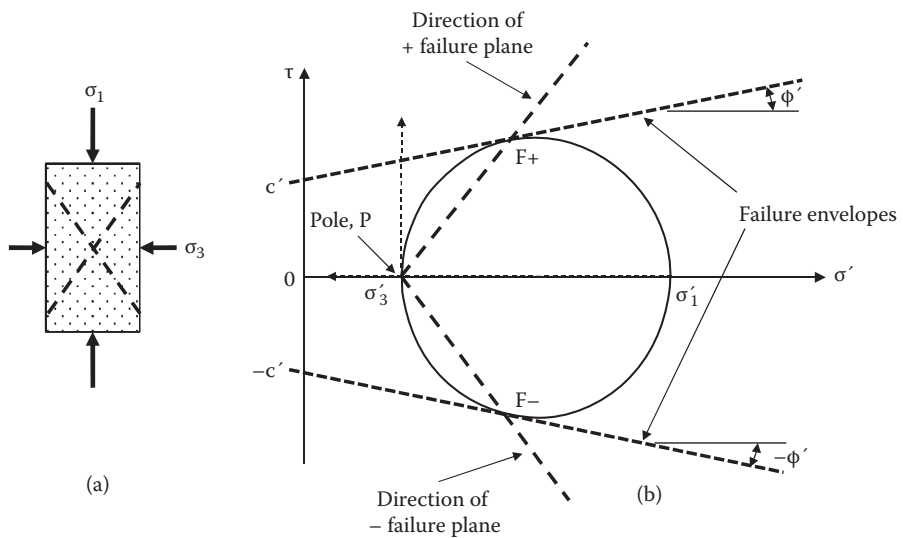


FIGURE 11.35 Directions of failure planes in triaxial specimen.

on the specimen as seen in Figure 11.35(b). Many failure lines parallel to PF+ and PF- are possible, but a few failure lines could be observed in real soil specimens.

As seen in a similar figure in Figure 11.36, the angle of the failure plane direction ψ is analytically related to ϕ' values as follows:

For a triangle $O'F\sigma'_3$, since $O'F = O'\sigma'_3$, $\angle O'F\sigma'_3 = \angle O'F\sigma'_3 = \psi$

Based on the triangle $O''O'F$, $\angle FO'\sigma'_1 = 90^\circ + \phi' = 2\psi$, and thus the direction of the failure plane from the horizontal $\psi = 45^\circ + \phi'/2$ (11.11)

Note that the preceding discussion of the failure plane direction should be applied only to the effective stress failure envelope, since the failure of soils is determined when Mohr's circle of failure in the effective stress just touches the unique effective stress failure envelope as discussed earlier in this chapter. For example, if this is applied to tests with a $\phi = 0$ failure envelope (UU test or unconfined compression test), the failure plane direction should be 45° from the horizontal, as seen in Figure 11.37. As a matter of fact, this is not true. Soil will

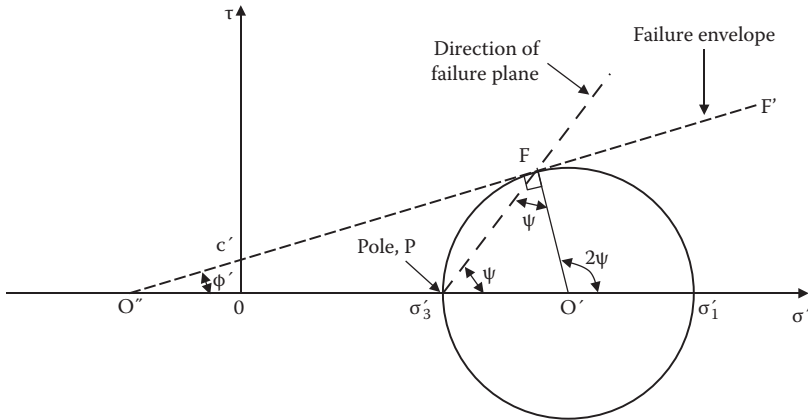


FIGURE 11.36 Analytical solution of failure plane direction.

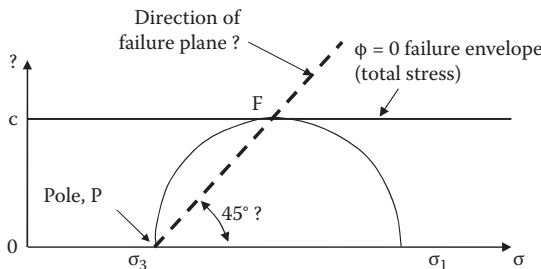


FIGURE 11.37 Questionable failure plane direction based on total stress Mohr's circle.

fail along a plane with $45^\circ + \phi'/2$ from the horizontal, instead. A mistake was made here because the $\phi = 0$ failure envelope is not in the effective stress mode; rather, it is in the total stress mode.

Exercise 11.4

A drained triaxial compression test for a normally consolidated clay specimen was conducted with $\sigma'_3 = 100$ kPa. After the test, the failure planes on the specimen were observed. The failure plane angle was measured as 55° inclined from the horizontal. Determine (a) the effective angle of internal friction ϕ' , and (b) σ'_{1f} value at failure.

SOLUTION

- (a) $\psi = 55^\circ$ and thus $\phi' = 20^\circ$ from Equation (11.11). ←
- (b) In Figure 11.38, the failure envelope is drawn with 20° from the origin (normally consolidated). A failure Mohr's circle with $\sigma'_3 = 100$ kPa is searched by trial and error, which just touches the failure envelope. Read σ'_{1f} value on the σ' axis as **204 kPa**. ←
Or, analytically, applying sine law to the triangle OO'F:

$$\begin{aligned} \sin\phi' &= O'F/OO' = [(\sigma'_{1f} - \sigma'_3)/2] / [(\sigma'_{1f} + \sigma'_3)/2] \\ &= (\sigma'_{1f} - 100) / (\sigma'_{1f} + 100) = \sin 20^\circ \end{aligned}$$

then,

$$\sigma'_{1f} = \mathbf{204.0 \text{ kPa}}$$
 is obtained. ←

In the figure, failure plane direction σ'_{1f} is also graphically seen.

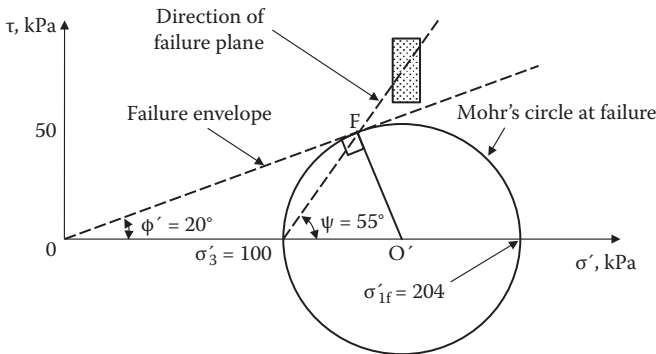


FIGURE 11.38 Exercise 11.4 solution.

11.11 SUMMARY

Shear strength determination is another important practice in many soil mechanics problems (foundation design, slope stability, retaining wall design, etc.). There are several shear testing procedures available and different shear strength parameters are obtained from these tests. Details of these were presented in this chapter. It is most important for engineers to understand which ones are the proper shear strength parameters for a given problem. They all depend on the anticipated field conditions in terms of preshearing condition (i.e., consolidated or unconsolidated) and shear failing mode (drained or undrained). They were discussed and summarized in detail in Sections 11.7 and 11.8. In Section 11.10, the concept of the pole of Mohr's circle (Chapter 10) was effectively utilized in the determination of the directions of the failure planes during shear.

REFERENCES

- Bjerrum, L., and Simons, N. E. (1960), Comparison of shear strength characteristics of normally consolidated clays, *Proceedings of Research Conference on Shear Strength of Cohesive Soils*, ASCS, 711–726.
- Bowles, J. E. (1996), *Foundation Analysis and Design*, 5th ed., McGraw–Hill, New York.
- Murthy, V. N. S. (2003), *Geotechnical Engineering*, Marcel Dekker, New York.
- Rendulic, L. (1936), Relation between void ratio and effective principal stresses for a remolded silty clay, *Proceedings of the 1st International Conference on Soil Mechanics and Foundation Engineering*, vol. III, 48–53.
- Skempton, A. W., and Henkel, D. J. (1953), The post-glacial clays of the Thames estuary at Tillbury and Shellhaven, *Proceedings of the 3rd International Conference on Soil Mechanics and Foundation Engineering*, Zurich, vol. I, 302–308.
- Taylor, R. N. (ed.) (1995), *Geotechnical Centrifuge Technology*, Blackie Academic and Professional, Glasgow.
- Terzaghi, K. (1925), *Erdbaumechanik*, Franz Deuticke.

Problems

- 11.1 Why does high water pressure not crush a soil element at a deep ocean bottom?
- 11.2 Why does the soil's high overburden stress not crush a soil element under a deep soil deposit?
- 11.3 Four direct shear tests were performed in a 10 cm × 10 cm square shear box for soil specimens with a similar density and the following data were obtained. Determine the angle of internal friction δ and the cohesion component, c , of the soil.

Applied Vertical Force F_v (Newton)	Measured Peak Shear Force F_h (Newton)
200	272
400	324
1000	487
1500	632

- 11.4 For a dry sandy soil, a direct shear test was performed. Its vertical normal force was 10 kgf and the measured shear force was 6.34 kgf. The specimen was prepared in a circular shear box of 10 cm in diameter.
- Determine the angle of internal friction ϕ of the soil.
 - When the normal stress of 150 kPa is applied, what will be the failure shear stress of this soil?
- 11.5 For normally consolidated clay specimens, drained direct shear tests were conducted and the following data were obtained. Determine the drained angle of internal friction ϕ' of the clay.

Applied Normal Stress, kPa	Measured Peak Shear Stress, kPa
150	22.4
300	44.6
400	59.8
500	71.6

- 11.6 For clay specimens, drained direct shear tests were conducted and the following data were obtained. Determine the drained angle of internal friction ϕ' and the cohesion component c' of the clay.

Applied Normal Stress, kPa	Measured Peak Shear Stress, kPa
100	66.2
200	87.2
300	105.1
400	116.4

- 11.7 An unconfined compression test was performed on a clayey specimen of 7.0 cm in diameter and 15.0 cm in height and the following data were obtained. Plot the stress and strain curve and determine the unconfined compression strength q_u and the cohesion C_u of the soil.

Vertical Deformation δ_v (mm)	Measured Axial Force F_v (kgf)
0	0
0.5	2.8
1.0	5.5
1.5	8.4
2.0	10.9
2.5	13.6
3.0	16.2
3.5	18.6
4.0	21.4
4.5	24.1
5.0	26.8
5.5	29.4
6.0	30.1

Continued

Vertical Deformation δ_v (mm)	Measured Axial Force F_v (kgf)
6.5	30.1
7.0	29.8
7.5	28.9
8.0	28.9

11.8 Consolidated drained triaxial tests were conducted for three similar specimens with different chamber pressures and the failure deviatoric stresses were measured as follows:

Test	Chamber Pressure σ_3 (kPa)	Measured Deviatoric Stress at Failure $(\sigma_1 - \sigma_3)_f$ (kPa)
I	50	92
II	100	127
III	150	166

- (a) Determine the angle of internal friction ϕ' and the cohesion component c' of the soil.
 - (b) Is this soil normally consolidated or overconsolidated?
- 11.9 A consolidated drained triaxial test was conducted for a normally consolidated clay. Its consolidation pressure was 80 kPa and the deviatoric stress at failure was 135 kPa. Determine the effective angle of internal friction ϕ' of the soil.
- 11.10 The effective angle of internal friction ϕ' was found to be 26° for a normally consolidated soil. If the soil is tested in consolidated drained triaxial test under $\sigma_3 = 60$ kPa, what would be the failure deviatoric stress $\sigma_1 - \sigma_3$?
- 11.11 For a soil, the effective angle of internal friction ϕ' was found to be 14° and the cohesion component c' was 46 kPa. The soil was tested in consolidated drained triaxial condition when the failure stress $(\sigma_1 - \sigma_3)$ was found to be 132 kPa. What was the confining pressure σ_3 for this test?
- 11.12 For the triaxial test in Problem 11.11:
- (a) Estimate the direction of potential failure planes in the specimen relative to the major principal stress plane.
 - (b) What are the normal stress σ_f and the shear stress τ_f on the failure plane in (a)?
- 11.13 Two similar specimens from the same site were tested under consolidated undrained triaxial conditions with pore water pressure measurements. The results are summarized here:

Specimen	Chamber Pressure (kPa)	Measured Deviatoric Stress at Failure (kPa)	Pore Water Pressure at Failure (kPa)
I	50	181	23
II	100	218	19

- (a) Plot the Mohr's circles at failure in both the total stress and the effective stress.
- (b) Determine the strength parameters ϕ and c in the total stress and ϕ' and c' in the effective stress.
- 11.14 If the specimen in Problem 11.13 is tested under the chamber pressure $\sigma_3 = 85$ kPa:
- (a) What will be the failure axial stress σ_1 ?
- (b) What will be the pore water pressure at failure?
- 11.15 Two similar specimens from the same site were tested under consolidated undrained triaxial conditions with pore water pressure measurements. The results are summarized here:

Specimen	Chamber Pressure (kPa)	Measured Deviatoric Stress at Failure (kPa)	Pore Water Pressure at Failure (kPa)
I	25	83	7.5
II	50	109	15

- (a) Plot the Mohr's circle at failure in both the total stress and the effective stress.
- (b) Determine the strength parameters ϕ and c in the total stress and ϕ' and c' in the effective stress.
- 11.16 If the specimen in Problem 11.15 is tested under the chamber pressure $\sigma_3 = 60$ kPa:
- (a) What will be the failure axial stress σ_1 ?
- (b) What will be the pore water pressure at failure?
- 11.17 A cohesive soil was tested in a consolidated undrained triaxial test with pore water pressure measurement and $\phi = 24^\circ$ and $c = 26$ kPa in the total stress and $\phi' = 27^\circ$ and $c' = 30$ kPa are obtained. If the similar specimen is tested under $\sigma_3 = 45$ kPa:
- (a) What will be the failure deviatoric stress?
- (b) What will be the pore water pressure at failure?
- 11.18 A consolidated undrained test was conducted on a clay specimen. The consolidation and chamber pressure was 50 kPa and the failure σ_1 was 86.2 kPa. If a similar specimen is first consolidated under 50 kPa consolidation pressure and then tested in an unconfined compression device, what will be the unconfined compression strength q_u ?
- 11.19 A normally consolidated clay had $\phi' = 25^\circ$. The same specimen is tested in an unconfined compression device and obtains the unconfined compression strength $q_u = 85$ kPa. How much pore water pressure is generated in this unconfined compression specimen at the failure?

Chapter 9 Bearing Capacity and Shallow Foundations

14.1 INTRODUCTION

Bearing capacity is the maximum pressure that the soil can support at foundation level without failure. This is a key design parameter for foundation design and also for design of retaining walls at the base level. This chapter deals with the basic theory and practice of this subject. In the later section, design techniques of shallow foundations are presented by directly utilizing the bearing capacity theory.

14.2 TERZAGHI'S BEARING CAPACITY THEORY

Terzaghi (1943) developed a bearing capacity solution for a continuous shallow foundation with a footing width B and an embedded depth D_f under a level ground as seen in Figure 14.1. He adapted the punching shear theory on metals by *Prandtl (1920)* to soils, including soil's gravitational force. He assumed that (1) soil shear strength is given by $\tau_f = c + \sigma_n \tan\phi$, (2) footing depth D_f is replaced by a surcharge load ($q = \gamma D_f$), and (3) the footing base has a rough surface.

In the model, when the footing load increases, the footing base pushes the triangle zone I downward. Then zone I pushes zone II sideways, and zone II pushes zone III further. Zone I behaves as a rigid elastic body during the entire process. Zone III is the Rankine's passive earth pressure zone and zone II is a transitional zone. From the force equilibria on these zones, Terzaghi obtained the following equation to determine the **ultimate bearing capacity** q_u as

$$q_u = cN_c + \gamma_1 D_f N_q + \frac{1}{2} \gamma_2 B N_\gamma \quad (14.1)$$

where, N_c , N_q , and N_γ are **bearing capacity factors** and functions of the effective angle of internal friction ϕ' of the soil. γ_1 is the unit weight of soil above the base of the footing level and γ_2 is the unit weight of soil below the base level. The cN_c term is a contribution to the bearing capacity from the cohesion resistance along the failure surface, and it is zero for $c = 0$ materials (noncohesive soils). The term $\gamma_1 D_f N_q$ is a contribution from the surcharge load $\gamma_1 D_f$ at the footing base level, and it is zero for foundations placed on the ground surface. The $\frac{1}{2} \gamma_2 B N_\gamma$ term is from the frictional resistance along the failure surface, and it is zero for $\phi' = 0$ materials (cohesive soils).

It is interesting to note that in Equation (14.1) the unit of q_u is the stress unit such as kN/m^2 . The bearing capacity q_u in the stress unit increases when the foundation width B increases by the contribution of the term $\frac{1}{2} \gamma_2 B N_\gamma$. This implies that in cases

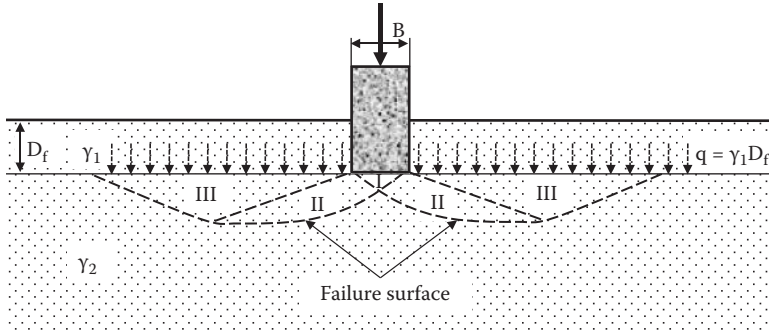


FIGURE 14.1 Terzaghi's bearing capacity model.

with $c = 0$ and $D_f = 0$, when the footing width B increases twice, the total footing load increases four times. This makes sense since when B increases, the shearing failure zone is enlarged and is extended to a deeper depth; thus, the shear resisting surface increases more than in proportion to the increased footing width B .

Although Terzaghi gave original N_c , N_q , and N_γ equations, they are not shown here to avoid confusion since several major modifications have been made since his original contribution. These values are, instead, given in the following section.

14.3 GENERALIZED BEARING CAPACITY EQUATION

Terzaghi's bearing capacity equation is valid for limited cases: that is, (1) shallow foundation, (2) two-dimensional strip footings, (3) no shearing resistance through the depth D_f zone, and (4) footing load applied in the vertical direction only. To extend the applicability of Equation (14.1) to more general situations, the following modified general bearing capacity is proposed by several researchers:

$$q_u = cN_c f_{cs} f_{cd} f_{ci} + \gamma_1 D_f N_q f_{qs} f_{qd} f_{qi} + \frac{1}{2} \gamma_2 B N_\gamma f_{\gamma s} f_{\gamma d} f_{\gamma i} \quad (14.2)$$

where

- f_{cs} , f_{qs} , $f_{\gamma s}$: **shape factors** for different footing shapes other than strip footings
- f_{cd} , f_{qd} , $f_{\gamma d}$: **depth factors** for deeper shallow foundations
- f_{ci} , f_{qi} , $f_{\gamma i}$: **inclination factors** for various directions of footing load than the vertical load

Although several researchers (*De Beer 1970; Hansen 1970; Vesic 1973; Hanna and Meyerhof 1981*, etc.) proposed modified bearing capacity factors, shape factors, depth factors, and inclination factors in Equation (14.2) based on experimental observations, the values of *Meyerhof (1963)* are presented here. Table 14.1 shows the bearing capacity factors N_c , N_q , and N_γ as functions of the effective angle of internal friction ϕ' , and Figure 14.2 plots these values.

Other modification factors by Meyerhof are summarized.

TABLE 14.1
Bearing Capacity Factors by Meyerhof

ϕ'	N_c	N_q	N_γ	ϕ'	N_c	N_q	N_γ
0	5.14	1.00	0.00	26	22.25	11.85	8.00
1	5.38	1.09	0.002	27	23.94	13.20	9.46
2	5.63	1.20	0.01	28	25.80	14.72	11.19
3	5.90	1.31	0.02	29	27.86	16.44	13.24
4	6.19	1.43	0.04	30	30.14	18.40	15.67
5	6.49	1.57	0.07	31	32.67	20.63	18.56
6	6.81	1.72	0.11	32	35.49	23.18	22.02
7	7.16	1.88	0.15	33	38.64	26.09	26.17
8	7.53	2.06	0.21	34	42.16	29.44	31.15
9	7.92	2.25	0.28	35	46.12	33.30	37.15
10	8.35	2.47	0.37	36	50.59	37.75	44.43
11	8.80	2.71	0.47	37	55.63	42.92	53.27
12	9.28	2.97	0.60	38	61.35	48.93	64.07
13	9.81	3.26	0.74	39	67.87	55.96	77.33
14	10.37	3.59	0.92	40	75.31	64.20	93.69
15	10.98	3.94	1.13	41	83.86	73.90	113.99
16	11.63	4.34	1.36	42	93.71	85.38	139.32
17	12.34	4.77	1.66	43	105.11	99.02	171.14
18	13.10	5.26	2.00	44	118.37	115.31	211.41
19	13.93	5.80	2.40	45	133.88	134.88	262.74
20	14.83	6.40	2.87	46	152.10	158.51	328.73
21	15.82	7.07	3.42	47	173.64	187.21	414.32
22	16.88	7.82	4.07	48	199.26	222.31	526.44
23	18.05	8.66	4.82	49	229.93	265.51	674.91
24	19.32	9.60	5.72	50	266.89	319.07	873.84
25	20.72	10.66	6.77				

Source: Meyerhof, G. G., 1963, *Canadian Geotechnical Journal*, vol. 1, no. 1, 16–26.

14.3.1 SHAPE FACTORS f_{cs} , f_{qs} , $f_{\gamma s}$

For other than strip footings, such as rectangular, square, and circular footings, this modification is needed. For rectangular footings, B (width) and L (length) shall always be assigned as $B \leq L$. For circular footings, B is the diameter of the footing and $B = L$ is assigned.

$$\text{for } \phi' = 0, \quad f_{cs} = 1 + 0.2(B/L) \tag{14.3}$$

$$f_{qs} = f_{\gamma s} = 1 \tag{14.4}$$

$$\text{for } \phi' \geq 10^\circ, \quad f_{cs} = 1 + 0.2(B/L)\tan^2(45^\circ + \phi'/2) \tag{14.5}$$

$$f_{qs} = f_{\gamma s} = 1 + 0.1(B/L)\tan^2(45^\circ + \phi'/2) \tag{14.6}$$

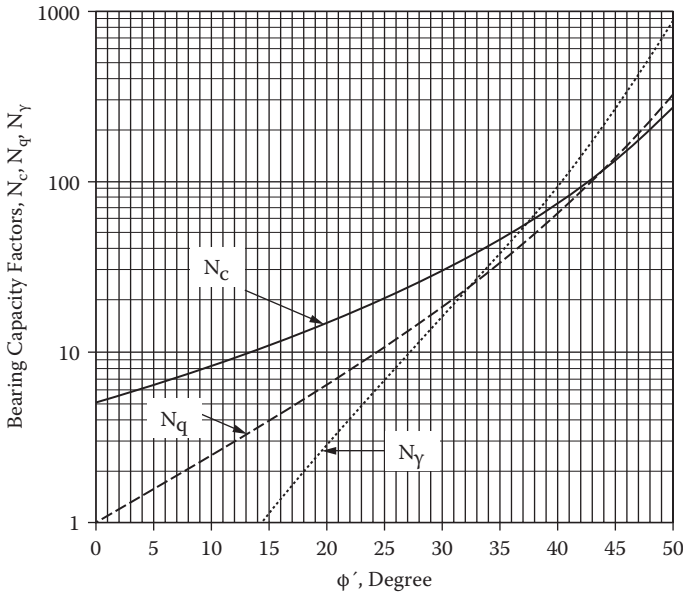


FIGURE 14.2 Bearing capacity factors N_c , N_q , and N_γ . (By *Meyerhof, G. G., 1963, Canadian Geotechnical Journal*, vol. 1, no. 1, 16–26.)

14.3.2 DEPTH FACTORS f_{cd} , f_{qd} , $f_{\gamma d}$

for $\phi' = 0$, $f_{cd} = 1 + 0.2(D_f/B)$ (14.7)

$f_{qd} = f_{\gamma d} = 1$ (14.8)

for $\phi' \geq 10^\circ$, $f_{cd} = 1 + 0.2(D_f/B)\tan(45^\circ + \phi'/2)$ (14.9)

$f_{qd} = f_{\gamma d} = 1 + 0.1(D_f/B)\tan(45^\circ + \phi'/2)$ (14.10)

14.3.3 INCLINATION FACTORS f_{ci} , f_{qi} , $f_{\gamma i}$

$f_{ci} = f_{qi} = (1 - \beta^\circ/90^\circ)^2$ (14.11)

$f_{\gamma i} = (1 - \beta/\phi')^2$ (14.12)

where β is the inclined angle of the footing load with respect to the vertical.

Note that all the modification factors approach 1.0 for Terzaghi's original conditions ($B/L = \infty$, $D_f = 0$, and $\beta = 0^\circ$) and Equation (14.1) can be used in that situation with the bearing capacity factors in Table 14.1.

Exercise 14.1

A strip footing with the soil's parameters is shown in Figure 14.3. Determine the bearing capacity of this foundation soil.

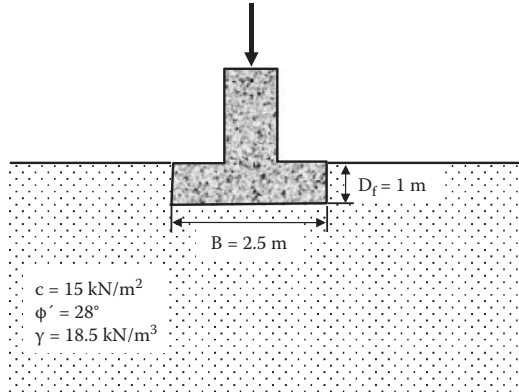


FIGURE 14.3 Footing for Exercise 14.1.

SOLUTION

For $\phi' = 28^\circ$, from Table 14.1 or Figure 14.2, $N_c = 25.80$, $N_q = 14.72$, and $N_\gamma = 11.19$

Shape factors f_{cs} , f_{qs} , $f_{\gamma s}$:

This is a strip footing so that $B/L = 0$; then, $f_{cs} = f_{qs} = f_{\gamma s} = 1$

Depth factors f_{cd} , f_{qd} , $f_{\gamma d}$:

$$\begin{aligned} \text{for } \phi' \geq 10^\circ, f_{cd} &= 1 + 0.2(D_f/B)\tan(45^\circ + \phi'/2) \\ &= 1 + 0.2(1/2.5)\tan(45^\circ + 28^\circ/2) = 1.133 \end{aligned}$$

$$\begin{aligned} f_{qd} = f_{\gamma d} &= 1 + 0.1(D_f/B)\tan(45^\circ + \phi'/2) \\ &= 1 + 0.1(1/2.5)\tan(45^\circ + 28^\circ/2) = 1.067 \end{aligned}$$

Inclination factors f_{ci} , f_{qi} , $f_{\gamma i}$:

Since $\beta = 0$, $f_{ci} = f_{qi} = f_{\gamma i} = 1.0$

From Equation (14.2) and $\gamma_1 = \gamma_2 = 18.5 \text{ kN/m}^3$:

$$\begin{aligned} q_u &= cN_c f_{cs} f_{cd} f_{ci} + \gamma_1 D_f N_q f_{qs} f_{qd} f_{qi} + \frac{1}{2} \gamma_2 B N_\gamma f_{\gamma s} f_{\gamma d} f_{\gamma i} \\ &= 15 \times 25.80 \times 1 \times 1.133 \times 1 + 18.5 \times 1 \times 14.72 \times 1 \times 1.067 \times 1 + \frac{1}{2} \times 18.5 \\ &\quad \times 2.5 \times 11.19 \times 1 \times 1.067 \times 1 = 438.5 + 290.5 + 276.1 = \mathbf{1005.1 \text{ kN/m}^2} \leftarrow \end{aligned}$$

Exercise 14.2

In Exercise 14.1, when **the footing width B is increased to 5.0 m**, what is the bearing capacity q_u ?

SOLUTION

Depth factors f_{cd} , f_{qd} , $f_{\gamma d}$:

$$\begin{aligned} \text{for } \phi' \geq 10^\circ, f_{cd} &= 1 + 0.2(D_f/B)\tan(45^\circ + \phi'/2) \\ &= 1 + 0.2(1/5.0)\tan(45^\circ + 28^\circ/2) = 1.067 \end{aligned}$$

$$\begin{aligned} f_{qd} = f_{\gamma d} &= 1 + 0.1(D_f/B)\tan(45^\circ + \phi'/2) \\ &= 1 + 0.1(1/5.0)\tan(45^\circ + 28^\circ/2) = 1.033 \end{aligned}$$

All other modification factors remain the same as in Exercise 14.1 and $B = 5.0$ m.

$$\begin{aligned} q_u &= cN_c f_{cs} f_{cd} f_{ci} + \gamma_1 D_f N_q f_{qs} f_{qd} f_{qi} + \frac{1}{2} \gamma_2 B N_\gamma f_{\gamma s} f_{\gamma d} f_{\gamma i} \\ &= 15 \times 25.80 \times 1 \times 1.067 \times 1 + 18.5 \times 1 \times 14.72 \times 1 \times 1.033 \times 1 \\ &\quad + \frac{1}{2} \times 18.5 \times 5.0 \times 11.19 \times 1 \times 1.033 \times 1 \\ &= 412.93 + 281.31 + 534.62 = \mathbf{1228.9 \text{ kN/m}^2} \leftarrow \end{aligned}$$

(22.3% increase over Exercise 14.1)

Exercise 14.3

In Exercise 14.1, if the footing is a **square footing with $B = L = 2.5$ m**, what is the bearing capacity q_u ?

SOLUTION

Shape factors f_{cs} , f_{qs} , $f_{\gamma s}$ for $B = L = 2.5$ m:

$$\begin{aligned} \text{For } \phi' \geq 10^\circ, f_{cs} &= 1 + 0.2(B/L)\tan^2(45^\circ + \phi'/2) \\ &= 1 + 0.2(2.5/2.5)\tan^2(45^\circ + 28^\circ/2) = 1.55 \end{aligned}$$

$$\begin{aligned} f_{qs} = f_{\gamma s} &= 1 + 0.1(B/L)\tan^2(45^\circ + \phi'/2) \\ &= 1 + 0.1(2.5/2.5)\tan^2(45^\circ + 28^\circ/2) = 1.28 \end{aligned}$$

Depth factors and inclination factors remain the same as in Exercise 14.1 and thus,

$$\begin{aligned} q_u &= cN_c f_{cs} f_{cd} f_{ci} + \gamma_1 D_f N_q f_{qs} f_{qd} f_{qi} + \frac{1}{2} \gamma_2 B N_\gamma f_{\gamma s} f_{\gamma d} f_{\gamma i} \\ &= 15 \times 25.80 \times 1.55 \times 1.133 \times 1 + 18.5 \times 1 \times 14.72 \times 1.28 \times 1.067 \times 1 \\ &\quad + \frac{1}{2} \times 18.5 \times 2.5 \times 11.19 \times 1.28 \times 1.067 \times 1 \\ &= 679.6 + 371.9 + 353.4 = \mathbf{1404.9 \text{ kN/m}^2} \leftarrow \end{aligned}$$

(39.8% increase over Exercise 14.1)

Exercise 14.4

In Exercise 14.1, if **the footing load is inclined by 5° from the vertical**, what is the bearing capacity q_u ?

SOLUTION

Inclination factors change from Exercise 14.1 with $\beta = 5^\circ$ and all others remain the same.

Inclination factors f_{ci} , f_{qi} , $f_{\gamma i}$:

$$f_{ci} = f_{qi} = (1 - \beta^2/90^\circ)^2 = (1 - 5^\circ/90^\circ)^2 = 0.892$$

$$f_{\gamma i} = (1 - \beta/\phi')^2 = (1 - 5^\circ/28^\circ)^2 = 0.675$$

$$\begin{aligned}
 q_u &= cN_c f_{cs} f_{cd} f_{ci} + \gamma_1 D_f N_q f_{qs} f_{qd} f_{qi} + 1/2 \gamma_2 B N_{\gamma} f_{\gamma s} f_{\gamma d} f_{\gamma i} \\
 &= 15 \times 25.80 \times 1 \times 1.133 \times 0.892 + 18.5 \times 1 \times 14.72 \times 1 \times 1.067 \times 0.892 \\
 &\quad + 1/2 \times 18.5 \times 2.5 \times 11.19 \times 1 \times 1.067 \times 0.675 \\
 &= 391.1 + 259.2 + 186.4 = \mathbf{836.7 \text{ kN/m}^2} \leftarrow
 \end{aligned}$$

(16.8% reduction from Exercise 14.1)

From the preceding four exercises, the effect of the footing width, the footing shapes, and the inclination angle of footing load on the bearing capacity intensity q_u is clearly observed.

14.4 CORRECTION DUE TO WATER TABLE ELEVATION

One more influential parameter on the bearing capacity is the water table elevation relative to the footing depth. In the bearing capacity equations (Equations 14.1 and 14.2), the unit weights of soils γ_1 and γ_2 are included. These are the ones for above the footing base level and below the base level, respectively. When soils are under the water table, the submerged unit weight $\gamma'_1 (= \gamma_1 - \gamma_w)$ and $\gamma'_2 (= \gamma_2 - \gamma_w)$ should be used.

To accommodate these unit weight changes in the bearing capacity equations, Figure 14.4 is prepared. In the figure, Γ_1 and Γ_2 are assigned as general unit weights of soils above the footing base level and below the base level, respectively, and they are used in place of γ_1 and γ_2 in Equations (14.1) and (14.2) when the ground water is

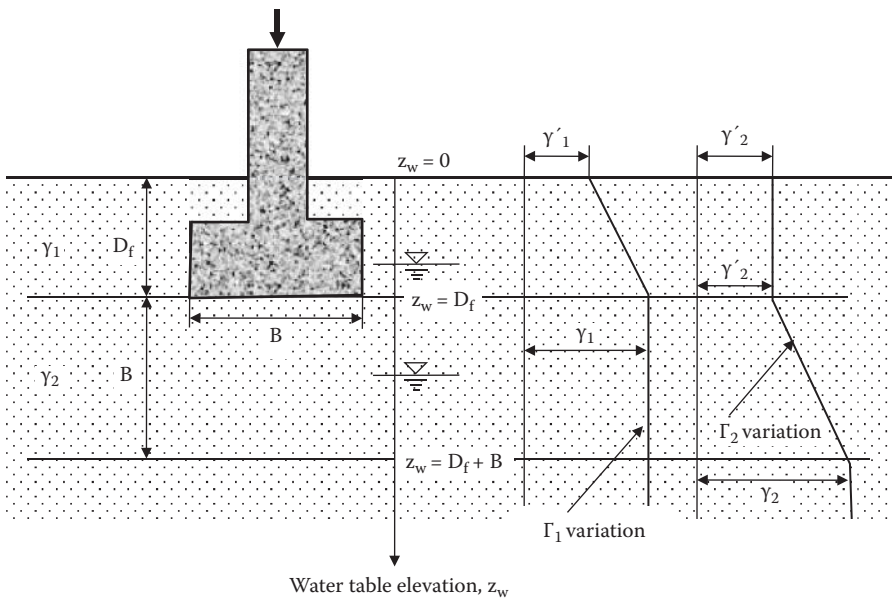


FIGURE 14.4 Effect of water table elevation on bearing capacity equations.

encountered in the bearing capacity computation. Also, z_w is defined as the ground water table elevation from the ground surface. It is assumed that when the water table is located beyond the depth B below the footing base level, there will be no effect by the water table at all on the bearing capacity. The depth B below the footing base is an approximated depth of anticipated bearing capacity failure. According to these observations and the assumption, the following boundary values of general unit weights of soils, Γ_1 and Γ_2 can be defined.

Water table at $z_w = 0$ (on the top of ground surface):

$$\Gamma_1 = \gamma'_1 \text{ and } \Gamma_2 = \gamma'_2$$

Water table at $z_w = D_f$ (at the footing base level):

$$\Gamma_1 = \gamma_1 \text{ and } \Gamma_2 = \gamma'_2$$

Water table at $z_w \geq D_f + B$ (beyond the depth B below the footing base):

$$\Gamma_1 = \gamma_1 \text{ and } \Gamma_2 = \gamma_2$$

The variations of Γ_1 and Γ_2 are plotted in Figure 14.4 by approximating the changes between $z_w = 0$ to D_f for Γ_1 , and between $z_w = D_f$ to $D_f + B$ for Γ_2 as linear lines. Accordingly, the following equations are obtained:

For $0 \leq z_w < D_f$:

$$\Gamma_1 = \gamma_1 + \gamma_w(z_w/D_f - 1) \quad (14.13)$$

$$\Gamma_2 = \gamma'_2 \quad (14.14)$$

For $D_f \leq z_w < D_f + B$:

$$\Gamma_1 = \gamma_1 \quad (14.15)$$

$$\Gamma_2 = \gamma_2 + \gamma_w[z_w - (D_f + B)]/B \quad (14.16)$$

For $z_w \geq D_f + B$:

$$\Gamma_1 = \gamma_1 \quad (14.17)$$

$$\Gamma_2 = \gamma_2 \quad (14.18)$$

Exercise 14.5

In Exercise 14.1, when the ground water table is located (a) at the ground surface, and (b) at 2 m below the ground surface, compute the bearing capacity q_u for each case. Assume that the total unit weights of soils $\gamma_1 = \gamma_2 = 18.5 \text{ kN/m}^2$ for both wet and dry conditions.

SOLUTION

- (a) Water table elevation $z_w = 0$ m

$$\Gamma_1 = \gamma_1 + \gamma_w (z_w/D_f - 1) = 18.5 + 9.81 (0/1 - 1) = 18.5 - 9.81 = 8.69 \text{ kN/m}^3$$

$$\Gamma_2 = \gamma'_2 = 18.5 - 9.81 = 8.69 \text{ kN/m}^3$$

All other values remain the same as in Exercise 14.1 and thus,

$$\begin{aligned} q_u &= cN_c f_{cs} f_{cd} f_{ci} + \gamma_1 D_f N_q f_{qs} f_{qd} f_{qi} + 1/2 \gamma_2 B N_{\gamma} f_{\gamma s} f_{\gamma d} f_{\gamma i} \\ &= 15 \times 25.80 \times 1 \times 1.133 \times 1 + 8.69 \times 1 \times 14.72 \times 1 \times 1.067 \times 1 \\ &\quad + 1/2 \times 8.69 \times 2.5 \times 11.19 \times 1 \times 1.067 \times 1 = 438.5 + 136.5 \\ &\quad + 129.7 = \mathbf{704.7 \text{ kN/m}^2} \leftarrow \end{aligned}$$

(29.9% deduction from Exercise 14.1)

- (b) Water table elevation $z_w = 2$ m ($D_f < z_w < D_f + B$)

$$\Gamma_1 = \gamma_1 = 18.5 \text{ kN/m}^3$$

$$\begin{aligned} \Gamma_2 &= \gamma_2 + \gamma_w [z_w - (D_f + B)]/B = 18.5 + 9.81[2 - (1 + 2.5)]/2.5 = 18.5 - 5.9 \\ &= 12.6 \text{ kN/m}^3 \end{aligned}$$

All other values remain the same as in Exercise 14.1 and thus,

$$\begin{aligned} q_u &= cN_c f_{cs} f_{cd} f_{ci} + \gamma_1 D_f N_q f_{qs} f_{qd} f_{qi} + 1/2 \gamma_2 B N_{\gamma} f_{\gamma s} f_{\gamma d} f_{\gamma i} \\ &= 15 \times 25.80 \times 1 \times 1.133 \times 1 + 18.5 \times 1 \times 14.72 \times 1 \times 1.067 \times 1 \\ &\quad + 1/2 \times 12.6 \times 2.5 \times 11.19 \times 1 \times 1.067 \times 1 = 438.5 + 290.6 \\ &\quad + 188.1 = \mathbf{917.2 \text{ kN/m}^2} \leftarrow \end{aligned}$$

(8.7% deduction from Exercise 14.1)

14.5 GROSS VERSUS NET BEARING CAPACITY

The bearing capacity q_u in the discussions so far is the ultimate **gross bearing capacity**, which is the ultimate stress value that the soil can carry at the base of the footing level, as seen in Figure 14.5. When the unit weight of concrete for the foundation is assumed to be the same as the unit weight of soils, the following vertical force equilibrium is obtained:

$$q_{u,\text{gross}} \cdot B = Q_{u,\text{net}} + \gamma_{\text{soil}} \cdot D_f \cdot B \quad (14.19)$$

and thus,

$$q_{u,\text{net}} = Q_{u,\text{net}}/B = q_{u,\text{gross}} - \gamma_{\text{soil}} \cdot D_f \quad (14.20)$$

where $q_{u,\text{net}}$ is defined as the ultimate **net bearing capacity**, which is the maximum carrying stress level at the ground surface level. This implies that $Q_{u,\text{net}} (= q_{u,\text{net}} \cdot B)$

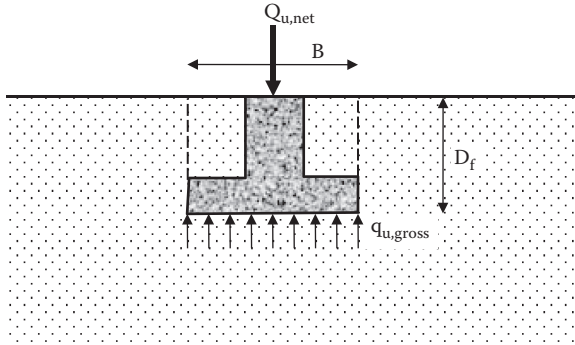


FIGURE 14.5 Gross and net bearing capacities.

is the superstructure's total load, which the soil can support. Accordingly, Equations (14.1) and (14.2) can be rewritten, in terms of the net bearing capacity, in Equations (14.21) and (14.22), respectively:

$$\begin{aligned} q_{u,\text{net}} &= q_u - \gamma_f D_f = (cN_c + \gamma_f D_f N_q + \frac{1}{2} \gamma_2 B N_\gamma) - \gamma_f D_f \\ &= cN_c + \gamma_f D_f (N_q - 1) + \frac{1}{2} \gamma_2 B N_\gamma \end{aligned} \quad (14.21)$$

$$\begin{aligned} q_{u,\text{net}} &= q_u - \gamma_f D_f = (cN_c f_{cs} f_{cd} f_{ci} + \gamma_f D_f N_q f_{qs} f_{qd} f_{qi} + \frac{1}{2} \gamma_2 B N_\gamma f_{\gamma s} f_{\gamma d} f_{\gamma i}) - \gamma_f D_f \\ &= cN_c f_{cs} f_{cd} f_{ci} + \gamma_f D_f (N_q f_{qs} f_{qd} f_{qi} - 1) + \frac{1}{2} \gamma_2 B N_\gamma f_{\gamma s} f_{\gamma d} f_{\gamma i} \end{aligned} \quad (14.22)$$

The gross and the net capacities should not be mixed up. *The net bearing capacity is the one used for designing superstructure above the foundation.* The settlement of soils is caused by increased stresses by new foundations, and thus the net pressure is used for settlement analysis.

14.6 FACTOR OF SAFETY ON BEARING CAPACITY

Since the bearing capacity equations utilize many empirical factors as discussed, in addition to uncertainties on material properties and spatial nonuniformity of soils, a proper value of the **factor of safety** (F.S.) is needed to obtain the **design (allowable) bearing capacity**, q_d . There are several possible ways to apply the F.S., among them: (1) apply for $q_{u,\text{gross}}$ equations, and (2) apply for material properties.

14.6.1 F.S. FOR GROSS BEARING CAPACITY

The design-bearing capacity is obtained by applying the F.S. in the gross-bearing capacity equations (Equation 14.1 or 14.2), and thus,

$$q_{d,\text{gross}} = q_{u,\text{gross}} / \text{F.S.} = (cN_c + \gamma_f D_f N_q + \frac{1}{2} \gamma_2 B N_\gamma) / \text{F.S.} \quad (14.23)$$

$$q_{d,\text{gross}} = q_{u,\text{gross}} / \text{F.S.} = (cN_c f_{cs} f_{cd} f_{ci} + \gamma_f D_f N_q f_{qs} f_{qd} f_{qi} + \frac{1}{2} \gamma_2 B N_\gamma f_{\gamma s} f_{\gamma d} f_{\gamma i}) / \text{F.S.} \quad (14.24)$$

Then, the design net-bearing capacity is obtained by subtracting $\gamma_1 D_f$ from $q_{d,\text{gross}}$ in Equation (14.23) or (14.24) as

$$q_{d,\text{net}} = q_{d,\text{gross}} - \gamma_1 D_f \quad (14.25)$$

Note that several papers in the literature apply the F.S. directly to $q_{u,\text{net}}$ in Equation (14.21) or (14.22). However, it is more logical to apply the F.S. to $q_{u,\text{gross}}$ first and then subtract $\gamma_1 D_f$ from it to obtain the design net value as in Equation (14.25), since $\gamma_1 D_f$ is a rather sure value and there is no need to apply the same level of the F.S. to it.

14.6.2 F.S. FOR STRENGTH PARAMETERS

Due to a certain degree of unreliability of soil shear strength parameters, the reduced design strength parameters are first obtained as

$$c_d = c/\text{F.S.} \quad (14.26)$$

$$\varphi'_d = \tan^{-1}(\tan \varphi'/\text{F.S.}) \quad (14.27)$$

where c and φ' are the measured cohesion and the angle of internal friction of soils and c_d and φ'_d are design values of them. Then, c_d and φ'_d values are inserted into the net bearing capacity equations (Equation 14.21 or 14.22) to obtain the design net bearing capacity value without further applying the F.S. in the equations, or with a marginal F.S. value in the equations.

In either method, the determination of the factor of safety is important, but it is not an easy task. It requires the best judgment of engineers based on a wealth of experience in the subject matter.

14.7 SHALLOW FOUNDATION DESIGN

When foundation soils are relatively strong and there are no problematic soils such as swelling/shrinking soils, highly compressive soils, etc., shallow foundations may be the solution due to economic advantages in comparison with deep foundations, which will be discussed in Chapter 15. In such cases, bearing capacity theory and practice in the earlier section of this chapter are directly utilized.

14.7.1 FOOTING DEPTH

In general practice, footings are placed at a certain depth from the ground surface. Embedded footing has an advantage of increasing the bearing capacity value—that is, increasing the D_f value in Equation (14.1) or Equation (14.2). In addition, footing should be placed below the frost line in the region to avoid possible frost heave actions of soils. Also, footings should be placed beneath the topsoils and weak surface soils, if they exist. The actual footing depth is determined by the design engineer based on a balance of the excavation cost and the cost of foundation design.

14.7.2 DESIGN METHOD

Foundations should be designed to be safe against two checking points: (1) footings that are *safe against bearing capacity failure*, and (2) superstructures that are *safe or functional against footing settlement* (immediate as well as time-delayed consolidation, and total as well as differential). Both safety criteria are equally important and should be done side by side. Settlement analysis can be performed based on the techniques in Chapter 9 and will not be discussed again here.

Bearing capacity analysis basically uses Equation (14.2) with all necessary modifications, including shape, depth, inclination, and water table elevation. Since the footing width B is to be determined and Equation (14.2) includes B , *this is a trial-and-error design process*. Exercise 14.6 shows this process as an example.

Exercise 14.6

Design the dimension of a square footing to carry a column load 1500 kN. The base of the footing will be placed at 1.2 m below the level of the surrounding ground surface and a minimum factor of safety 2.5 is required. Soil around the footing is clay with unconfined compression strength of 130 kN/m² and the total unit weight of soil $\gamma_t = 18.5$ kN/m³. The water table is at the footing base level (Figure 14.6).

SOLUTION

Use Equation (14.2):

$$q_u = cN_c f_{cs} f_{cd} f_{ci} + \gamma_1 D_f N_q f_{qs} f_{qd} f_{qi} + 1/2 \gamma_2 B N_\gamma f_{\gamma s} f_{\gamma d} f_{\gamma i}$$

$$c = (\text{unconfined compression strength } q_u)/2 = 130/2 = 65 \text{ kN/m}^2$$

$$\gamma_1 = 18.5 \text{ kN/m}^3, \gamma_2 = 18.5 - 9.81 = 8.69 \text{ kN/m}^3$$

$$\phi' = 0^\circ$$

Thus, from Table 14.1,

$$N_c = 5.14, N_q = 1.0, N_\gamma = 0, \text{ and}$$

$$f_{cs} = 1 + 0.2(B/L) = 1 + 0.2 (B/B) = 1.2$$

$$f_{qs} = 1.0$$

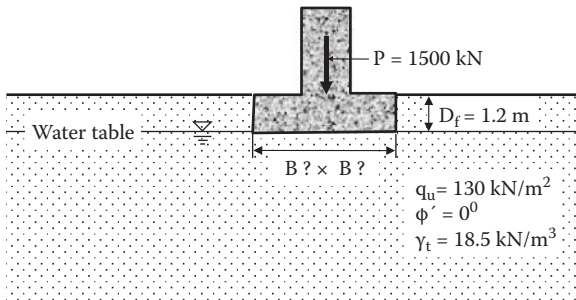


FIGURE 14.6 Exercise 14.6 problem.

$$\begin{aligned}
 f_{cd} &= 1 + 0.2(D_f/B) = 1 + 0.2(1.2/B) \\
 f_{qd} &= 1.0 \\
 f_{ci} &= f_{qi} = 1.0
 \end{aligned}$$

First, **assume $B \times B = 2.0 \text{ m} \times 2.0 \text{ m}$** as the first trial; then,

$$\begin{aligned}
 q_u &= cN_c f_{cs} f_{cd} f_{ci} + \gamma_1 D_f N_q f_{qs} f_{qd} f_{qi} \\
 &= 65 \times 5.14 \times 1.2 \times (1.0 + 0.2(1.2/2.0)) \times 1 \times 1 + 18.5 \times 1.2 \times 1.0 \times 1.0 \times 1.0 \\
 &= 449.0 + 22.2 = 471.2 \text{ kN/m}^2
 \end{aligned}$$

$$q_{d,net} = q_{d,gross} - \gamma_1 D_f = q_{u,gross}/F.S. - \gamma_1 D_f = 471.2/2.5 - 18.5 \times 1.2 = 166.3 \text{ kN/m}^2$$

$$Q_{d,net} = q_{d,net} \times B \times B = 166.3 \times 2 \times 2 = \mathbf{655.1 \text{ kN} < 1500 \text{ kN}}$$
 (applied load)

Thus, **$B \times B = 2.0 \text{ m} \times 2.0 \text{ m}$ footing is not large enough.**

Next, **assume $B \times B = 3.0 \text{ m} \times 3.0 \text{ m}$** as the second trial; then,

$$\begin{aligned}
 q_u &= cN_c f_{cs} f_{cd} f_{ci} + \gamma_1 D_f N_q f_{qs} f_{qd} f_{qi} \\
 &= 65 \times 5.14 \times 1.2 \times (1.0 + 0.2(1.2/3.0)) \times 1 \times 1 + 18.5 \times 1.2 \times 1.0 \times 1.0 \times 1.0 \\
 &= 433.0 + 22.2 = 455.2 \text{ kN/m}^2
 \end{aligned}$$

$$q_{d,net} = q_{d,gross} - \gamma_1 D_f = q_{u,gross}/F.S. - \gamma_1 D_f = 455.2/2.5 - 18.5 \times 1.2 = 159.9 \text{ kN/m}^2$$

$$Q_{d,net} = q_{d,net} \times B \times B = 159.9 \times 3 \times 3 = \mathbf{1439 \text{ kN} < 1500 \text{ kN}}$$
 (applied load)

Thus, **$B \times B = 3.0 \text{ m} \times 3.0 \text{ m}$ is still not quite satisfactory, but close.**

Next, **try $B = 3.1 \text{ m} \times 3.1 \text{ m}$ footing.**

$$\begin{aligned}
 q_u &= cN_c f_{cs} f_{cd} f_{ci} + \gamma_1 D_f N_q f_{qs} f_{qd} f_{qi} \\
 &= 65 \times 5.14 \times 1.2 \times (1.0 + 0.2(1.2/3.1)) \times 1 \times 1 + 18.5 \times 1.2 \times 1.0 \times 1.0 \times 1.0 \\
 &= 432.0 + 22.2 = 454.2 \text{ kN/m}^2
 \end{aligned}$$

$$q_{d,net} = q_{d,gross} - \gamma_1 D_f = q_{u,gross}/F.S. - \gamma_1 D_f = 454.2/2.5 - 18.5 \times 1.2 = 159.5 \text{ kN/m}^2$$

$$Q_{d,net} = q_{d,net} \times B \times B = 159.5 \times 3.1 \times 3.1 = \mathbf{1532 \text{ kN} > 1500 \text{ kN}}$$
 (applied load)

Thus, **$B \times B = 3.1 \text{ m} \times 3.1 \text{ m}$ footing is satisfactory. ←**

Note that the settlement analysis for a chosen footing should be conducted according to Chapter 9 and, when anticipated settlement is found to be larger than the allowable limit (such as in Table 9.9), resizing of the footing is required.

14.8 SUMMARY

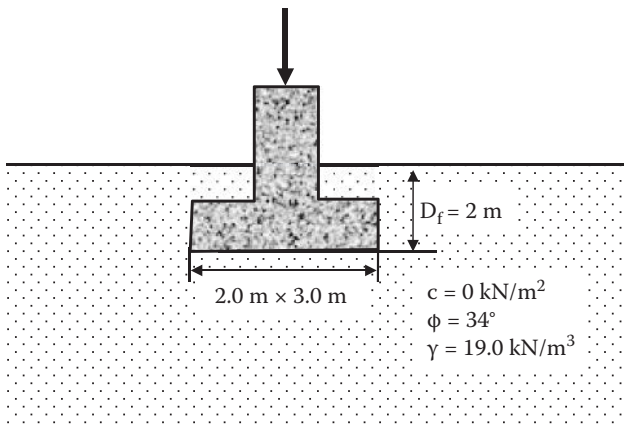
Bearing capacity is the first requirement for shallow foundation designs. In this chapter only the basic concepts of the bearing capacity and shallow foundation design techniques were presented. There are many alternative solutions and additional cases in this subject area. Readers should refer to these detailed discussions in the foundation engineering literature.

REFERENCES

- De Beer, E. E. (1970), Experimental determination of the shape factors and bearing capacity factors of sand, *Geotechnique*, vol. 20, no. 4, 387–411.
- Hanna, A. M., and Meyerhof, G. G. (1981), Experimental evaluation of bearing capacity of footings subjected to inclined loads, *Canadian Geotechnical Journal*, vol. 18, no. 4, 599–603.
- Hansen, J. B. (1970), *A revised and extended formula for bearing capacity*, Bulletin 28, Danish Geotechnical Institute, Copenhagen.
- Meyerhof, G. G. (1963), Some recent research on the bearing capacity of foundations, *Canadian Geotechnical Journal*, vol. 1, no. 1, 16–26.
- Prandtl, L. (1920), über die Härte plastischer Körper, *Nachr. Kgl. Ges. Wiss. Göttingen, Math. Phys. Klasse*.
- Terzaghi, K. (1943), *Theoretical Soil Mechanics*, John Wiley & Sons, New York.
- Vesic, A. S. (1973), Analysis of ultimate loads of shallow foundations, *Journal of the Soil Mechanics and Foundations Division, ASCE*, vol. 99, no. SM1, 45–73.

Problems

- 14.1 A rectangular footing (2.0 m × 3.0 m) is placed on a granular soil at 2 m below the ground surface as seen in the figure. Determine:
- Gross ultimate bearing capacity
 - Net ultimate bearing capacity
 - Net design bearing capacity with factor of safety = 2.5; use Equation (14.25)



Chapter 10 Deep Foundations

15.1 INTRODUCTION

When foundation soils are problematic, such as having low bearing capacity, being highly compressive, having high swelling/shrinking potential, etc., shallow foundations cannot be effectively used. In such situations, going for **deep foundations** may be one of the solutions. There are several types of deep foundations: pile, pier, drilled shaft, caisson, etc. Basic differences between these deep foundation types are their diameters and the installation techniques. All of these are inserted into ground to a certain sufficient depth and supported partially by the tip resistance and partially by skin resistance around the perimeter. The design concepts are similar for all kinds of deep foundations, and thus in this chapter the theory and practice of only pile foundation is described.

Piles can also resist lateral load to a certain degree and design procedures are available. However, this chapter only covers axially loaded vertical piles. The pile design practice uses many empirical and experimental results, and there are many alternative solutions in this field. Readers are referred to other foundation engineering literature for the details of these additional topics.

15.2 TYPES OF PILES

Shapes of piles vary depending on the materials and installation techniques. Figure 15.1 shows some of those variations. Since pile resistance comes from the tip bearing resistance at the pile tip and from the skin resistance around the perimeter of the pile, the differences in material and installation techniques influence the pile's total load carrying capacity, as will be discussed later in this chapter. Several different materials are used for piles: timber, concrete, steel, and hybrid materials such as concrete-filled steel piles, fiber-reinforced plastic piles, and many others. For concrete piles, there are cast-in-place, precast, and prestressed piles. For steel piles, there are steel pipe piles and H-section piles with many different dimensions. Table 15.1 shows a general guideline for typical load bearing capacity and length of these various piles.

15.3 LOAD CARRYING CAPACITY BY STATIC ANALYTICAL METHODS

The vertical load applied on the pile top Q is supported partially by the **tip bearing capacity** Q_t and partially by the **skin friction** Q_s , as seen in Figure 15.2. Thus,

$$Q = Q_t + Q_s \quad (15.1)$$

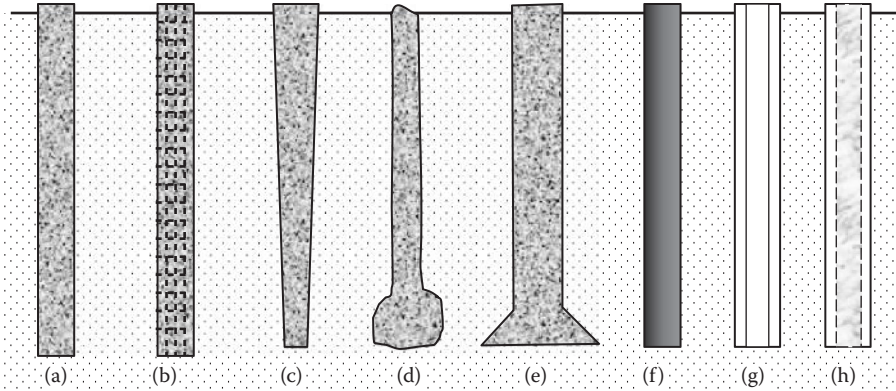


FIGURE 15.1 Shapes and materials of piles. (a) straight non-reinforced concrete pile, (b) straight reinforced concrete pile, (c) tapered pile, (d) uncased Franki pile, (e) concrete pile with enlarged base, (f) steel pipe pile, (g) steel H-section pile, (h) concrete-filled steel pipe pile.

TABLE 15.1
Typical Length and Load Capacity of Various Piles

Type of Pile	Typical Section (m) Diameter or Others	Typical Length (m)	Average Load Capacity (kN)
Timber	0.125–0.45	12–35	250
Cast-in-place concrete	0.15–1.5	≤35	600
Precast concrete with rebar	0.15–0.3	≤35	750
Precast concrete prestressed	0.15–0.6	≤35	1000
Steel pipe	0.2–1	<35	900
Steel H-pile	Web: 1–3 Flange: 0.2–0.35	<60	900
Concrete-filled steel pipe	0.2–1	<35	900

Source: After **Budhu, M., 2010, Soil Mechanics and Foundations**, 3rd ed., John Wiley & Sons, New York.

Depending on the depth and soil types, piles can be classified as (a) **tip bearing pile**, (b) **friction pile**, and (c) both of these, as shown in Figure 15.3. Note that tip bearing piles hold some frictional resistance and friction piles also hold small tip bearing resistance.

Ultimate tip bearing resistance $Q_{t,u}$ can be determined by utilizing the bearing capacity Equation (14.1) in Chapter 14.

$$Q_{t,u} = A_p q_u = A_p \left[cN_c^* + \gamma_1 D_f N_q^* \left(+ \frac{1}{2} \gamma_2 B N_\gamma^* \right) \right] \tag{15.2}$$

where

A_p : pile’s cross-sectional area at the tip

N_c^*, N_q^*, N_γ^* : modified bearing capacity factors

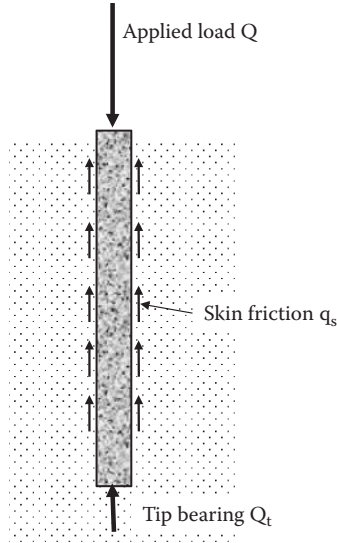


FIGURE 15.2 Load transfer mechanism of pile.

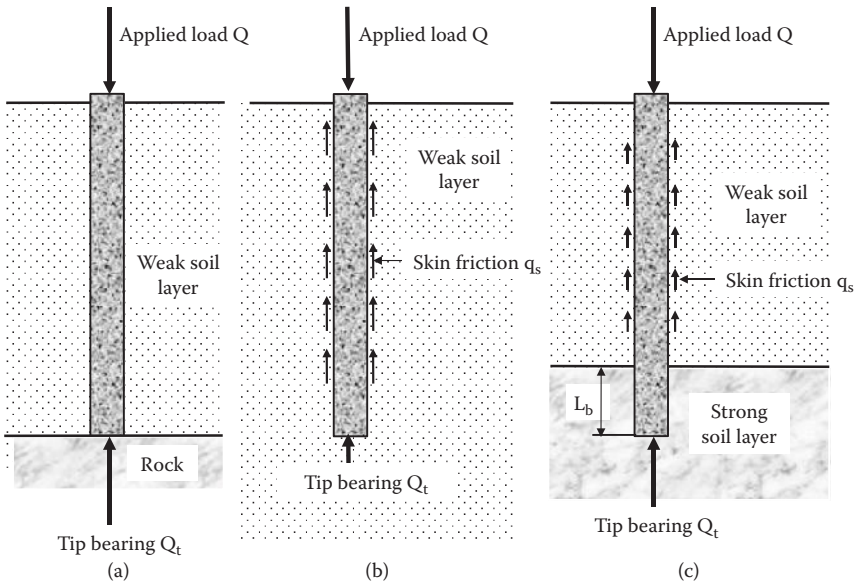


FIGURE 15.3 Types of piles; (a) tip bearing piles, (b) friction pile, (c) combination.

Modified bearing capacity factors that include shape factor and depth factor were discussed in Chapter 14. When Equation (15.2) is utilized in pile foundations, the pile depth L is large in comparison with the embedded depth D_f in case of shallow foundations. And thus the shape of the failure zone that will be developed around the tip will be different from Figure 14.1 in cases of shallow foundation, as seen

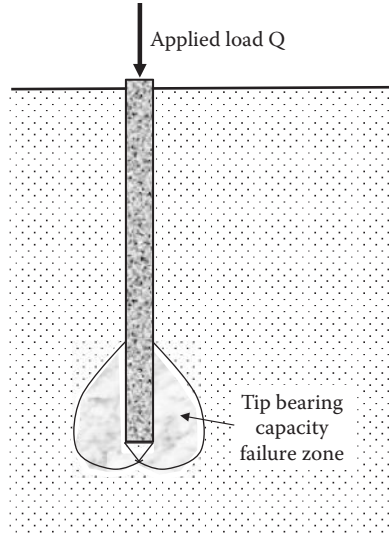


FIGURE 15.4 Bearing capacity failure at pile tip.

in Figure 15.4. In addition, quite often the third term $\frac{1}{2}\gamma_2BN_q^*$ in Equation (15.2) is ignored since pile width (or diameter) B is much smaller than pile depth D_f . The semi-empirical values of modified bearing capacity factors N_c^* and N_q^* were presented by *Meyerhof (1976)* in Figure 15.5, where L_b is the embedded length of piles as seen in Figure 15.3(c).

Skin friction resistance Q_s is written as

$$Q_s = \sum q_f \cdot \Delta L = \sum f \cdot p \cdot \Delta L \quad (15.3)$$

where

$q_f = f \cdot p$: skin resistance per unit depth of pile (kN/m)

f : unit skin friction resistance (kN/m²)

p : perimeter of pile (m)

ΔL : incremental length of pile (m)

The values of “ f ” and “ p ” might change throughout the pile length, and thus the total resistance should be a summation of those values for individual depths. The value of unit skin resistance “ f ” depends on the depth, soil type (adhesion for clayey soils and friction for sandy soils), pile material, installation method (driven, drilled, cast in place, etc.) and maybe others. It is not an easy task to determine an appropriate value for each case. That is the main reason why so many different methods and design parameters are available on pile design procedure.

15.3.1 TIP AREA A_p AND PERIMETER OF PILE “ p ”

Tip area A_p and perimeter of pile “ p ” need special attention for pipe piles and H-section piles.

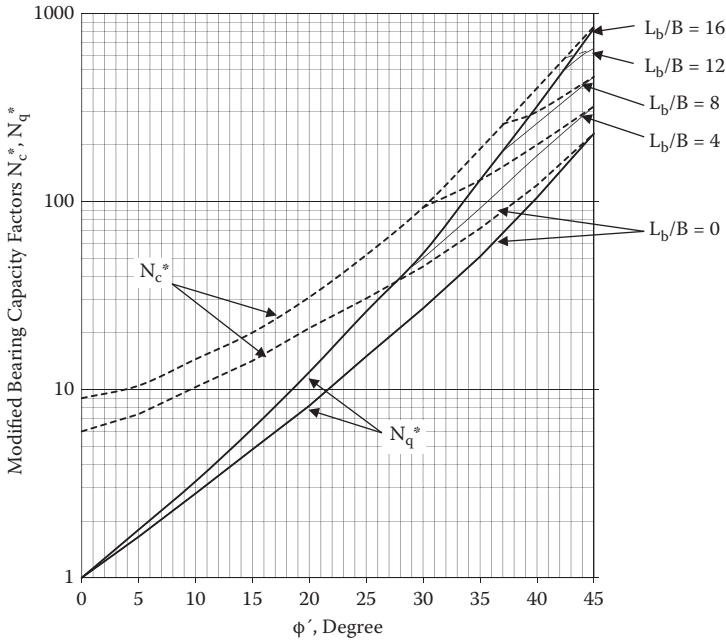


FIGURE 15.5 Modified bearing capacity factors. (Meyerhof, G. G., 1976, *Journal of Geotechnical Engineering Division, ASCE*, vol. 102, no. GT3, 197–228.)

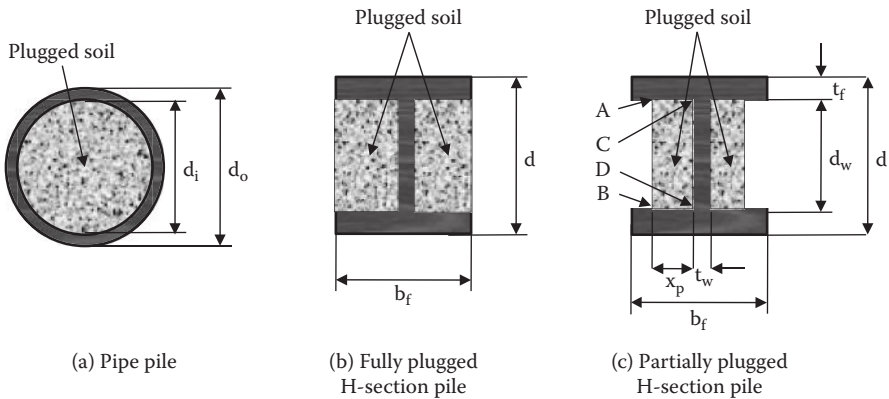


FIGURE 15.6 Plugged piles.

As can be seen in Figure 15.6(a), the inside of relatively small diameter **hollow pipe piles** would be plugged totally with soils when piles are driven into the ground. Thus, both for sandy soils and for clayey soils, the area A_p for tip resistance is treated as $\pi(d_o)^2/4$ instead of $\pi(d_o^2 - d_i^2)/4$.

For **H-section piles**, in clayey ground, most of the space between upper and lower flanges would be plugged with the soil upon pile driving. Thus, $A_p = d \cdot b_f$ and

the perimeter $p = 2(d + b_f)$ (Figure 15.6(b)). In sandy soils, however, the space between flanges would not be fully plugged as seen in Figure 15.6(c). In this case, by equating the frictional resistance between the flange-web section of pile and the soil, that is, along A–C–D–B and soil's own frictional resistance along A–B, the plugged distance x_p can be estimated from

$$2K\sigma'_v(2x_p + d_w)\tan\delta = 2K\sigma'_v d_w \tan\phi' \quad (15.4)$$

where K is the coefficient of lateral earth pressure, σ'_v is the effective vertical stress at the depth, δ is the soil–pile frictional resistance angle, and ϕ' is the effective angle of internal friction angle of the soil. Equation (15.4) is solved for x_p as follows:

$$x_p = d_w (\tan\phi'/\tan\delta - 1)/2 < (b_f - t_w)/2 \quad (15.5)$$

Accordingly, for sandy soils, A_p becomes $d \cdot b_f - d_w(b_f - t_w - 2x_p)$ and the perimeter “p” becomes $2(d + b_f) + 2(b_f - t_w - 2x_p)$ after the soil plug.

In addition, the weight of the plugged soil inside or around piles should be included in the weight of pile computation.

Exercise 15.1

An H-section pile (HP 360 × 174) is driven into sandy soil layer ($\gamma_f = 18.0 \text{ kN/m}^3$, $\phi' = 35^\circ$ and $\delta = 20^\circ$). (a) Estimate the plugged distance x_p in Figure 15.6(c), (b) compute the adjusted A_p and perimeter p for plugged pile, and (c) compare these with the unplugged value. (d) How much is the increased weight of 10 m long pile in comparison with the non-plugged pile?

Note that HP 360 × 174 has $d = 361 \text{ mm}$, $b_f = 378 \text{ mm}$, flange and web thickness $t_f = t_w = 20.45 \text{ mm}$, and linear weight = 174 kgf/m.

SOLUTION

Referring to Figure 15.6(c), $d = 0.361 \text{ m}$, $b_f = 0.378 \text{ m}$, and $t_f = t_w = 0.02045 \text{ m}$.

$$d_w = d - 2 t_f = 0.361 - 2 \times 0.02045 = 0.3201 \text{ m}$$

$$\begin{aligned} \text{(a) From Equation (15.5), } x_p &= d_w (\tan\phi'/\tan\delta - 1)/2 = 0.3201 \times (\tan 35^\circ/\tan 20^\circ - 1)/2 \\ &= 0.1479 \text{ m} < (b_f - t_w)/2 = 0.179 \text{ m, O.K., } x_p = 0.1479 \text{ m} \leftarrow \end{aligned}$$

$$\begin{aligned} \text{(b) } A_p \text{ with soil plug} &= d \cdot b_f - d_w (b_f - t_w - 2x_p) \\ &= 0.361 \times 0.378 - 0.3201 \times (0.378 - 0.02045 - 2 \times 0.1479) = 0.1167 \text{ m}^2 \leftarrow \end{aligned}$$

$$\begin{aligned} \text{Perimeter "p" with soil plug} &= 2(d + b_f) + 2(b_f - t_w - 2x_p) \\ &= 2 \times (0.361 + 0.378) + (0.378 - 0.02045 - 2 \times 0.1479) = 1.602 \text{ m} \leftarrow \end{aligned}$$

$$(c) A_p \text{ without soil plug} = d \cdot b_f - d_w (b_f - t_w)$$

$$= 0.361 \times 0.378 - 0.3201 \times (0.378 - 0.02045) = 0.0220 \text{ m}^2 \leftarrow$$

$$\text{Perimeter "p" without soil plug} = 2(d + b_f) + 2(b_f - t_w)$$

$$= 2 \times (0.361 + 0.378) + 2 \times (0.378 - 0.02045) = 2.1931 \text{ m} \leftarrow$$

$$A_{p \text{ with plug}}/A_{p \text{ w/o plug}} = 0.1167/0.0220 = 5.30 \text{ (530\% increase from non-plug)}$$

$$p_{\text{with plug}}/p_{\text{w/o plug}} = 1.602/2.1931 = 0.730 \text{ (27\% decrease from non-plug)}$$

$$(d) W_{\text{steel}} = w \cdot L = 174 \text{ kg/m} \times 0.00981 \text{ m/s}^2 \times 10 \text{ m} = 17.069 \text{ kN} \leftarrow$$

$$W_{\text{with plug}} = W_{\text{steel}} + 2\gamma_{\text{soil}} \cdot x_p \cdot d_w \cdot L = 17.069 + 2 \times 18.0 \times 0.1479 \times 0.3201 \times 10$$

$$= 17.069 + 17.043 = 34.11 \text{ kN (100\% increase from non-plug)} \leftarrow$$

15.4 STATIC PILE CAPACITY ON SANDY SOILS

15.4.1 TIP RESISTANCE

The estimation of tip resistance Q_t in sandy soils ($c = 0$) uses Equation (15.2) by dropping the third term since the value of B in the third term is much smaller than D_f value in the second term in piles, and thus the following equation is obtained. Note that D_f in Equation (15.2) is replaced with L (total pile length) in Equation (15.6) and hereafter:

$$Q_t = A_p \cdot (\gamma_1 LN_q^*) = A_p \cdot \sigma'_v \cdot N_q^* \leq A_p \cdot q_l = Q_l \quad (15.6)$$

The N_q^* value can be read from Figure 15.5. σ'_v is the effective overburden stress around the pile tip (an average value at several pile diameters above and below the tip). A_p is described in the previous section. In sandy soils, field observations showed that for shallow pile depth, Q_t increases with the increase of pile depth L (or increase of σ'_v value) as in Equation (15.6) up to a certain depth. However, as the pile depth becomes larger, Q_t does not increase any more linearly with the increase of L , and there is a certain limiting value of Q_t . **Meyerhof (1976)** suggested the following limiting value q_l ($= Q_l/A_p$):

$$q_l \text{ (kN/m}^2\text{)} = 50 N_q^* \tan \varphi' \quad \left(\text{or } q_l \text{ (1b/ft}^2\text{)} = 1000 N_q^* \tan \varphi' \right) \quad (15.7)$$

According to Meyerhof, beyond the critical depth, the tip resistance remains the same with q_l regardless of the effective overburden pressure and water table condition.

15.4.2 SKIN FRICTION RESISTANCE

Skin friction resistance Q_s in sandy soil requires the estimation of unit skin friction "f" in Equation (15.3). Similarly to the tip resistance in sandy soil, the unit skin

friction “ f ” also increases with the depth z to a limiting depth L_1 and remains constant thereafter. The limiting depth L_1 is approximately from $15B$ to $20B$, where B is the diameter (or width) of the pile. Accordingly, the unit skin friction is expressed by

$$f = K \cdot \sigma'_v \cdot \tan \delta \leq f_1 \quad (15.8)$$

where K is the lateral earth pressure coefficient, σ'_v is the effective overburden stress at depth z , and δ is the friction angle between soil and pile surface. *Meyerhof (1976)* suggested the following f_1 values based on field observations:

$$f_1 \text{ (kN/m}^2\text{)} = 1.91 N_{\text{avg}} \text{ for driven piles} \quad (15.9)$$

$$f_1 \text{ (kN/m}^2\text{)} = 0.955 N_{\text{avg}} \text{ for bored piles} \quad (15.10)$$

where N_{avg} is the measured average standard penetration value.

Determination of K and δ in Equation (15.8) is not easy. Typical values of friction angle δ relative to soil's effective internal friction angle ϕ' are shown in Table 15.2.

The lateral earth pressure coefficient K is very sensitive to boundary displacement, as discussed in Chapter 12, and depends on several parameters: type of pile installation method (driven or jetted), high-displacement versus low-displacement piles (that is, closed-end piles versus H-piles), soil's density, etc. Table 15.3 shows typical values of K relative to K_0 , which is the coefficient of lateral earth pressure at rest (i.e., Equation 12.5).

TABLE 15.2
Typical δ/ϕ' Values

Pile Shaft Surface Material	δ/ϕ' Ratio
Rough concrete	1.0
Smooth concrete	0.8–1.0
Rough steel	0.7–0.9
Smooth steel	0.5–0.7
Timber	0.8–0.9

Source: After *Kulhawy, F. H. et al. (1983)*, report no. EL-2870, Electrical Power Research Institute, Palo Alto, CA.

TABLE 15.3
Typical K/K_0 Values

Pile Type and Installation Method	K/K_0 Ratio
Jetted piles	0.5–0.67
Driven piles, small displacement	0.75–1.25
Driven piles, large displacement	1.0–2.0

Source: After *Kulhawy, F. H. et al., 1983*, report no. EL-2870, Electrical Power Research Institute, Palo Alto, CA.

Exercise 15.2

A 20 m long concrete pile with 0.3 m diameter is driven into a uniform sandy soil layer. The average properties of the sand are $\phi' = 38^\circ$, $\gamma_1 = 18.2 \text{ kN/m}^3$, and standard penetration value N of 30. The water table is at 5 m below the ground surface. Determine the design load carrying capacity of this pile with factor of safety = 3.0.

SOLUTION

$$A_p = \pi (B/2)^2 = \pi (0.3/2)^2 = 0.0707 \text{ m}^2, \quad p = \pi B = \pi \times 0.3 = 0.942 \text{ m}$$

$$\sigma'_{\text{at tip}} = 5 \text{ m} \times 18.2 \text{ kN/m}^3 + 15 \text{ m} \times (18.2 \text{ kN/m}^3 - 9.81 \text{ kN/m}^3) = 216.9 \text{ kN/m}^2$$

TIP RESISTANCE

From Figure 15.5, $N_q^* = 220$ for $\phi' = 38^\circ$ and $L_b/B = 20/0.3 = 66.7$ (since there is no weak soil layer, $L_b = L = 20 \text{ m}$ and use the value for $L_b/B = 16$ as the maximum value in the figure).

From Equation (15.6),

$$Q_t = A_p (\gamma_1 L N_q^*) = A_p \sigma'_{\text{at tip}} N_q^* = 0.0707 \times 216.9 \times 220 = 3374 \text{ kN}$$

$$Q_1 = A_p \cdot q_1 = A_p \cdot 50 N_q^* \tan \phi' = 0.0707 \times 50 \times 220 \times \tan 38^\circ = 607 \text{ kN},$$

which is smaller than $Q_t = 3374 \text{ kN}$; thus, **$Q_t = 607 \text{ kN}$** ←

SKIN FRICTION RESISTANCE

$\delta/\phi' = 1.0$ and $K/K_o = 1.5$ are chosen from Tables 15.2 and 15.3 for rough concrete pile and for large displacement driven pile, respectively. Then,

$$\delta = \phi' = 38^\circ \text{ and } K = 1.5 \times K_o = 1.5 \times (1 - \sin \phi') = 1.5 \times (1 - \sin 38^\circ) = 0.577$$

$$\text{From Equation (15.9), } f_1 \text{ (kN/m}^2\text{)} = 1.91 N_{\text{avg}} = 1.91 \times 30 = 57.3 \text{ kN/m}^2.$$

$$\text{From Equation (15.8), } f = K \cdot \sigma'_v \cdot \tan \delta \leq f_1.$$

Table 15.4 is created to compute “ f ” for several depths. Table 15.4 suggests that the critical depth L_c to reach the f_1 value (57.3 kN/m²) is in the 5 m < z < 10 m range and L_c can be found from

$$f_1 = 57.3 = K \tan \delta (91 + (L_c - 5) (18.2 - 9.81)), \text{ then } \mathbf{L_c = 9.30 \text{ m}}$$

Accordingly, distribution of f value in Table 15.4 is plotted in Figure 15.7.

From Equation (15.3),

$$Q_s = \Sigma q_i \cdot \Delta L = \Sigma f \cdot p \cdot \Delta L = p \Sigma f \cdot \Delta L = 0.942 \times (41 \times 5/2 + (41 + 57.3) \times (9.3 - 5)/2 + 57.3 \times (20 - 9.30)) = 0.942 \times 1138.3 = \mathbf{1073 \text{ kN}} \leftarrow$$

where $\Sigma f \cdot \Delta L$ is obtained from the area of corrected “ f ” distribution in Figure 15.7. Therefore, **total design load carrying capacity** is

$$Q = (Q_t + Q_s)/F.S. = (607 + 1073)/3.0 = \mathbf{560 \text{ kN}} \leftarrow$$

TABLE 15.4
Unit Skin Friction f with Depth

Depth z (m)	σ'_v (kN/m ²)	$f = K \cdot \sigma'_v \cdot \tan\delta$ (kN/m ²)	Corrected f with f_i (kN/m ²)
0	0	0	0
5	91	41.0	41.0
10	132.95	59.93	57.3
15	174.9	78.8	57.3
20	216.9	97.78	57.3

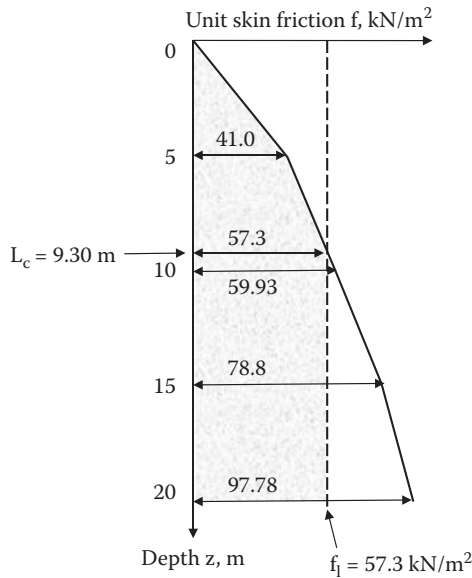


FIGURE 15.7 Unit skin friction distributions.

15.5 STATIC PILE CAPACITY IN COHESIVE SOILS

15.5.1 TIP RESISTANCE

Piles into clayey soil expect that the pile capacity is mostly supported by skin friction. There may exist, however, small tip resistance for large diameter piles. In cohesive soils ($\phi' = 0$ case), $N_q^* = 0$ from Figure 15.5 and using Equation (15.2) we obtain

$$Q_{t,u} = A_p q_u = A_p c N_c^* \tag{15.11}$$

$N_c^* \approx 6$ can be read in Figure 15.5 for $\phi' = 0$.

According to *O'Neill and Reese (1999)*,

$$N_c^* = 6.7 \text{ for } c = 25 \text{ kN/m}^2$$

$$N_c^* = 8.0 \text{ for } c = 50 \text{ kN/m}^2$$

$$N_c^* = 9.0 \text{ for } c \geq 100 \text{ kN/m}^2$$

Undrained shear strength C_u between the tip and 2B below the tip are used for cohesion “c” in Equation (15.11).

15.5.2 SKIN FRICTIONAL RESISTANCE

This is the most complex practice to determine proper unit skin friction “f” around piles in a clayey soil stratum. Three methods are used in practice—namely, the α -method, β -method, and λ -method.

α -Method (total stress method): *Tomlinson (1971)* initially proposed this method, which assumes that the skin friction of piles in clayey soils is related to clay’s undrained shear strength. Thus, this method is also categorized as the **total stress method**. Since the undrained shear strength is related to the short-term strength without pore water pressure dissipation (Chapter 11), this method is usually used to estimate the short-term skin friction capacity of piles in cohesive soils. Equation (15.12) is used to express the unit skin friction.

$$f = \alpha \cdot C_u \quad (15.12)$$

where

f: unit skin friction

C_u : undrained shear strength of clay (see Chapter 11)

α : adhesion factor

Then, the total skin friction capacity Q_s is obtained from Equation (15.3) for stratified soil profiles. The α value has been empirically determined from many field observations. Figure 15.8 shows such data. The average curve can be used as representative values of α . However, as can be seen in the figure, the data scatter is large and the α value determination in this method is approximate in nature.

Sladen (1992) related the α value to the undrained shear strength C_u and the effective overburden stress σ'_v of soils, and proposed the following approximate solution.

$$\alpha = C \cdot (\sigma'_v/C_u)^{0.45} \quad (15.13)$$

where the coefficient $C = 0.5$ for bored concrete piles and driven steel piles and $C \geq 0.5$ for driven piles in very stiff soils.

β -Method (effective stress method): *Burland (1973)* originally proposed this method in which, when piles are driven into clayey soils, soils around piles are disturbed and remolded and excess pore water pressure is generated locally around the pile perimeters. Since this remolded zone is rather thin, the generated pore

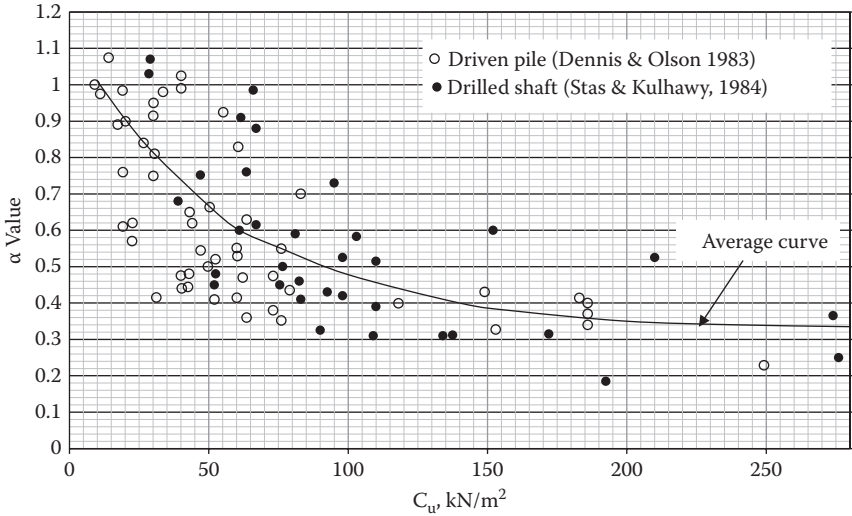


FIGURE 15.8 Measured α value versus C_u relations. (After *Terzaghi, K., Peck, R. B. and Mesri, G., 1996, Soil Mechanics in Engineering Practice*, 3rd ed., John Wiley & Sons, New York.)

water pressure dissipates in a short time. Skin friction resistance is controlled by the effective stress parameters of remolded soils after the dissipation of pore water pressure, and thus this method is normally used for the long-term skin friction capacity of piles in cohesive soils. Accordingly, the unit skin friction resistance “f” can be expressed by

$$f = \beta \cdot \sigma'_v = (K \cdot \tan \delta) \cdot \sigma'_v \tag{15.14}$$

where

β : skin friction parameter (= $K \cdot \tan \delta$)

K: lateral earth pressure coefficient

δ : frictional angle at pile–soil interface

σ'_v : effective overburden stress at the section of the pile

For the lateral earth pressure coefficient K, the **at-rest coefficient** (Equation 12.5 in Chapter 12) can be used here, and is relisted again as Equation (15.15):

$$K_0 = (1 - \sin \phi') (\text{OCR})^{\sin \phi'} \tag{15.15}$$

where ϕ' is the effective angle of internal friction of remolded clay. Frictional angle δ at the pile–soil interface depends on the soil’s angle of internal friction ϕ' and pile materials. The δ/ϕ' ratios in Table 15.2 can be used for this purpose. The OCR (overconsolidation ratio; Equation 9.26) = 1.0 for normally consolidated clays.

The ϕ' value could be obtained from a laboratory drained shear test or effective stress analysis of an undrained shear test as discussed in Chapter 11. Typical ϕ' values

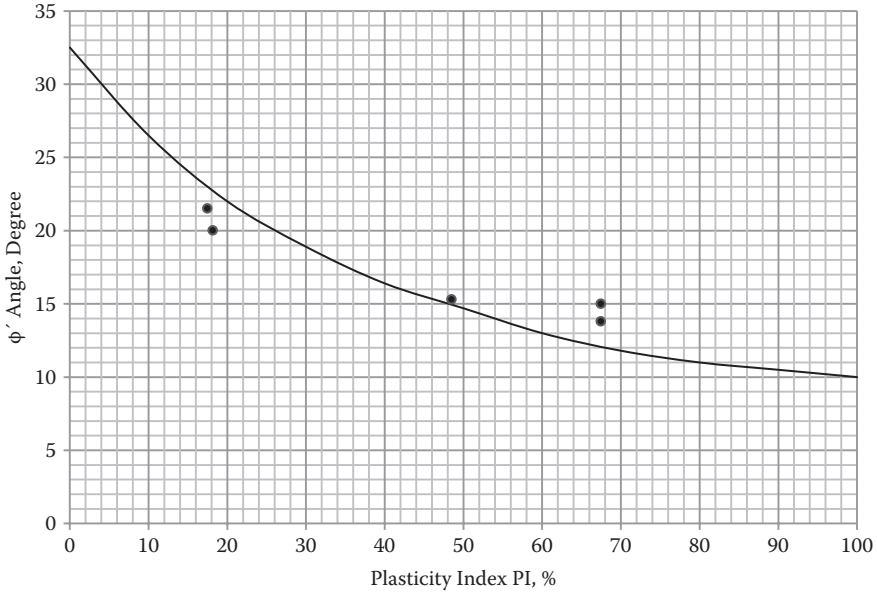


FIGURE 15.9 Typical ϕ' values of remolded clays. (After *Bjerrum, L. and Simons, N. E., 1960, Proceedings of Research Conference on Shear Strength of Cohesive Soils*, ASCE, Boulder, CO, 711–726.)

of remolded clays were reported between 10° and 33° , depending on PI value, as shown in Figure 15.9.

From Figure 15.9, for example, if $\phi' = 33^\circ$ for $PI = 0$ and $\phi'/\delta = 1.0$ are selected for a non-plastic clay, $K_0 = 0.455$ for $OCR = 1.0$ and $\beta = 0.295$ is obtained. In the case of $\phi' = 10^\circ$ for $PI = 100$ and $\phi'/\delta = 1.0$ for high plastic soft clay, $K_0 = 0.826$ for $OCR = 1.0$ and $\beta = 0.145$ is obtained.

After “f” is properly determined, Equation (15.3) is used to estimate the total side friction capacity Q_s for stratified profiles. Again, it should be noted that this method provides only a rough estimation of side friction capacity of piles.

λ -Method: Originally proposed by *Vijayvergiya and Focht (1972)* for offshore piles (long piles), this method uses the average effective overburden stress and cohesion through the pile length to obtain the mean unit skin friction “f” in the following equation:

$$f = \lambda \cdot (\sigma'_v + C_u) \tag{15.16}$$

where

f: mean unit skin friction

σ'_v : mean effective overburden stress

C_u mean undrained shear strength

The λ parameter changes with the pile depth and is plotted in Figure 15.10 based on measured load test values.

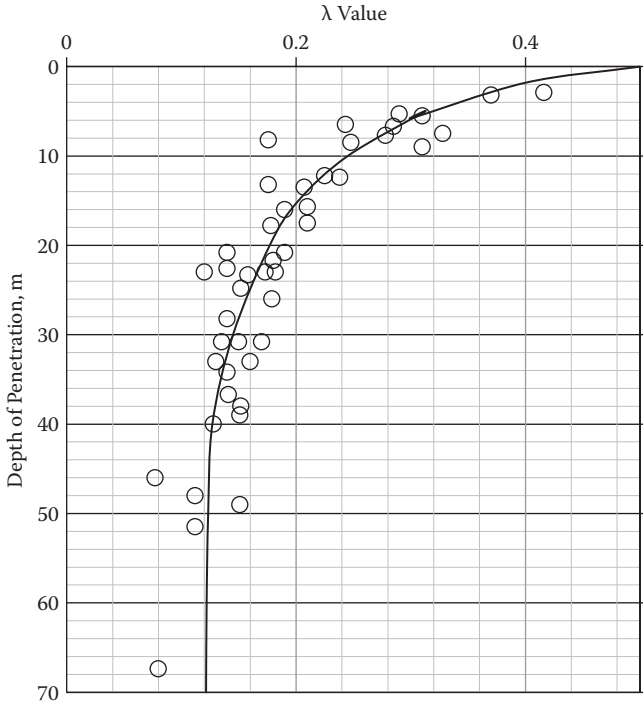


FIGURE 15.10 Variation of the λ parameter with pile depth. (After *Vijayvergiya, V. N., and Focht, J. A., Jr., 1972*, OTC paper 1718, *4th Offshore Technology Conference*, Houston, TX, 865–874.)

For stratified clay layers, weighted average values for σ'_v and C_u will be computed. See the example in Exercise 15.3 for this computation. After obtaining the average unit skin friction “f,” Equation (15.17) is used to compute the total skin friction capacity Q_s .

$$Q_s = f \cdot p \cdot L \quad (15.17)$$

where

f: average unit skin friction resistance (kN/m^2)

p: perimeter of pile (m)

L: total length of pile (m)

Exercise 15.3

A 20 m long steel pile with 0.5 m diameter is driven into layered clay soils as shown in Figure 15.11. Estimate the pile’s ultimate tip resistance Q_t , skin friction resistance Q_s , and total capacity Q_u by the (a) α -method, (b) β -method, and (c) λ -method.

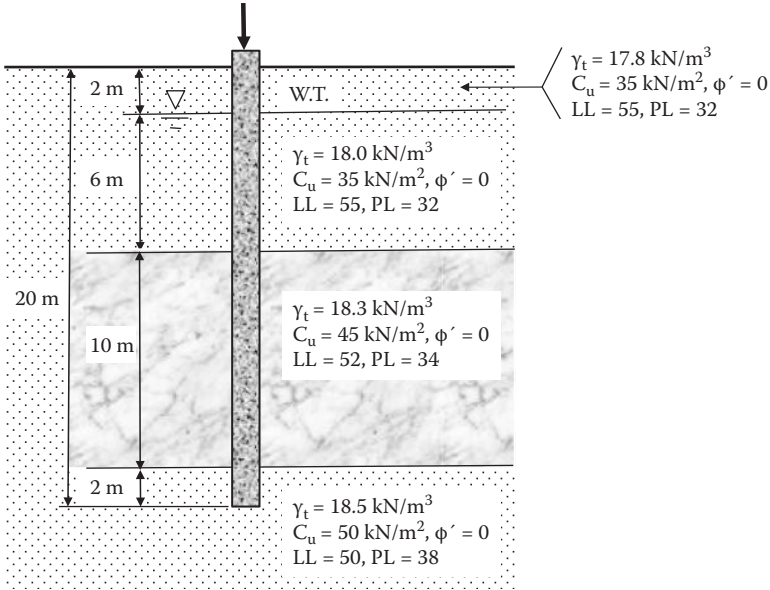


FIGURE 15.11 Exercise 15.3 problem.

SOLUTION

$$A_p = \pi (B/2)^2 = \pi (0.5/2)^2 = 0.196 \text{ m}^2$$

$$p = \pi B = \pi \times 0.5 = 1.570 \text{ m}$$

The pile's tip resistance $Q_{t,u}$ is obtained from Equation (15.11) and using $N_c^* = 8.0$ for $c = 50 \text{ kN/m}^2$.

$$Q_{t,u} = A_p q_u = A_p c N_c^* = 0.196 \times 50 \times 8.0 = \mathbf{78.4 \text{ kN} \leftarrow}$$

(a) α -method

Equation (15.12) is utilized for the average values of C_u for each sublayer in Figure 15.11 and α -values are read from Figure 15.8. The spreadsheet in Table 15.5 is prepared for the computation.

The total ultimate pile capacity is then

$$Q_u = Q_{u,t} + Q_s = 78.4 + 940.6 = \mathbf{1019 \text{ kN} \leftarrow}$$

(b) β -method

Equations (15.14) and (15.15) are utilized for each sublayer in Figure 15.11 and ϕ' values are read from Figure 15.9 and $\delta = \phi'$ is assumed.

Effective overburden stresses σ'_v at each mid-sublayer are

$$\sigma_{v \text{ at } 1\text{m}} = 17.8 \times 1 = 17.8 \text{ kN/m}^2$$

$$\sigma_{v \text{ at } 5\text{m}} = 17.8 \times 2 + (18.0 - 9.81) \times 3 = 60.2 \text{ kN/m}^2$$

TABLE 15.5
Computation of Side Friction by the α -Method

z (m)	ΔL (m)	C_u (kN/m ²)	α	f (= $\alpha \cdot C_u$) (kN/m ²)	$Q_{f,i}$ (= f · p · ΔL) (kN)
0–2	2	35	0.775	27.13	85.2
2–8	6	35	0.775	27.13	255.6
8–18	10	45	0.70	31.5	494.6
18–20	2	50	0.67	33.5	105.2
Total Q_s =					940.6

TABLE 15.6
Computation of Side Friction by the β -Method

Z (m)	ΔL (m)	σ'_v at Mid-layer (kN/m ²)	PI (%)	φ' (degree)	K = $1 - \sin \varphi'$	β = K · tan δ ($\delta = \varphi'$)	f = $\beta \cdot \sigma'_v$ (kN/m ²)	$Q_{f,i}$ = f · p · ΔL (kN)
0–2	2	17.8	23	21	0.642	0.246	4.4	13.8
2–8	6	60.2	23	21	0.642	0.246	14.8	139.4
8–18	10	127.2	18	22.5	0.617	0.255	32.4	508.7
18–20	2	178.3	12	25.5	0.569	0.272	48.4	152.0
Total Q_s =							813.9	

$$\sigma_{v \text{ at } 13 \text{ m}} = 17.8 \times 2 + (18.0 - 9.81) \times 6 + (18.3 - 9.81) \times 5 = 127.2 \text{ kN/m}^2$$

$$\sigma_{v \text{ at } 19 \text{ m}} = 17.8 \times 2 + (18.0 - 9.81) \times 6 + (18.3 - 9.81) \times 10 + (18.5 - 9.81) \times 1 = 178.3 \text{ kN/m}^2$$

The spreadsheet in Table 15.6 is prepared for the rest of the computation. The total ultimate pile capacity is then

$$Q_u = Q_{u,t} + Q_s = 78.4 + 813.9 = \mathbf{892.3 \text{ kN} \leftarrow}$$

(c) λ -method

Equation (15.17) is utilized for weighted average values of σ'_v and C_u . Figure 15.12 is the plots of σ'_v and C_u with the depth. σ'_v values at each sublayer boundary are calculated as follows:

$$\sigma_{v \text{ at } 2 \text{ m}} = 17.8 \times 2 = 35.6 \text{ kN/m}^2$$

$$\sigma_{v \text{ at } 8 \text{ m}} = 17.8 \times 2 + (18.0 - 9.81) \times 6 = 84.74 \text{ kN/m}^2$$

$$\sigma_{v \text{ at } 18 \text{ m}} = 17.8 \times 2 + (18.0 - 9.81) \times 6 + (18.3 - 9.81) \times 10 = 169.64 \text{ kN/m}^2$$

$$\sigma'_{v \text{ at } 20 \text{ m}} = 17.8 \times 2 + (18.0 - 9.81) \times 6 + (18.3 - 9.81) \times 10 + (18.5 - 9.81) \times 2 = 187.02 \text{ kN/m}^2$$

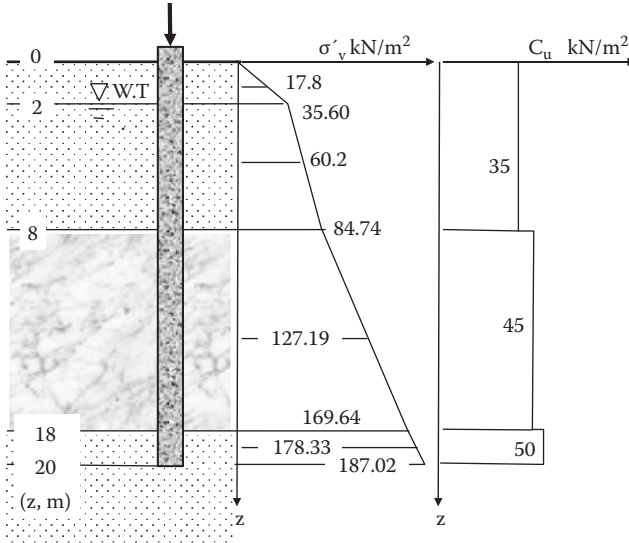


FIGURE 15.12 Distributions of σ'_v and C_u with the depth.

From Figure 15.12, the areas of both distribution curves are used to calculate weighted mean values as

$$\sigma'_{v, \text{mean}} = (\text{area of } \sigma'_v \text{ distribution})/L = [35.6 \times 2/2 + (35.6 + 84.74) \times 6/2 + (84.74 + 169.64) \times 10/2 + 169.64 + 187.02) \times 2/2]/20 = 101.3 \text{ kN/m}^2$$

$$C_{u, \text{mean}} = (\text{area of } C_u \text{ distribution})/L = [35 \times 8 + 45 \times 10 + 50 \times 2]/20 = 41.5 \text{ kN/m}^2$$

From Figure 15.6, $\lambda = 0.175$ is obtained for $L = 20 \text{ m}$.

$$\text{From Equation (15.16), } f = \lambda(\sigma'_v + 2C_u) = 0.175 \times (101.3 + 2 \times 41.5) = 32.2 \text{ kN/m}^2$$

$$Q_s = f \cdot p \cdot L = 32.2 \times 1.570 \times 20 = \mathbf{1011 \text{ kN}}$$

The total ultimate pile capacity is then

$$Q_u = Q_{u,t} + Q_s = 78.4 + 1011 = \mathbf{1089 \text{ kN} \leftarrow}$$

15.6 OTHER METHODS OF PILE CAPACITY ESTIMATION

The preceding pile capacity determination techniques are called the “static analytical method” since they use soils’ static parameters such as C_u , ϕ' , δ , and σ'_v . Although these techniques are routinely used during the preliminary design phase of pile foundation, the reliability of the result is not great due to the uncertainty of soil parameters and its static nature. This is one of the reasons why there are many variations in static analytical methods. Pile driving is dynamic in nature and more reliable evaluation procedures are available based on the pile’s